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EFFECTIVENESS OF SECURITIES WITH FUZZY PROBABILISTIC RETURN

The generalized fuzzy present value of a security is defined here as fuzzy valued utility of cash flow. The generalized fuzzy present value cannot depend on the value of future cash flow. There exists such a generalized fuzzy present value which is not a fuzzy present value in the sense given by some authors. If the present value is a fuzzy number and the future value is a random one, then the return rate is given as a probabilistic fuzzy subset on a real line. This kind of return rate is called a fuzzy probabilistic return. The main goal of this paper is to derive the family of effective securities with fuzzy probabilistic return. Achieving this goal requires the study of the basic parameters characterizing fuzzy probabilistic return. Therefore, fuzzy expected value and variance are determined for this case of return. These results are a starting point for constructing a three-dimensional image. The set of effective securities is introduced as the Pareto optimal set determined by the maximization of the expected return rate and minimization of the variance. Finally, the set of effective securities is distinguished as a fuzzy set. These results are obtained without the assumption that the distribution of future values is Gaussian.

Keywords: *behavioural present value, fuzzy present value, random future value, fuzzy probabilistic return, effective financial security*

1. Introduction

Typically, the analysis of properties of any security is conducted as an analysis of return rate properties. Any return rate is an increasing function of future value (*FV*) and a decreasing function of present value (*PV*).

In classical financial approach *PV* is defined as discounted cash flow. This cash flow may be a present or future one. Ward [35] defines fuzzy *PV* as discounted fuzzy

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cash flow. The fuzzy cash flow used here is interpreted as an imprecise forecast of future crisp cash flow. The Ward's definition is generalized to the case of fuzzy duration by Greenhut et al. [11]. Sheen [29] generalizes the Ward's definition to the case of fuzzy interest rate. Buckley [1], [2], Gutierrez [12], Kuchta [19] and Lesage [22] discuss some problems connected with the application of fuzzy arithmetic for calculating fuzzy *PV*.

In agreement with uncertainty thesis expressed by Mises [24] and Kaplan et al. [16], each future cash flow is under uncertainty. In crisp case this uncertainty is usually modeled in such a way that cash flow is described as a random variable. Therefore Huang [14] generalizes the Ward's definition for the case when future cash flow is given as a fuzzy random variable in the sense given by Kwakernaak [20], [21]. A more general definition of fuzzy *PV* is proposed by Tsao who assumes that future cash flow is a fuzzy probabilistic set in the sense given by Hiroto [13].

All kinds of fuzzy *PV* defined above may be used for the determination of fuzzy net present value (N*PV*) defined as a sum of fuzzy *PV*.

In recent years the concept of cash flow utility has played an important part in the behavioral finance research. This problem is discussed for example by Frederick et al. [33], Dacey et al. [6], Killeen [17], Zauberman et al. [37], Kontek [18] and Doyle [7]. *PV* is defined there as the utility of cash flow. Thus generalized fuzzy *PV* is defined as fuzzy valued utility of cash flow. The generalized fuzzy *PV* is more general than the definitions of fuzzy *PV* proposed by Greenhut et al. [11] and Sheen [29].

In financial market theory current market price of security is interpreted as a crisp *PV*. Piasecki [27] considers the impact of chosen behavioural factors on *PV*. The formal model of behavioural *PV* is offered as a result of these considerations. Behavioural *PV* is dependent on equilibrium price, current market price and degree of investor's susceptibility to changes. It implies that the present value can deviate from its observed market price out of influence of behavioural factors. The model so obtained explains the mechanism of maintaining the balance between supply and demand for efficient financial market. States of the behavioural environment are defined imprecisely. Therefore behavioural *PV* is described by trapezoidal fuzzy number in the sense given by Campos et al. [3]. It means that behavioural *PV* is at imprecision risk. Behavioural *PV* is an example of generalized fuzzy *PV* defined above. On the other hand, in a general case behavioural *PV* does not depend on the value of future cash flow. It means that there exists such behavioural *PV* which is not fuzzy *PV* in the sense given by Ward [35] or by Huang [14].

