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POLISH STOCK MARKET AND SOME FOREIGN MARKETS – DEPENDENCE ANALYSIS BY COPULAS

By applying copulas the examination was carried out to find out whether trading volume, stock return and return volatility are pairwise dependent. In the investigations it was shown that there exists a close relationship between these variables on the domestic market and between Polish stock returns and the returns of foreign stock market indexes. A similar significant relationship concerns also trading volumes. In addition, stock returns (returns volatility) of the Austrian and especially of the German stock market influence Polish trading volume. The lack of significant DJIA returns impact on the trading volume on WSE on the same day is probably caused by the fact that changes of DJIA lead changes on the European stock markets.

Keywords: *Copulas, dependences, stock returns, trading volume*

1. Introduction

In the literature it is widely accepted that stock prices reflect investor's beliefs about possible future of company development. The level of prices is based on the available information about the respective firm (Gurgul et al. (2003a), Gurgul et al. (2003b), Mestel et al. (2003)), Mestel and Gurgul (2003). The new pieces of information change investors expectations and cause price movements. Usually investors

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differ in their evaluation of new information. Therefore pieces of information which reach the stock market might have no effect on stock prices. This can take place if some investors interpret the same news quite differently or if they start from different expectations. Thus, it is easy to derive from these facts that prices represent the average reaction of investors to incoming information. Changes in prices therefore reflect the average reaction of investors to news (Clark (1993), Copeland (1976)). As in the case of prices, trading volume and trading volume changes adjust mainly to the available set of relevant information on the market. Contrary to stock prices, a revision in investor's expectations always leads to an increase in trading volume. Trading volume reflects the sum of investor's reactions to news. Therefore investigation of the joint behaviour of stock prices and trading volume improves the understanding of the microstructure of a stock market. This fact can also have implications for research on options and futures markets.

A proper answer to the question as to whether knowledge of some variables on one financial market (e.g. volatility) can support short-run forecasts of others is very important not only for researchers but also for market participants (Gallant et al. [11]). The short-run forecasts of one variable by mean of another variable are a function of the level of dependency between these two variables. Therefore it is not surprising that there has recently been published an increased number of contributions concerning the level of dependence between trading volume, stock return and return volatility. Previous empirical contributions have focused on the contemporaneous relationship between price changes and volume (see overview by Karpoff [18])[1]. The contributors investigated a contemporaneous relationship between trading volume and volatility approximated by absolute value or squared stock returns. They observe positive feedback between trading volume and stock prices, i.e. high trading volume is associated with an increase in stock prices and vice versa. In this context the literature widely disccusses an asymmetry in stock price response to good and bad news. Empirical evidence that supports such an asymmetry in market reactions e.g. on the Austrian stock market, can be also found in Gurgul et al. [13], Gurgul et al. [14], Mestel et al. [21], Ariel [1], Frech [9], Jain and Joh [16] and Jennings et al. [17]. Some of the earlier studies look at contemporaneous and dynamic price–volume relationships and the impact of one market on another market[2].

Applying linear and nonlinear Granger causality tests (Granger [12]) to daily Dow Jones stock returns, and percentage changes in NYSE trading volume, Hiemstra and Jones [15] established a significant bi-directional nonlinear causality between returns

[1] In this extensive review of theoretical and empirical research concerning the price-volume relation, Karpoff [18] gives several causes why this relationship is important.

[2] A few previous studies explicitly test for causality between stock prices and trading volume, e.g. Rogalski [24], Smirlock and Starks [25], Jain and Joh [16].

and volume[3]. The contribution by Chordia and Swaminathan [4] provides evidence that the daily returns of stock with high trading volume outperform the daily returns of stock with low trading volume. The authors tried to convince the reader that the reason for this observation is the tendency of stock with a high trading volume to respond promptly to rewly released information. These studies focus on domestic relationships in a dynamic context between trading volume and stock returns. The contribution of Lee and Rui [19] gives new insight into this subject by examining inter-country dynamic relations for the three largest stock markets, namely US, UK and Japanese. The authors established that US financial market variables have predictive power for UK and Japanese financial market variables.

At the end of the 80s Poland started the transition process from a centrally planned economy to a market economy. It was the first transformation of this kind in history and there was no economic theory of such a process. The early 90s were extremely difficult for Poland and other central European countries.

Stock quotations on the WSE started on 16th April, 1991. One can assume that on this day the WSE was re-established (after more than a 50-year break) as the exclusive place of trading on the Polish stock market. Continuous trading begun in 1996, but only the most liquid stocks were included in this system. The question arises whether on the Polish stock market the same mechanisms can be identified as on a developed capital market and a subsequent question about the degree of dependence of the Polish stock market on three developed stock markets, namely US (DJIA), German (DAX) and Austrian (ATX).

The question of dependency among stock markets at the time of globalization is quite an important topic. The level of dependence between the stock markets can be measured through such variables as stock return, trading volume and volatility. Because of deficiences in traditional dependency measures like correlation, in the recent finance literature copulas are applied. The shortcomings of correlation which can be avoided by means of copula application are briefly listed in paragraph 3.2.

