LOWER-LIMIT BARRIER IN THE PROBLEM OF THE IDENTIFICATION OF A BARRIER IN THE FUNCTIONING OF A CERTAIN INVENTORY STORAGE AND ISSUE SYSTEM

The paper investigates a certain inventory system whose input is a non-aggregated dynamic-parameter process. The authors derive equations that define the distribution of conditional probabilities for the case of a lower-limit barrier in subsystem $L$. They depend on the parameters of the functioning of transport subsystem and the parameters of the process of product supply to finite-volume storage.

Keywords: *system, barrier, inventory*

1. Introduction

Paper [1] defined the notion of a barrier in the functioning of an inventory storage and issue system with non-aggregated dynamic-parameter input and presented the general operating principles of such a system and the conditional distributions for intermediate states of the subsystem $L$. These distributions were used in paper [2] to derive relations satisfied by the density functions for intermediate states.

Continuing the investigations reported in [1], [2], this paper contains derivation of the analytical forms of the conditional probabilities in the case of a lower-limit barrier in a process controlled by a dynamic-parameter input.

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2. Conditional probabilities in the case of a lower-limit barrier in the subsystem $L$

The lower-limit barrier of the subsystem $L$ at time $t \in T_1$ is defined by the random event $z(t) = 0$. The state can be characterized by probabilities of the form:

$$Q_{k}^{ui}(\{0\}, t) = P(z(t) = 0, x(t) = x_k, v(t) = u), \quad t \in T_1, \quad u = 1,0.$$  \hspace{1cm} (1)

These probabilities satisfy the following relations:

for $u = 0$,

$$Q_{k}^{ui}(\{0\}, t + \tau) = P(z(t + \tau) = 0, x(t + \tau) = x_k, v(t + \tau) = 0) =
\sum_{i} \int_{V_i} q_{ik}^{0ui}(z, \{0\}; \tau, t)Q_{k}^{ui}(dz, t) + \int_{0}^{V_i} q_{ik}^{1ui}(z, \{0\}; \tau, t)Q_{k}^{ui}(dz, t),$$  \hspace{1cm} (2)

where

$$q_{ik}^{0ui}(z, \{0\}; \tau, t) = P(x(t) \text{ has one jump for } t + \theta \in (t, t + \tau), v(s) = 0 \text{ for } s \in [t, t + \tau],
\begin{align*}
z - d\tau \leq 0 \big| x(t) &= x_i, \quad v(t) = 0 \big) + o_i^{(\tau)}(\tau); \\
i \neq k, z = z(t) \in [0, V_1], \quad t \in T_1, \quad t + \tau \in T_1, \tag{3}
\end{align*}$$

$$q_{ik}^{0ui}(z, \{0\}; \tau, t) = P(x(s) = x_k, s \in [t, t + \tau], v(s) = 0 \text{ for } s \in [t, t + \tau],
\begin{align*}
z - d\tau \leq 0 \big| x(t) &= x_k, \quad v(t) = 0 \big) + o_i^{(\tau)}(\tau); \\
z = z(t) \in [0, V_1], \quad t \in T_1, \quad t + \tau \in T_1, \tag{4}
\end{align*}$$

$$q_{ik}^{0ui}(z, \{0\}; \tau, t) = P(x(s) = x_k, \text{ for } s \in [t, t + \tau], v(t) \text{ has one jump for } t + \theta \in (t, t + \tau),
\begin{align*}
h(z + x_k \theta) - d(\tau - \theta) \leq 0 \big| x(t) &= x_k, \quad v(t) = 1 \big) + o_i^{(\tau)}(\tau); \\
z = z(t) \in [0, V_1], \quad t \in T_1, \quad t + \tau \in T_1, \tag{5}
\end{align*}$$

for $u = 1$,

$$Q_{k}^{ui}(\{0\}, t + \tau) = P(z(t + \tau) = 0, x(t + \tau) = x_k, v(t + \tau) = 1) =
\sum_{i} \int_{V_i} q_{ik}^{1ui}(z, \{0\}; \tau, t)Q_{k}^{ui}(dz, t) + \int_{0}^{V_i} q_{ik}^{0ui}(z, \{0\}; \tau, t)Q_{k}^{ui}(dz, t),$$  \hspace{1cm} (6)