In agreement with the thesis cited above as expressed by Mises [24] and Kaplan et al. [16], each *FV* is under uncertainty. Therefore the security *FV* is usually presented as a random variable. Distribution of this random variable is a formal image of the uncertainty risk. Vast uncertainty risk information is gathered in this way. Papers [8], [30]–[33] and [36] are examples of this knowledge.

Piasecki [27] noted that if *PV* is a fuzzy number and *FV* is a random variable then the return rate is given as a probabilistic fuzzy subset in the real line. Thus this return rate will be called fuzzy probabilistic return. Despite a careful preliminary survey of library holdings the author has not found any similar model of return rate. When we apply fuzzy probabilistic return for assessment of the security with generalized fuzzy *PV* then we can use, without any changes, all the rich empirical knowledge about probability distributions of return rate which has been gathered. This fact expands the possibility of real applications. It is a highly advantageous feature of the proposed model.

Investment in effective security (ES) is a standard investor's goal in normative theories of financial market. Therefore the main goal of our considerations is to distinguish the family of ESs with fuzzy probabilistic return.

Achieving this goal requires the study of basic parameters characterizing fuzzy probabilistic return. In Section 1 fuzzy expected value and variance are determined for this case of return. The expected return rate is replaced there by fuzzy return rate which takes into account also behavioural aspects of decision making in finance. However, such increase in cognitive value has a price. This price is the imprecision risk disclosure.

Imprecision is composed of ambiguity and indistinctness. Moreover, fuzzy probabilistic return is at uncertainty risk. Hence the three-dimensional image of risk described in Section 2.

When we have used imprecise images of security we cannot precisely indicate the recommended investment alternative. Each investment alternative is then recommended to some extent. Investor shifts some of the responsibility to advisers or the forecasting tool applied. For this reason, the investor restricts his choice of investment decisions to alternatives recommended in the greatest degree. In this way the investor minimizes his individual responsibility for financial decision making. It shows that imprecision risk assessment is relevant for the analysis of investment processes. This problem was widely discussed in [25].

In this paper we consider the case when each effective security is indicated as recommended investment alternative. In comparison with the classical Markowitz theory, imprecision is a new aspect of risk assessment. We ask a question here whether such an extension of risk assessment is appropriate. The usefulness of taking into account imprecision in risk study is well justified by the following three arguments.

Firstly, it is always possible to reduce the uncertainty risk of forecast with an appropriate manipulation by lowering the forecast precision.

Secondly, if we take into account the imprecision risk we can reject investment alternatives which are attractive from the viewpoint of the classical Markowitz theory, but, unfortunately, the information gathered about them is highly imprecise.

Thirdly, from the viewpoint of the classical Markowitz theory and its implications, we witness many anomalies in financial markets practice. Seeing these paradoxes is the starting point for the development of behavioural finance.

In Section 3, a set of effective securities is defined as the Pareto optimum determined by the maximization of fuzzy expected return rate and the minimization of risk assessments. Two cases of risk management are taken into account there. The minimization of uncertainty risk and the simultaneous minimization of uncertainty risk and imprecision risk are considered here.

This article is addressed to two groups of readers. The results obtained may be of interest to financial market theorists and to practitioners constructing investment decision support systems.

2. Imprecise assessment of return rate

Let us assume that the time horizon $t > 0$ of an investment is fixed. Then, the security considered here is determined by two values:

– anticipated future value (FV) $V_t \in \mathbb{R}^+$,

– assessed present value (PV) $V_0 \in \mathbb{R}^+$.

The basic characteristic of benefits from owning this instrument is a return rate r_t given by the identity

$$r_t = r\left(V_0, V_t\right) \tag{1}$$

In the general case, the function: r: $\mathbb{R}^+ \times \mathbb{R}^+ \to \mathbb{R}$ is a decreasing function of PV and an increasing function of FV. It implies that for any V_t we can determine inverse function $r_0^{-1}\left(\bullet, V_t\right) : \mathbb{R} \to \mathbb{R}^+$. In a special case we have here:

- simple return rate

$$r_t = \frac{V_t - V_0}{V_0} = \frac{V_t}{V_0} - 1 \tag{2}$$

- logarithmic return rate

$$r_t = \ln \frac{V_t}{V_0} \tag{3}$$

The FV is at risk of uncertainty. A formal model of this uncertainty is the presentation of FV V_t as a random variable $\tilde{V}_t : \Omega = \{\omega\} \to \mathbb{R}^+$. The set Ω is a set of elementary states of the financial market. In the classical approach to the problem of return rate determination, PV of a security is identified with the observed market price $\breve{C}$. The return rate is then a random variable which is at uncertainty risk. This random variable is determined by the identity

$$\tilde{r}_t(\omega) = r\left(\breve{C}, \tilde{V}_t(\omega)\right) \tag{4}$$

In practice of financial markets analysis, the uncertainty risk is usually described by probability distribution of return rates. At the moment, we have an extensive knowledge on this subject. Let us assume that this probability distribution is given by cumulative distribution function $F_r\colon \mathbb{R} \to [0;1]$. Then, the probability distribution of FV is described by cumulative distribution function $F_V\colon \mathbb{R} \to [0;1]$ given as follows

$$F_V(x) = F_r\left(r_t\left(\breve{C}, x\right)\right) \tag{5}$$

Assessment of security FV is based on objective measurement only. It means that the cumulative distribution function of future value is independent of how the present value is determined.

It is shown in [27] that security PV may be at imprecision risk. The above--mentioned imprecision risk was determined by behavioural premises. Imprecisely assessed PV is described as generalized fuzzy PV which is represented by its membership function $\mu\colon \mathbb{R}^+ \to [0;1]$. The return rate is then at risk of coincidence uncertainty and imprecision. According to the Zadeh extension principle, for each fixed elementary state $\omega \in \Omega$ of financial market, membership function $\rho(\bullet, \omega)\colon \mathbb{R} \to [0;1]$ of return rate is determined by the identity

$$\rho(r, \omega) = \max\left\{\mu(y) : y \in \mathbb{R}^+, r = r\left(y, \tilde{V}_t(\omega)\right)\right\} = \mu\left(r_0^{-1}\left(r, \tilde{V}_t(\omega)\right)\right) \tag{6}$$

This means that the return rate considered here is represented by fuzzy probabilistic set defined by Hiroto [13]. For this reason, this return rate is called fuzzy probabilistic return. In special cases we have here:

- for the simple return rate

$$\rho(r, \omega) = \mu\left((1+r)^{-1} \tilde{V}_t(\omega)\right) \tag{7}$$

- for the logarithmic return rate

$$\rho(r, \omega) = \mu\left(\mathrm{e}^{-r} \tilde{V}_t(\omega)\right) \tag{8}$$

For any fuzzy probabilistic return, we determine the parameters of its distribution. We have here:

- distribution of expected return rate

$$\rho(r) = \int_{-\infty}^{+\infty} \mu\left(r_0^{-1}\left(r, \tilde{V}_t(\omega)\right)\right) dF_V(y) \tag{9}$$

- expected return rate

$$\frac{\int_{-\infty}^{+\infty} r\rho(r)\,dr}{\int_{-\infty}^{+\infty} \rho(r)\,dr} \tag{10}$$

Distribution of expected return rate $\rho \in [0, 1]^{\mathbb{R}}$ is a membership function of fuzzy subset $\tilde{R}$ in the real line. This subset $\tilde{R}$ is called the fuzzy expected return rate. This rate represents both rational and behavioural aspects in the approach to estimate the expected benefits. We will use the following variance of return rate as the assessment of the risk uncertainty

$$\sigma^2 = \left(\int_0^{+\infty}\int_{-\infty}^{+\infty} v(r, y)\,dF_V(y)\,dr\right)^{-1} \int_0^{+\infty}\int_{-\infty}^{+\infty} r\, v(r, y)\,dF_V(y)\,dr \tag{11}$$

where

$$v\left(x, \tilde{V}_t(\omega)\right) = \begin{cases} \max\left(\rho\left(\overline{r} + \sqrt{x}, \omega\right), \rho\left(\overline{r} - \sqrt{x}, \omega\right)\right), & x \geq 0 \\ 0 x < 0 \end{cases} \tag{12}$$

A detailed analysis of these relationships shows that the variance so determined describes both the rational and behavioural aspects of safety assessment of capital employed.