Applying copulas, in this paper we examine whether trading volume depends on stock return as well as return volatility based on data which come from the Warsaw Stock Exchange (WSE). Besides domestic relationships between stock returns and trading volume and volatility, we are also interested in the question of relationships between WIG and the foreign stock markets mentioned above. We checked if WIG returns (logvolume of WIG) are somewhat related to stock returns (logvolumes) of the US, German and Austrian stock markets. In addition, we tested the impact of returns (volatility) from the US, German and Austrian stock markets on the trading volume of the Polish stock market. We found a significant relationship between trading volume

[3] These results seem to be not in line with the empirical work of Gallant et al. [11], who finds a strong nonlinear impact of lagged stock returns on current and future trading volume, but only weak nonlinear effects from lagged trading volume to current and future stock returns by using daily S&P 500 index stock returns and NYSE trading volume.

from Warsaw Stock Exchange and Vienna Stock Exchange returns (volatility), and more especially German returns (volatility).

The remainder of this paper is organized as follows, Sections 2 and 3 contain a brief description of our data set and an outline of the methodology applied. In Section 4 test results for dependence between trading volume and return and volatility, both domestically and internationally, are presented. Section 5 concludes the paper.

2. Data

Our data set comprises the daily market price index (WIG), and trading volume measured by the number of shares traded within one trading day on the WSE. Additionally, we included in our sample the daily market price index and trading volume for three foreign stock exchanges, namely New York (DJIA), Frankfurt (DAX) and Vienna (ATX). The investigation covers the period 22/06/1998–05/11//2004. Each variable contains 1648 observations. The sources of these data include the data banks collected by PARKIET, REUTERS, BLOOMBERG, DEUTSCHE BOERSE and additional publications of WSE. All these series are matched.

We applied continuous stock returns given by:

$$R_t = \log(PI_t) - \log(PI_{t-1}) = \log[1 + r(t-1, t)] = \widetilde{r}(t-1, t) \qquad (1)$$

where PI_t stands for the value of a price index on date t, PI_{t-1} stands for the value of a price index on date $t - 1$, and log denotes the natural logarithm. As can be easily justified, $\widetilde{r}(t-1, t)$ is an approximation of the theoretical definition of returns given by discrete stock returns. Taking Maclaurin's expansion of (1) by the order of two, we can state that this formula undervalues the exact return (i.e. discrete stock return). In spite of this we decided to compute stock returns by using continuous returns (equation (1)), because the use of a natural logarithm improves some statistical properties of financial time series distribution. Furthermore, since the logarithmic transformation belongs to the Box–Cox transformation, it can stabilize the variance.

3. Methodology

3.1. Stationarity and expected trading volume

We applied a methodology based on copula theory. Any inference on the basis of this theory rests on the assumption that the variables to which this methodology is applied are stationary. Therefore in the first phase of our study we tested the hypothesis whether the time series of trading volume, returns and return volatility can be assumed to be stationary. In consistence with the empirical work of Gallant et al. [11], who documented evidence of both linear and nonlinear time trends in trading volume series, we made attempts to estimate the regression defined by equations (2) to check whether trading volume series need to be detrended.

$$V_t = a_0 + a_1 t + \varepsilon_t$$

$$V_t = a_0 + a_1 t + a_2 t^2 + \varepsilon_t \tag{2}$$

where V_t is raw trading volume, $t = 1, ..., N$ (N stands for the number of observations).

After controlling the presence of linear and nonlinear trends in trading volume series, we continued the testing of stationarity by using an augmented Dickey-Fuller (ADF) test. This test is based upon the regression:

$$\Delta y_t = \alpha_0 + \alpha_1 y_{t-1} + \sum_{i=1}^{k} \delta_i \Delta y_{t-i} + \varepsilon_t \tag{3}$$

where y denotes the respective time series.

The unit root test is carried out by testing the null hypothesis $\alpha_1 = 0$ against the one-sided alternative $\alpha_1 < 0$. It is not surprising that the t-Student statistic of the estimated parameter α_1 does not have a conventional t-distribution under the null hypothesis of a unit root. Instead of this, we use the critical values recommended by Charemza and Deadman [3]. If the ADF t-statistic for α_1 lies to the left of these values, the null can be rejected.

Conducting ADF tests for each time series of stock returns and trading volume included in our sample, we find parameter α_1 to be negative and statistically significant at common significance levels. Hence we come to the conclusion that our time series can be assumed to be stationary.

The considered financial return series exhibit some degree of autocorrelation and, more importantly, heteroskedasticity (detected by Engle's ARCH test). Copula modelling requires that the data are approximately independent and identically distributed (i.i.d.). We use ARMA models to describe conditional means, GARCH/GJR for conditional variances of returns and volatilities and EGARCH for conditional variances of detrended trading volumes (see Appendix A for a formulation of these models, the estimation results are available upon request from the authors). Standardized (by the corresponding conditional standard deviations) residuals from the models applied are approximately i.i.d. and are used for modelling the structure of dependence by copulas.

3.2. Copulas

Classic correlation as a dependence measure exhibits some shortcomings; the variances of X and Y must be finite or the linear correlation is not defined. This is not ideal for a dependency measure and causes problems when we work with heavy-tailed distributions. Moreover, the independence of two random variables implies that they are uncorrelated, but zero correlation does not in general imply independence. Only in the case of multivariate normal distribution there is a lack of correlation equivalent to independence. In addition, linear correlation has the another serious deficiency. It is not invariant under nonlinear strictly increasing transformations, i.e. if T: $IR \to IR$, then for two real-valued random variables X and Y it doesn't in general holds true that

$$\rho(T(X), T(Y)) \neq \rho(X, Y) .$$

Correlation is also a source of statistical problems. It is not a robust measure. This means that a single observation can have an arbitrarily high influence on the linear correlation. In addition the correlation is defined if variances of both random variables are finite.