where

$$q_{ik}^{1ui}(z, \{0\}; \tau, t) = P(x(t) \text{ has one jump for } t + \theta \in (t, t + \tau), v(s) = 1 \text{ for } s \in [t, t + \tau],
\begin{align*}
h(z + x_k \theta) - (\tau - \theta)x_k \leq 0 \big| x(t) &= x_k, \quad v(t) = 1 \big) + o_i^{(\tau)}(\tau); \\
i \neq k, z = z(t) \in [0, V_1], \quad t \in T_1, \quad t + \tau \in T_1, \tag{7}
\end{align*}$$
\[ q_{kk}^{1U}(z; \{0\}; \tau, t) = P(x(s) = x_k, v(s) = 1 \text{ for } s \in [t, t + \tau], \]
\[ z + \pi_k < 0 \left| x(t) = x_k, v(t) = 1 + o_t^{(1)}(\tau) \right; \]
\[ = z(t) \in [0, V_1], \ t + \tau \in T, \] (8)
\[ q_{kk}^{0U}(z; \{0\}; \tau, t) = P(x(s) = x_k \text{ for } s \in [t, t + \tau], v(t) \text{ has one jump for } t + \theta \in (t, t + \tau),\]
\[ h(z - d \theta) + x_k (\tau - \theta) \leq 0 \left| x(t) = x_k, v(t) = 0 + o_t^{(1)}(\tau) \right; \]
\[ = z(t) \in [0, V_1], \ t + \tau \in T. \] (9)

In order to find the probabilities \( Q_k^{1U}(\{0\}; t + \tau) \) and \( Q_k^{0U}(\{0\}; t + \tau) \), we have to find the conditional probabilities \( q_{kk}^{1U}(z; \{0\}; \tau, t), q_{kk}^{1U}(z; \{0\}; \tau, t), q_{kk}^{0U}(z; \{0\}; \tau, t), q_{kk}^{0U}(z; \{0\}; \tau, t) \). Taking into account relations (3) and (6) from [1], we obtain
\[ q_{kk}^{1U}(z; \{0\}; \tau, t) = \exp(-\pi^{(1)}_k \tau) \int \frac{\pi^{(i)}_k}{\pi^{(1)}_k} d(1 - \exp(-\pi^{(1)}_k \theta)) + o_t^{(1)}(\tau), \] (10)
where
\[ B = \{ \theta; 0 < \theta < \tau; h(z + \theta x_k) + (\tau - \theta) x_k \leq 0 \}, \]
\[ i \neq k, z = z(t) \in [0, V_1], \ t + \tau \in T. \] (11)

If \( x_i > 0, x_k < 0, \tau \) small, \( 0 < \theta < \tau, \ t \in T, \ t + \tau \in T \), then
\[ \{ \theta; h(z + \theta x_k) + (\tau - \theta) x_k \leq 0 \} \cap \{ \theta; z + \theta x_i + (\tau - \theta) x_k \leq 0 \} = \]
\[ = \begin{cases} \theta; 0 \leq \theta \leq \frac{-z + \pi_k}{x_k - x_i}, & z \leq -\pi_k, \\
\emptyset, & \text{other } z. \end{cases} \]

Thus
\[ B = \begin{cases} \theta; 0 < \theta \leq \frac{z + \pi_k}{x_k - x_i}, & 0 \leq z \leq -\pi_k, \\
\emptyset, & \text{other } z. \end{cases} \]

Hence, from (10), by integration, we get
\[ q_{kk}^{1U}(z; \{0\}; \tau, t) = \begin{cases} (1 - \pi^{(1)}_k \tau) \pi^{(i)}_k \frac{z + \pi_k}{x_k - x_i} + o_t^{(1)}(\tau; z), & 0 \leq z \leq -\pi_k, \\
0, & \text{other } z. \end{cases} \] (12)
Similarly it can be shown that the conditional probabilities are given by the following formulas:

for $x_k > 0$, $x_i < 0$,

$$q_{ik}^{11}(z,\{0\}; \tau, t) - o_{1i}(\tau) = \begin{cases} (1 - \pi_i^{(1)} \tau)\pi_{ik}^{(1)}(\tau) + o^{(1)}(\tau), & 0 \leq z < -\bar{\pi}_i, \\ 0, & \text{other } z, \end{cases} \tag{18}$$

for $x_i < 0$, $x_k > 0$,

$$q_{ik}^{11}(z,\{0\}; \tau, t) - o_{1i}(\tau) = \begin{cases} (1 - \pi_i^{(1)} \tau)(1 - \pi_k^{(1)} \tau) + o^{(1)}(\tau), & 0 \leq z < -\bar{\pi}_i, \\ 0, & \text{other } z, \end{cases} \tag{19}$$
for \( x_k = 0 \),

\[
q_{kk}^{0\ell}(z, \{0\}; \tau, t) - o_1^{(i)}(\tau) = \begin{cases} 
(1 - \pi_k^{(i)}(\tau)) \pi_0^{\eta} \left( \tau - \frac{z}{d} \right) + o^{(1)}(\tau; z), & 0 \leq \tau < d, \\
0, & \text{other } z,
\end{cases}
\]  