Similarly as in the case of precisely defined return rate, there are such probability distributions of future value for which the return rate variance does not exist. We then replace this distribution with a distribution truncated on both sides, for which the variance always exists. This procedure finds its justification in the theory of perspective [15]. Among other things, this theory describes the behavioural phenomenon of the rejection of extremes.

3. Three-dimensional image of risk

In the classical portfolio theory given by Markowitz [23], the normative investment strategy is the maximization of expected return rate $\bar{r}$, while its variance ζ is minimized. In this situation, each security is represented by a pair $(\bar{r}, \zeta^2)$. This pair represents rational premises for securities evaluation. It is implicitly assumed that the returns have Gaussian distribution.

In this section, the expected return rate r is replaced by the fuzzy return rate $\tilde{R}$ which takes into account also behavioural aspects of decision making in finance. In this way, we disclose imprecision risk. Imprecision is composed of ambiguity and indistinctness.

Ambiguity is the lack of clear recommendation of one alternative from among various alternatives. In accordance with the suggestion given in [1], we will evaluate the ambiguity risk by energy measure $d(\tilde{R})$ of the fuzzy expected return rate $\tilde{R}$. This measure is determined by the identity

$$\delta = d(\tilde{R}) = \frac{\int_{-\infty}^{+\infty} \rho(x)\,dx}{1 + \int_{-\infty}^{+\infty} \rho(x)\,dx} \tag{13}$$

Indistinctness is the lack of explicit distinction between the information provided and its negation. According to the suggestion given in [3], we will evaluate the indistinctness risk by entropy measure $e(\tilde{R})$ of fuzzy expected return rate $\tilde{R}$. This measure is described as follows

$$\varepsilon = e(\tilde{R}) = \frac{\int_{-\infty}^{+\infty} \min\big(\rho(x), 1-\rho(x)\big)\,dx}{1 + \int_{-\infty}^{+\infty} \min\big(\rho(x), 1-\rho(x)\big)\,dx} \tag{14}$$

The uncertainty follows from lack of investor knowledge about future states of the financial market. The lack of this knowledge implies that no investor is sure of future profits or losses. Properties of this risk are discussed in a rich body of literature. In this paper, we will evaluate the uncertainty risk by variance σ^2 given by identity (11).

In this situation, for each fuzzy expected return rate $\tilde{R}$ we assign a three-dimensional vector $(\sigma^2, \delta, \varepsilon)$. This vector is an image of the risks which is understood to be composed of the risks of uncertainty, ambiguity and indistinctness.

An increase of ambiguity risk means that the number of recommended investment alternatives increases too. This increases the chance of selecting the recommended alternative, which involves the opportunity cost. An increase in indistinctness risk means that distinctions between recommended and unrecommended alternatives are more blurred. It implies a higher probability of choosing unrecommended alternatives.

These observations show that an increase in imprecision risk makes investment conditions noticeably worse. Using the three-dimensional image of risk (σ^2, δ, ε) facilitates imprecision risk management. It is desirable here to minimize each of the three risk assessments.

Using the three-dimensional image of risk enables investigation of relationships between different types of risk. Here we can observe empirical interaction between risks. Moreover, there is a formal correlation between the uncertainty risk and ambiguity risk. The number of recommended alternatives increases with ambiguity risk. In this way, there is more certainty that the recommended alternatives include the best investment decision. This means that the uncertainty risk decreases. In summary, the uncertainty risk and the ambiguity risk are negatively correlated.

4. Financial effectiveness

A security is called effective (ES) if it has, for a given variance, a maximum expected return rate. In the classical portfolio theory Markowitz assumed that the distribution of return rates is Gaussian. Then the set of ESs is given as the upper branch of the Markowitz curve which is called ESs curve.

The set of ESs can also be specified by means of the multicriteria comparison theory. Using this approach we can dispense with the assumption that the probability distribution of return rates is Gaussian. We define two preorders on the set of all securities. These preorders are the maximization of expected return rates and the minimization of variance. The set of ESs is then described as the Pareto optimum set for multicriteria comparison defined by the above preorders. If we additionally assume here that the return rates distribution is Gaussian, the set of ESs will coincide with the upper branch of Markowitz curve. This means that the set of ESs is a generalization of the concept of ESs curve defined on the basis of the classical Markowitz theory.