A low dependency between two markets implies a good opportunity for an investor to diversify their investments risk. Suppose that the annual returns in a domestic market and in a foreign market have a linear correlation coefficient of 0.10. Under the normal distribution assumption, the probability that returns in both markets are in their lowest 5^{th} percentiles is less than 0.0025. Thus, based on the Gaussian assumption, an investor can significantly reduce his risk by balancing his portfolio with stocks from a foreign stock market. However, it has been observed that market crashes and financial crises often happen in different countries in about the same time period, even if the dependency measured by correlation is very low between these markets. Researchers have raised the question of a different dependence structure between markets with the same (pairwise) correlations. These dependence structures could increase or decrease the diversification benefit compared to the normal distribution assumption.

Another approach that has been used in empirical studies is computing conditional correlation. It is well known that correlations calculated with different conditions could exhibit essential differences. It has been found that correlations conditional on large price or trading volume movements are higher than those conditional on small movements. This observation is known in the literature as "correlation breakdown". As Bouye et al. [2] stressed, in these situations the correlation coefficient does not explain the actual dependency. The reason is that even in the case of normal distribution a stronger dependence is forecasted in hectic periods and a weaker dependence in quiet periods. To conclude, although conditional correlation provides more information concerning dependence between market variables than unconditional, the results of correlation analysis should be evaluated with caution. These results may be confusing and misleading.

In order to take into account the structure of the stochastic dependence between financial variables have been frequently used in recent years copulas. The copulas reflect the structure of dependence between variables, whereas classical correlation coefficients do not (see e.g. Embrechts et al. [7], Embrechts et al. [8], Frees and Valdez [10]).

From a mathematical point of view copulas are multidimensional cumulative distribution functions with a uniform distribution on [0, 1] margins. Copulas are invariant in respect to strict monoton and continuous transformations.

In the following part we are concerned with 2-dimensional copulas. The definition and properties of d-dimensional copulas, where $d > 2$ are quite similar to those for the 2-dimensional copula.

Now, we give a definition of 2-dimensional copula function.

Definition 1. A two-dimensional copula (or briefly, a copula) is a function with the following properties:

1. For every u_1, u_2 in [0, 1]

$$C(u_1,0) = 0 = C(0,u_2)\,.$$

2. For every u_1, u_2 in [0, 1]

$$C(u_1,1) = u_1 \quad \text{and} \quad C(1,u_2) = u_2\,.$$

3. For every u_{11}, u_{12}, u_{21}, u_{22} in [0, 1] such that $u_{11} \le u_{12}$ and $u_{21} \le u_{22}$, the following condition is satisfied:

$$C(u_{12},u_{22}) - C(u_{12},u_{21}) - C(u_{11},u_{22}) + C(u_{11},u_{21}) \ge 0\,.$$

From this definition it follows that a 2-dimensional copula C is a distribution function defined on $[0.1]^2$ with uniformly distributed margins on [0, 1]. Property 1 says that C is grounded, property 3 is called 2-increasing. These properties regarded together imply that copula C satisfies the Lipschitz condition (so copulas are uniformly continuous). The most important result regarding copulas is Sklar's Theorem presented below (see Nelsen [22], Embrechts et al. [8]):

Theorem 1. *Let H be a joint distribution function with margins F_1 and F_2. Then there exists a copula C such that for all x_1 and x_2 in* $\overline{R}$

$$H(x_1, x_2) = C(F_1(x_1), F_2(x_2))\,. \tag{4}$$

If F_1 and F_2 are continuous, then C is unique; otherwise, C is uniquely determined on $\mathrm{Ran}F_1 \times \mathrm{Ran}F_2$. *Conversely, if C is a copula and F_1 and F_2 are distribution functions, then the function H defined by* (1) *is a joint distribution with margins F_1 and F_2.*

Corollary 1. *Let H be a distribution function with margins F_1 and F_2 and copula C. Then for every* (u_1,u_2) *in* $[0, 1]^2$

$$C(u_1,u_2) = H(F_1^{-1}(u_1), F_2^{-1}(u_2)),$$

where F_i^{-1} *are quantile functions.*

There are three special copulas mentioned in the literature. $\Pi(u_1,u_2) = u_1 u_2$ is called the product copula (copula of independency). The functions $W(u_1, u_2) = \max(u_1 + u_2 - 1.0)$ and $M(u_1,u_2) = \min(u_1,u_2)$ are called Fréchet–Hoeffding bounds and in the case of bivariate copulas represent perfect negative and perfect positive dependencies, respectively. In Figure 1 we present the contours of these copulas.