(20)

for \( x_k < 0 \),

\[
q_{kk}^{0\ell}(z, \{0\}; \tau, t) - o_1^{(i)}(\tau) =
\]

\[
\begin{cases} 
(1 - \pi_k^{(i)}(\tau)) \pi_0^{\eta} \tau + o^{(1)}(\tau), & d + x_k \leq 0, 0 \leq z < d, \\
0, & d + x_k \leq 0, \text{ other } z, \\
(1 - \pi_k^{(i)}(\tau)) \pi_0^{\eta} \left( \tau - \frac{z + x_k}{d + x_k} \right) + o^{(1)}(\tau; z), & d + x_k > 0, 0 \leq z < x_k, \\
0, & d + x_k > 0, \text{ other } z,
\end{cases}
\]  

(21)

for \( x_k < 0 \),

\[
q_{kk}^{1\ell}(z, \{0\}; \tau, t) - o_1^{(i)}(\tau) =
\]

\[
\begin{cases} 
(1 - \pi_k^{(i)}(\tau)) \pi_1^{\eta} \tau + o^{(1)}(\tau), & d + x_k \leq 0, 0 \leq z < d, \\
0, & d + x_k \leq 0, \text{ other } z, \\
(1 - \pi_k^{(i)}(\tau)) \pi_1^{\eta} \tau + o^{(1)}(\tau), & d + x_k > 0, 0 \leq z < x_k, \\
(1 - \pi_k^{(i)}(\tau)) \pi_1^{\eta} \frac{d \tau - z}{d + x_k} + o^{(1)}(\tau; z), & d + x_k > 0, -x_k < z < d, \\
0, & d + x_k > 0, \text{ other } z,
\end{cases}
\]  

(22)

for \( x_k > 0 \),

\[
q_{kk}^{1\ell}(z, \{0\}; \tau, t) - o_1^{(i)}(\tau) = \begin{cases} 
(1 - \pi_k^{(i)}(\tau)) \pi_1^{\eta} \frac{d \tau - z}{d + x_k} + o^{(1)}(\tau; z), & 0 \leq z < d, \\
0, & \text{other } z,
\end{cases}
\]  

(23)

for \( x_k = 0 \),

\[
q_{kk}^{1\ell}(z, \{0\}; \tau, t) - o_1^{(i)}(\tau) = \begin{cases} 
(1 - \pi_k^{(i)}(\tau)) \pi_1^{\eta} \frac{d \tau - z}{d} + o^{(1)}(\tau; z), & 0 \leq z < d, \\
0, & \text{other } z,
\end{cases}
\]  

(24)
for \( x_k > 0 \),

\[
q_{dik}^{1u}(z, \{0\}; \tau, t) - q_{il}^{(l)}(\tau) = 0, \; 0 \leq z \leq V_1. \tag{25}
\]

The components \( o^{(l)}(\tau; \ldots), o^{(l)}(\tau) \), which appear in equations (12)–(25) are in the vicinity of \( \tau = 0 \) infinitely small quantities of an order higher than \( \tau \). The relations (12)–(25) will be used in the authors’ next paper to derive relations satisfied by the probabilities \( Q^l_\tau(\{0\}; t + \tau) \). These will be used for quantitative identification of a barrier in the functioning of the system under consideration.

References


Bariera dolna procesu w zagadnieniu identyfikacji i bariery funkcjonowania pewnego systemu gromadzenia i wydawania zapasów

Badana jest bariera działania pewnego systemu gospodarki zapasami, którego wejście jest procesem niezagregowanym o dynamicznych parametrow. Wyprowadzono wzory wyrażające warunkowe rozkłady prawdopodobieństwa w przypadku bariery dolnej podsystemu \( L \). Zależą one od parametrów funkcjonowania podsystemu transportowego oraz parametrów procesu podaży produktu do magazynu o skończonej objętości.

Słowa kluczowe: system, bariera, zapas