Any investment in ES is an investment in security guarantying maximum returns with minimal risk of capital loss. This is a standard investor's goal in normative theories of financial market. It poses some difficulties with application since investors typically invest in securities which are outside of the ES set. Accordingly, from the viewpoint of these theories, they invest in ineffective securities. At the same time, these investors declare investing in ESs to be their normative goal. In this way we find a paradox of the real financial market.

The above-mentioned paradox is very common. This fact cannot be explained by lack of sufficient knowledge of the real processes occurring in the financial markets and economic environment. Increasing professionalization of investor activity and fast development of informatics imply that full access to market information and its processing capacity is available to all investors who manage the vast majority of exchange trading volume.

The paradox may be explained in the following way. The normative aim of investing in ESs is declared by investors who invest only in securities similar to the effective one. The degree of effectiveness of a given security is equal to the degree of its similarity to ES. In practice this means that almost every commercially available security is effective to some extent. On the other hand, an ES is no longer traded on the stock exchange. All this explains the paradox of divergence between the normative investor's purpose and the real goal of investment strategy. Investors always act in a manner similar to effective course of action.

Let us consider the normative model of investors' activity. The set of all securities is denoted by $\mathbb{Y}$. The security $\breve{Y} \in \mathbb{Y}$ is represented by the pair $(\tilde{R}_Y, (\sigma_Y^2, \delta_Y, \varepsilon_Y))$, where individual symbols have the following meanings:

- $\tilde{R}_Y$ is the fuzzy expected rate of return on security $\breve{Y}$,
- σ_Y^2 is the variance rate of return on security $\breve{Y}$,
- δ_Y is the energy measure of fuzzy expected return rate $\tilde{R}_Y$,
- ε_Y is the entropy measure of fuzzy expected return rate $\tilde{R}_Y$.

The fuzzy expected return rate $\tilde{R}_Y$ is defined by distribution of expected return rate $\rho_Y \in [0,1]^{\mathbb{R}}$. On the set of fuzzy real numbers $F(\mathbb{R})$ we define the relation $\tilde{K} \geq \tilde{L}$, which reads:

Fuzzy real number $\tilde{K}$ is greater or equal to fuzzy real number $\tilde{L}$

This relation is a fuzzy preorder defined by such membership function v_Q: $F(\mathbb{R}) \times F(\mathbb{R}) \to [0,1]$ which fulfils the condition

$$v_Q\left(\tilde{R}_Y, \tilde{R}_Z\right) = \sup\left\{\min\left\{\rho_y(u), \rho_Z(v)\right\} : u \geq v\right\} \tag{15}$$

for any pair $\left(\tilde{R}_Y, \tilde{R}_Z\right)$ of fuzzy expected return rates.

In the next step, we determine multicriteria comparison $W \subset \mathbb{Y} \times \mathbb{Y}$ by maximization of fuzzy expected return rates and by the minimization of variance. We describe the relation formed in this way as the predicate $\breve{Y} \supseteq \breve{Z}$ which reads

$$\textit{Security } \breve{Y} \textit{ is no more effective than security } \breve{Z} \tag{16}$$

In a formal way this multicriteria comparison is defined by equivalence

$$\breve{Y} \supseteq \breve{Z} \Leftrightarrow \tilde{R}_Y \geq \tilde{R}_Z \wedge \sigma_Y \leq \sigma_Z \tag{17}$$

In this situation, the relation W is fuzzy preorder defined by its membership function $v_W: \mathbb{Y}\times\mathbb{Y} \to [0,1]$. For any pair of securities $\breve{Y}, \breve{Z} \in \mathbb{Y}$ thee above-mentioned membership function is represented by the identity

$$v_W\left(\breve{Y}, \breve{Z}\right) = \begin{cases} v_Q\left(\tilde{R}_Y, \tilde{R}_Z\right)\sigma_Y \leq \sigma_Z \\ 0 \end{cases}, \qquad \sigma_Y > \sigma_Z \tag{18}$$