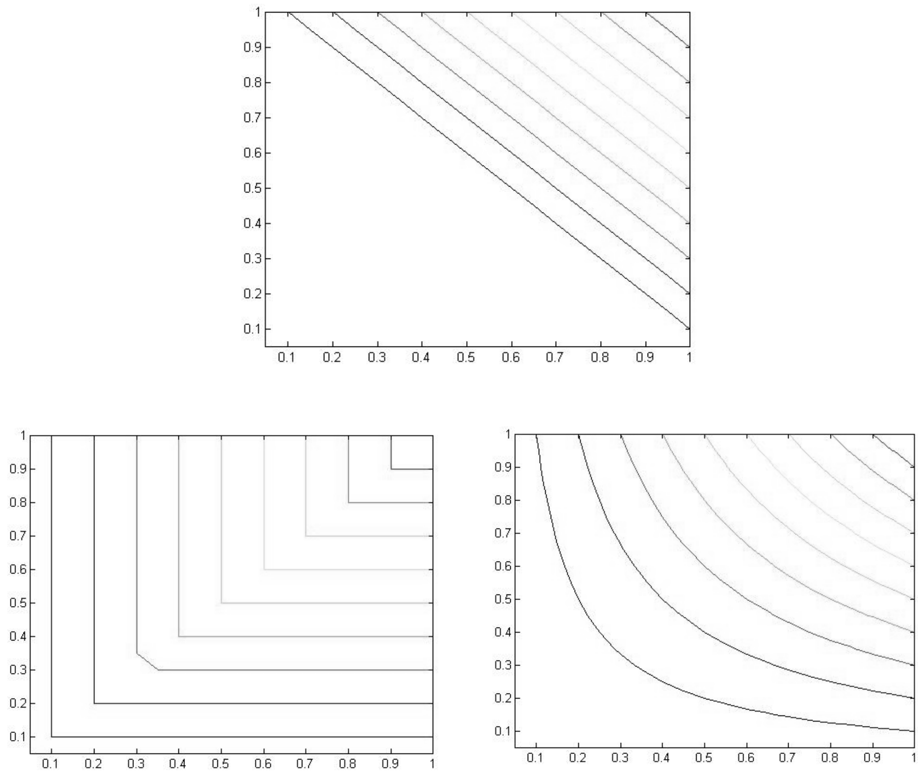

Fig. 1. Contours of W (top), M (centre) and Π (bottom)

Now, we give a lower and upper restriction for each copula $C(u_1, u_2)$

Theorem 2. *Let C be a copula. Then for every* u_1, u_2 *in* [0, 1]

$$W(u_1, u_2) \le C(u_1, u_2) \le M(u_1, u_2).$$

Copula modelling is a natural way to study the tail dependence of multivariate distributions. Thus, tail dependences structures are often modelled by copulas. They briefly describe the limiting proportion that one margin exceeds a certain threshold given that the second margin has already exceeded that threshold. Below we define upper and lower tail dependence in terms of copulas (see e.g. Patton [23]).

Definition 2. If a bivariate copula C is such that

$$\lim_{u\uparrow 1}\frac{1-2u+C(u,u)}{1-u}=\lambda_U$$

exists, then C has upper tail dependence if $\lambda_U \in (0,1]$, and upper tail independence if $\lambda_U = 0$.

If a bivariate copula C such that

$$\lim_{u\downarrow 0}\frac{C(u,u)}{u}=\lambda_L$$

exists, then C has a lower tail dependence if $\lambda_L \in (0, 1]$, and a lower tail independence if $\lambda_L = 0$.

In the literature some classes of copulas are delineated. One of the most important classes are called the elliptical copula. Two important examples of elliptical copulas are a normal copula based on Gaussian distribution and a t-copula based on t-Student distribution.

A normal copula is described by the equation

$$C_R^{Ga}(u_1,u_2)=\Phi_R(\Phi^{-1}(u_1),\Phi^{-1}(u_2);R),$$

where Φ_R denotes the joint distribution of the 2-variate standard normal distribution function with linear correlation matrix R and Φ denotes the distribution function of the univariate standard normal distribution.

For t-copula the equality

$$C_{R,\upsilon}^{t}(u_1,u_2)=t_{R,\upsilon}(t_\upsilon^{-1}(u_1),t_\upsilon^{-1}(u_2);R,\nu),$$

holds true. In this equality $t_{R,\nu}$ denotes the 2-dimensional t-Student distribution characterized by correlation matrix R, and ν degrees of freedom, t_ν stands for the univariate distribution function of t-Student distribution with ν degrees of freedom.

Another important class we discuss in this paper are Archimedean copulas.

Before we define these copulas we give a definition of pseudo-inverse function.

Definition 3. Let be given a continuous, strictly decreasing function from

[0, 1] to [0, ∞], such that $\varphi(1) = 0$. The pseudo-inverse of φ is the function $\varphi^{[-1]}:[0,\infty]\to[0,1]$ given by

$$\varphi^{[-1]} = \begin{cases} \varphi^{-1}(t), & 0 \le t \le \varphi(0) \\ 0 & \varphi(0) \le t \le \infty. \end{cases} \tag{5}$$

Now we characterize an important class of copulas.

Theorem 3. *Let there be a continuous, strictly decreasing function φ such that $\varphi(1) = 0$, and let $\varphi^{[-1]}$ be a pseudo-inverse of φ defined by* (5). *Let C:$[0, 1]^2 \to [0, 1]$ be a function given by*

$$C(u_1, u_2) = \varphi^{[-1]}(\varphi(u_1) + \varphi(u_2)) . \tag{6}$$

Then the function C is a copula if and only if φ is convex.

Copulas of form (6) are called Archimedean copulas. Function φ is called the generator of the copula. If $\varphi(1) = \infty$ (this is the case if $\varphi^{[-1]} = \varphi^{-1}$), we say that φ is a strict generator and the copula is called strict too.

Except for the symmetrized Joe-Clayton copula all Archimedean copulas used in this paper are one-parameter copulas.

We use also the Plackett copula for which a measure of dependence is the cross product ratio (in 2×2 contingency table). More details about these copulas (their definitions, range of parameters, tail dependence coefficients) may be found in Nelsen [22] and Patton [23].

Before estimating the parameters of copulas we perform a test of independence based on Kendall's tau. The sample version of Kendall's tau is defined as:

$$\hat{\tau} = \binom{N}{2}^{-1} \sum_{i<j} \operatorname{sgn}[(X_1^i - X_1^j)(X_2^i - X_2^j)] .$$

Under the null hypothesis H_0:$C = \Pi$ of independence between X_1 and X_2, the distribution of $\hat{\tau}$ is close to normal with zero mean and variance $2(2N + 5)/[9N(N - 1)]$.

We use the semi-parametric method called CML (Canonical Maximum Likelihood, see for example Bouye et al. [2]) to estimate parameters of copulas. The idea of the CML is to transform the data $(\mathbf{x}_1^t, \mathbf{x}_2^t)$ (where $\mathbf{x}_i$ are vectors of standardized residuals obtained from models for margins) into uniform variates $(\hat{\mathbf{u}}_1^t, \hat{\mathbf{u}}_2^t)$ using empirical marginal distribution and then to estimate the parameters in the following way:

$$\hat{\boldsymbol{\alpha}}_{CML} = \arg\max \sum_{t=1}^{N} \ln c(\hat{\mathbf{u}}_1^t, \hat{\mathbf{u}}_2^t; \boldsymbol{\alpha})$$

where $c(u_1, u_2) = \dfrac{\partial C(u_1, u_2)}{\partial u_1 \partial u_2}$ denotes density of copula, and α is a set of estimated parameters.

After having estimated the parameters of the copulas, we selected the copula that fitted best the data set using Akaike (AIC) and Bayesian (BIC) information criteria.

4. Empirical results

At the very beginning of empirical investigations we computed descriptive statistics. In Table 1 we present these statistics. One can easily see that the time series of returns, trading volume and volatility are highly nonnormal.

They exhibit particularly considerable kurtosis and skewness.

Table 1. Descriptive statistics of data

PANEL A : Daily stock returns				
	WIG	ATX	DAX	DJIA
1	**2**	**3**	**4**	**5**
Mean	0.000297	0.000235	–0.0002	0.000107
SD	0.015156	0.010071	0.018106	0.012044
Kurtosis	6.620991	6.758212	4.763626	5.347789
Skewness	–0.21151	–0.61766	–0.09723	–0.01333
Min	–0.09974	–0.06426	–0.08875	–0.06578
1st Quartile	–0.00789	–0.00483	–0.01025	–0.00655
Median	0.000435	0.000528	0.000322	0.000159
3rd Quartile	0.008089	0.005468	0.010182	0.006739
Max	0.078933	0.042064	0.075527	0.061547
PANEL B: Daily detrended trading volume				
	WIG	ATX	DAX	DJIA
Mean	5.60E-10	2.29E-10	3.40E-09	2.42E-08
SD	3322606	890952.4	25731298	67034711
Kurtosis	7.912583	92.02762	56.36092	6.070101
Skewness	1.606895	6.98681	4.997853	1.150952
Min	–6449467	–1161424	–8.4E+07	–2E+08
1st Quartile	–2224637	–476888	–1.2E+07	–4.9E+07
Median	–691630	–136136	–2817399	–9083732
3rd Quartile	1443311	256911.7	7348702	37316790
Max	23313005	14899104	3.65E+08	4.27E+08

cd. tab. 1

PANEL C: Daily volatility (absolute values of returns)				
	WIG	ATX	DAX	DJIA
Mean	0.010872	0.007216	0.013455	0.008914
SD	0.01056	0.007027	0.012113	0.008098
Kurtosis	11.4549	11.43908	7.173584	8.958387
Skewness	2.206391	2.288446	1.732841	1.922723
Min	5.18E-06	0	9.23E-06	0
1st Quartile	0.003474	0.002395	0.004622	0.003073
Median	0.007942	0.00519	0.010193	0.006682
3rd Quartile	0.015207	0.009887	0.018327	0.012403
Max	0.099741	0.064261	0.088747	0.065782

The dataset covers the period 22/06/1998–05/11/2004. Each variable consists of 1648 observations. The sources of these data include the data banks collected by PARKIET, REUTERS, BLOOMBERG, DEUTSCHE BOERSE and additional publications of WSE. All these time series are matched. The descriptive statistics of daily stock returns, log-volume and volatility exhibit significant kurtosis and skewness. This indicates non-normality of the listed variables.

4.1. Domestic market

The empirical procedure in this section tests the relation between stock returns and trading volume, and the relation between volatility and trading volume on Warsaw Stock Exchange (WSE). We find Kendall's tau equal to 0.1308 and 0.1813, respectively. This indicates that dependence between returns and trading volume is essentially smaller than between trading volume and volatility. We reject the hypothesis of independence at a significance level of 5% in both cases. Table 2 reports results of copula estimation procedures.

Table 2. Relationships on Polish stock market (WSE)

Relationship	"Best" copula	Parameter(s)		λ_L	λ_U
$R \leftrightarrow V$	symmetrized Joe-Clayton	0.2310	0.0000	0.0000	0.2310
$\|R\| \leftrightarrow V$	rotated Clayton	0.3801		0.0000	0.1615

The best copulas fitted to the domestic data of the pairs returns – trading volume and volatility – trading volume show some dependence asymmetry. Result exhibit upper tail dependence and lower tail independence. SJC stands for symmetrized Joe-Clayton copula.