The set $\tilde{\Phi}$ of ESs is equal to the Pareto optimum defined by the multicriteria comparison (17). The set $\tilde{\Phi}$ is represented by its membership function φ: $\mathbb{Y} \to [0,1]$ determined by the identity

$$\varphi(\tilde{Y}) = \inf\left\{\max\left\{v_W\left(\tilde{Y}, \tilde{Z}\right), 1 - v_W\left(\tilde{Z}, \tilde{Y}\right)\right\} : \tilde{Z} \in \mathbb{Y}\right\} \tag{19}$$

The value $\varphi(\breve{Y})$ is interpreted as the truth value of the sentence:

$$\textit{The security } \breve{Y} \textit{ is effective} \tag{20}$$

We described above the case where the investor determined effective securities taking into account only the risk of uncertainty. Now let us focus on the case where the investor simultaneously takes into account the uncertainty risk and the imprecision risk. Let us consider now the multicriteria comparison $L \subset \mathbb{Y} \times \mathbb{Y}$ determined by maximization of fuzzy expected return rates and three criteria for minimization of the risk measures described above. We describe the relation formed in this way as the predicate $\breve{Y} \sqsupseteq \breve{Z}$ which reads

$$\textit{Security } \breve{Y} \textit{ is no more strictly effective than security } \breve{Z} \tag{21}$$

In a formal way this multicriteria comparison is defined by equivalence

$$\breve{Y} \sqsupseteq \breve{Z} \Leftrightarrow \tilde{R}_Y \geq \tilde{R}_{\dot{Z}} \wedge \delta_Y \leq \delta_Z \wedge \varepsilon_Y \leq \varepsilon_Z \tag{22}$$

In this case, the relation L is fuzzy preorder defined by its membership function v_L: $\mathbb{Y} \times \mathbb{Y} \to [0, 1]$. For any pair of financial instruments $\breve{Y}, \breve{Z} \in \mathbb{Y}$ the above-mentioned membership function is represented by the identity

$$v_L\left(\breve{Y},\breve{Z}\right)=\begin{cases} v_Q\left(\tilde{R}_Y,\tilde{R}_Z\right)\sigma_Y \le \sigma_Z \wedge \delta_Y \le \delta_Z \wedge \varepsilon_Y \le \varepsilon_Z \\ 0 \sim \left(\sigma_Y \le \sigma_Z \wedge \delta_Y \le \delta_Z \wedge \varepsilon_Y \le \varepsilon_Z\right) \end{cases} \tag{23}$$

The set $\tilde{\Psi}$ of strictly effective securities is determined as the Pareto optimum defined by multicriteria comparison (22). The set $\tilde{\Psi}$ is represented by its membership function ψ: $\mathbb{Y} \to [0, 1]$ determined by the identity

$$\psi\left(\tilde{Y}\right)=\inf\left\{\max\left\{v_L\left(\tilde{Y},\tilde{Z}\right),1-v_L\left(\tilde{Z},\tilde{Y}\right)\right\}:\tilde{Z}\in\mathbb{Y}\right\} \tag{24}$$

The value $\psi\left(\tilde{Y}\right)$ is interpreted as the truth value of the sentence:

$$\textit{The security } \breve{Y} \textit{ is strictly effective} \tag{25}$$

If investors consider purchasing or selling the security $\breve{Y}$ they can take into account the values $\varphi\left(\tilde{Y}\right)$ and $\psi\left(\tilde{Y}\right)$. Investors should limit the area of their investment to securities characterized by a relatively high value of these indicators. Also, investors should limit the sale of their securities to those for which the indicators mentioned above have low values. The considerations presented in [11] suggest that individual investors use different values of these indicators at the same time. Such diversification follows from the diversification of subjective behavioural reasons for investment decisions.

5. Conclusions

It has been shown that an increase in imprecision risk makes investment conditions noticeably worse. Accordingly, imprecision should be considered as a risk relevant to the investment process.

The paper applies behavioural reasons for investment decision making to describe the similarity of individual securities to effective ones (ES). Such result is obtained without the assumption that the probability distribution of return rates is Gaussian. The normative theory presented here explains that the divergence between the normative investor's purpose and the real goal of investment strategy is implied by behavioural aspects of financial market perception. Each of the paradoxes explained is apparent. This formal theory allows one to control the choice of securities similar to ES. It follows from the fact that using this theory we can determine the truth value of sentence (20) or (25).