In Figure 2 the density and contours of the rotated Clayton copula are presented.

It follows from this table that returns and trading volume are dependent in the upper tail. The same is true for trading volume and volatility dependence.

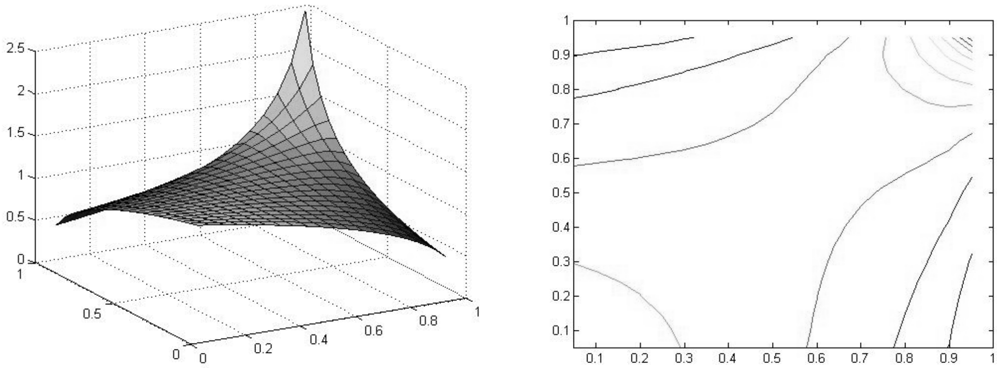

Fig. 2. The density (left) and contours (right) of the rotated Clayton copula for $|R| \leftrightarrow V$ from WSE with parameter $\alpha = 0.3801$

4.2. Cross-Country Evidence

Beyond the domestic market we also examined the relationship between some WSE variables and stock market variables from Vienna (ATX), Frankfurt (DAX) and New York (DJIA). The results are presented in the following three sections.

4.2.1. Dependencies between returns of Polish stock market and returns (return volatility) of foreign stock markets

The null hypotheses of independence are rejected in all cases. The coefficients of the rank correlation of Kendall are highly significant and are equal to 0.1621, 0.2253 and 0.1173 for Austrian, German and US stock returns, respectively. The table below contains results of estimation procedures.

Table 3. Dependencies between returns on stock markets

Relationship	"Best" copula	Parameter(s)		λ_L	λ_U
R denotes returns of Polish stock market (WIG)					
$R \leftrightarrow R_{ATX}$	SJC	0.0152	0.1836	0.1836	0.0152
$R \leftrightarrow R_{DAX}$	Frank	48.809		0	0
$R \leftrightarrow R_{DJIA}$	SJC	0.0088	0.0887	0.0087	0.0888

Dependencies between WIG returns (R) and returns of ATX, DAX and DJIA are significant. In the case of WIG and ATX the dependence in lower tail (λ_L) is stronger pronounced than in upper tails (λ_U). In the other cases dependencies in the tails are low and they do not exhibit significant asymmetry.

In Figure 3 we present the density and contours of symmetrized Joe-Clayton (SJC) copula for $R \leftrightarrow R_{ATX}$ (see Table 3).

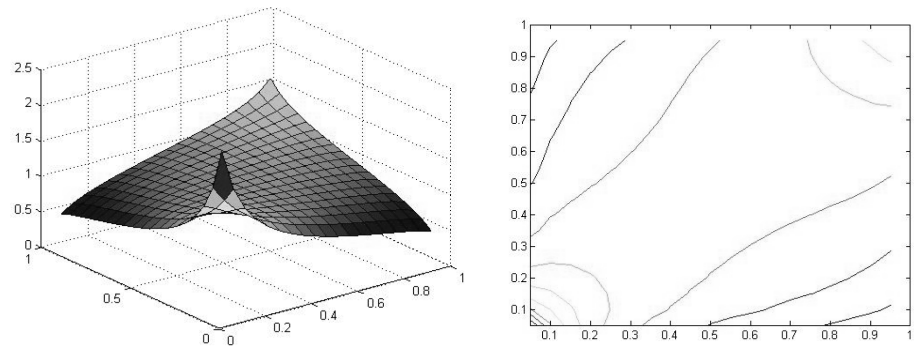

Fig. 3. Density (left) and contours of density of symmetrized Joe-Clayton copula with $\alpha = [0.0152; 0.1836]$

The shape of best copulas and values of parameters λ_L and λ_U convinced us that dependencies of returns are significant and approximately symmetric in lower and upper tails (except for ATX data, where dependence in lower tail is stronger).

4.2.2. Trading volume of Polish stock market versus trading volumes of foreign stock markets

This section contains results of testing dependencies between Polish trading volume and the trading volumes of some developed stock markets. We find that significant rank correlations (even at 1% level) between the volume of WIG and ATX, DAX, DJIA volumes, respectively exist. The coefficients of Kendall's tau are rather small and amount to 0.0455, 0.0861 and 0.1479, respectively. The copulas that fit these relationships best are reported in the table below.

Table 4. Dependencies between trading volumes on stock markets

Relationship	"Best" copula	Parameter(s)		λ_L	λ_U
V denotes trading volume of Polish stock market (WIG)					
$V \leftrightarrow V_{ATX}$	Plackett	1.2299		0	0
$V \leftrightarrow V_{DAX}$	*t*-student	0.2281	21.065	0.0012	0.0012
$V \leftrightarrow V_{DJIA}$	Plackett	1.4761		0	0

The volume dependencies do not differ in lower (λ_L) and upper (λ_U) tails. These empirical observations follow also from theoretical properties of the "best fit" copulas, which are symmetric by definition.

We visualize the relationship between WIG and DJIA volumes by Figure 4.

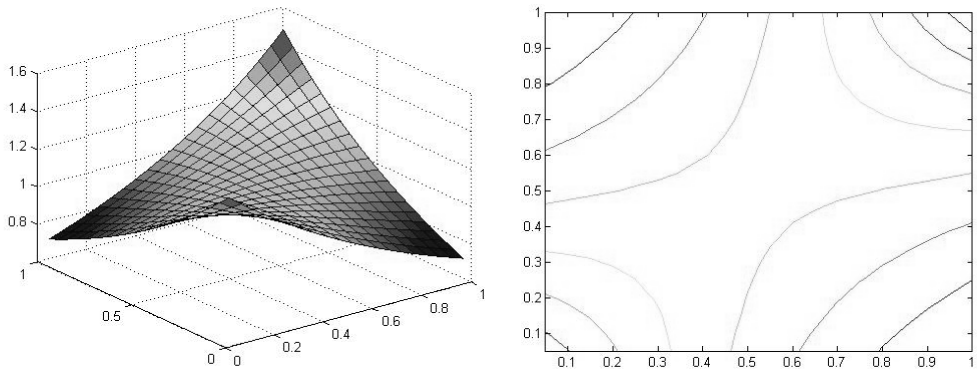

Fig. 4. Density (left) and contours of density of Plackett copula for $V \leftrightarrow V_{DJIA}$ with $\alpha = 1.4761$

These results concerning trading volume dependencies are symmetric in lower and upper tails. In Figure 4 we can see symmetric independence in the tails.

4.2.3. Polish trading volume and foreign stock returns (returns volatilities) dependencies

In this paragraph the relationship between the returns (volatilities) of foreign stock markets and the volume of the Polish stock market is modelled. The coefficients of Kendall's tau between the volume of WIG and ATX stock returns (ATX volatility) is equal to 0.0558 and is significant at a 5% level (0.0319, significant at a 10% level) as in the case of the German market (coefficients are 0.0449 and 0.0322, respectively). In the case of the DJIA index we cannot reject the null hypotheses of independence. Kendall's tau coefficients are equal to 0.014933748 and 0.004200046 (with p-values of 0.3636 and 0.7983, respectively).

Table 5. Dependencies between Polish trading volume (V) and foreign returns (volatilities)

Relationship	"Best" copula	Parameter(s)	λ_L	λ_U
R denotes returns of Vienna Stock Exchange (ATX)				
$R \leftrightarrow V$	Plackett	1.2867	0	0
$\lvert R\rvert \leftrightarrow V$	Plackett	1.1582	0	0
R denotes returns of Frankfurt Stock Exchange (DAX)				
$R \leftrightarrow V$	rotated Clayton	0.0908	0	0.0005
$\lvert R\rvert \leftrightarrow V$	Plackett	1.1573	0	0
R denotes returns of New York Stock Exchange (DJIA)				
$R \leftrightarrow V$	normal	0.0240	0	0
$\lvert R\rvert \leftrightarrow V$	normal	0.0138	0	0

Polish trading volume depends (from statistical point of view) weakly on ATX and DAX returns and returns volatility and does not depend on DJIA returns and returns volatility. All relations are symmetric in the ranges of the market variables considered (compare (λ_L) and (λ_U)).

Conclusions

This study analyses the relationships between trading volume and stock returns and return volatility for the Polish stock market and its dependence on three developed stock markets, namely the US, German and Austrian.

Applying a dependence measure based on copulas, we observed:

- a significant relationship between returns and trading volume on the domestic stock market,
- a significant relationship between volatility and trading volume on the domestic market,
- a significant relationship between Polish stock returns and the returns of foreign stock market indexes,
- a significant relationship between Polish trading volume and the trading volumes of the foreign markets under study,
- a significant relationship between Polish trading volume and stock return and stock return volatility from the German (DAX) and Austrian (ATX) stock markets,
- the lack of a significant relationship between Polish trading volume (WIG) and stock return and stock return volatility on the American stock market (DJIA).

In our opinion there are good reasons why in the case of the Warsaw Stock Exchange a significant relationship between return volatility and trading volume takes place. Considering the size of the Polish stock market and the well-known fact that initial stock price movements attract additional equity investments on the WSE, we come to the conclusion that the significant relationship between volatility and trading volume is quite obvious.

It results from our investigations that there exists a close relationship between Polish stock returns and the returns of foreign stock market indexes. A similar significant relationship concerns trading volumes. In addition, stock returns (return volatility) of the Austrian and especially the German stock market considerably influence Polish trading volume. The lack of significant DJIA impact on the WSE on the same day is probably caused by the fact that changes of DJIA lead changes on the European stock markets.

Since trading volumes data exhibits significant autocorrelation it follows from our findings that knowledge of stock price movements on the German as well as the Austrian stock market can improve short–run forecasts of the current and future trading volume of the Polish stock market. This may be important for so called noisy traders who make investment decisions solely upon the observations of prices movements and trading volume development on domestic and foreign stock markets.

In addition, the similar level of impact of the German and Austrian stock markets on the Polish stock market is not a surprise although the Frankfurt Stock Exchange is much bigger than the Vienna Stock Exchange. The fact is that the Austrian stock market depends very strongly on the German stock market. A very important reason for the close relation between these stock markets is the significant dependence of the

Austrian economy on the German economy (i.e. exports from Austria to Germany in 2006 amounted to 32% of total Austrian exports).

Our investigations are based on reliable data which come from the period 22/06/1998–05/11//2004. Future investigations should be concerned with the stability over time of the relations between domestic and foreign stock market variables taking into account more recent data.

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Polski rynek akcji a wybrane rynki zagraniczne – analiza zależności za pomocą kopul

W literaturze finansowej zajmowano się w znacznie mniejszym stopniu związkami między wielkością obrotów, stopami zwrotu i ich wariancją warunkową w przypadku rynków „wschodzących" niż w przypadku rozwiniętych rynków kapitałowych. Do pomiaru zależności pomiędzy zmiennymi finansowymi stosuje się przede wszystkim współczynniki korelacji: Pearsona, Spearmana i Kendalla.

Okazuje się, że są to miary, które w przypadku rynków kapitałowych nie oddają struktury zależności między zmiennymi finansowymi. W przypadku tych samych rozkładów brzegowych współczynniki korelacji są takie same. Obserwuje się, że mimo nieistotnej lub słabej korelacji między różnymi rynkami finansowymi dochodzi na nich do niekiedy bardzo poważnych kryzysów dokładnie w tym samym czasie. Można stąd wysnuć hipotezę, że prawdopodobnie zależności w tzw. ogonach rozkładów stóp zwrotu, zwłaszcza w lewym ogonie (ang. *left tail*) są znacznie silniejsze niż można by wnioskować z wartości globalnych współczynników korelacji. Poza tym w przypadku zarówno współczynnika korelacji Pearsona jak i rangowych współczynników korelacji wartość zerowa tych współczynników nie oznacza na ogół braku zależności. Dlatego w ostatnich latach coraz powszechniej stosuje się alternatywne miary zależności między zmiennymi finansowymi, przede wszystkim kopule. Stosując teorię kopul autorzy tej pracy starali się odpowiedzieć na pytanie, czy na podstawie wielkości stóp zwrotu i ich zmienności (ang. *volatility*) można wnioskować o wielkości obrotów? Przedstawione wyniki pozwalają na wyciągnięcie wniosku, że znajomość ruchu cen pozwala w krótkim okresie przewidzieć, przynajmniej w pewnym stopniu, wzrost obrotów. Poza badaniem związków między obrotami, stopami zwrotu i ich zmiennością na rynku krajowym zbadano związek stóp zwrotu i ich zmienności na rynkach zagranicznych z wielkością obrotów na Warszawskiej Giełdzie Papierów Wartościowych dla okresu 22.06.1998–05.11.2004. Wykorzystano indeksy giełdowe WIG, ATX, DAX oraz DJIA. Taki związek (wpływ na wielkość obrotów spółek objętych WIG) został potwierdzony statystycznie w przypadku rynków niemieckiego i austriackiego.

Słowa kluczowe: *kopule, zależności, stopy zwrotu, obroty*

Appendix. Definitions of applied models

ARMA(r, m):

$$y_t = c + \sum_{i=1}^{r} \phi_i y_{t-1} + \varepsilon_t + \sum_{j=1}^{m} \theta_j \varepsilon_{t-j}$$

GARCH(p, q):

$$\sigma_t^2 = \kappa + \sum_{i=1}^{p} G_i \sigma_{t-1}^2 + \sum_{j=1}^{q} A_j \varepsilon_{t-j}^2 ,$$

where $\kappa > 0$, $\sum_{i=1}^{p} G_i + \sum_{i=1}^{q} A_i < 1$, $G_i \geq 0$, $i = 1, 2, ..., p$, $A_j \geq 0, j = 1, 2, ..., q$.

GJR(p, q):

$$\sigma_t^2 = \kappa + \sum_{i=1}^{p} G_i \sigma_{t-i}^2 + \sum_{j=1}^{q} A_j \varepsilon_{t-j}^2 + \sum_{j=1}^{q} L_j S_{t-j}^{-} \varepsilon_{t-j}^2 ,$$

where:

$S_{t-j}^{-} = \begin{cases} 1, & \varepsilon_{t-j} < 0, \\ 0, & \varepsilon_{t-j} \geq 0. \end{cases}$ and $\sum_{i=1}^{p} G_i + \sum_{j=1}^{q} A_j + \frac{1}{2} \sum_{j=1}^{q} L_j < 1$, where

$\kappa > 0$,

$G_i \geq 0$, $i = 1, 2, ..., p$,

$A_j \geq 0$, $j = 1, 2, ..., q$,

$A_j + L_j \geq 0$, $j = 1, 2, ..., q$.

EGARCH(p, q):

$$\log \sigma_t^2 = \kappa + \sum_{i=1}^{p} G_i \log \sigma_{t-i}^2 + \sum_{j=1}^{q} A_j \left(\frac{|\varepsilon_{t-j}|}{\sigma_{t-j}} - E\left\{ \frac{|\varepsilon_{t-j}|}{\sigma_{t-j}} \right\} \right) + \sum_{j=1}^{q} L_j \left(\frac{|\varepsilon_{t-j}|}{\sigma_{t-j}} \right).$$