Firstly, the results so obtained may be applied in behavioural finance theory as the normative model. Investing only in strictly efficient securities can be recognized as normative investor's goal. This strategy results in rejection of those investment alternatives which are admittedly attractive from the viewpoint of classical Markowitz theory, but the information gathered about them is unfortunately imprecise.

Secondly, the results presented in the paper may provide theoretical foundations for constructing investment decision support system.

Applications of the normative model presented involve several difficulties. The main difficulty is high formal and computational complexity of the tasks of determining the membership function of effective securities set. Computational complexity of the normative model is the price which we pay for the lack of detailed assumptions about the return rate. On the other hand, low logical complexity is an important good point of the formal model presented in this paper.

The problem of finding a membership function of effective securities set can also be solved using econometric analysis of financial markets. Examples of such solutions are presented in [26] and [28].

The author's main contribution in this paper is to propose two models of effective security with fuzzy probabilistic return. Moreover, the paper also offers an original three-dimensional image of the risk affecting fuzzy probabilistic return.

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References

[1] BUCKLEY J.J., *Fuzzy mathematics of finance*, Fuzzy Sets and Systems, 1987, 21, 257–273.

[2] BUCKLEY J.J., *Solving fuzzy equations in economics and finance*, Fuzzy Sets and Systems, 48, 1992, 289–296

[3] CAMPOS L.M., GONZALES A., *A subjective approach for ranking fuzzy numbers*, Fuzzy Sets and Systems, 1989, 29, 145–153.

[4] CHIU C.Y., PARK C.S. , *Fuzzy cash analysis using present worth criterion*, Eng. Econom., 1994, 39 (2), 113–138.

[5] CZOGAŁA E., GOTTWALD S., PEDRYCZ W., *Contribution to application of energy measure of fuzzy sets*, Fuzzy Sets and Systems, 1982, 8, 205–214.

[6] DACEY R., ZIELONKA P., *A detailed prospect theory explanation of the disposition effect*, Journal of Behavioral Finance, 2008, 9 (1), 43–50.

[7] DOYLE, J.R., *Survey of time preference, delay discounting models*, Working Paper, Cardiff Business School, Cardiff University, 2010 [online], http://ssrn.com/abstract=1685861.

[8] ECHAUST K., TOMASIK E., *Wybrane rozkłady prawdopodobieństwa w modelowaniu empirycznych stóp zwrotu akcji notowanych na GPW w Warszawie*, Zeszyt Naukowy Akademii Ekonomicznej w Poznaniu, 2008, 104, 34–66.

[9] FREDERICK S., LOEWENSTEIN G., O'DONOGHUE T., *Time Discounting and Time Preference: A critical Review*, Journal of Economic Literature, 2002, 40, 351–401.
[10] GOTTWALD S., CZOGAŁA E., PEDRYCZ W., *Measures of fuzziness and operations with fuzzy sets*, Stochastica, 1982, 6, 187–205.
[11] GREENHUT J.G., NORMAN G., TEMPONI C., *Towards a fuzzy theory of oligopolistic competition*, IEEE Proceedings of ISUMA-NAFIPS 1995, 1995, 286–291.
[12] GUTIERREZ I., *Fuzzy numbers and net present value*, Scand. J. Mgmt., 1989, 5 (2), 149–159.
[13] HIROTO K., *Concepts of probabilistic sets*, Fuzzy Sets and Systems, 1981, 5, 31–46.
[14] HUANG X., *Two new models for portfolio selection with stochastic returns taking fuzzy information*, European Journal of Operational Research, 12007, 80 (1), 396–405.
[15] KAHNEMAN D., TVERSKY A., *Prospect theory: an analysis of decision under risk*, Econometrica, 1979, 47, 263–292.
[16] KAPLAN S., BARISH N.N., *Decision-making allowing uncertainty of future investment opportunities*, Management Science, 1967, 13 (10), 569–577.
[17] KILLEEN P.R., *An additive-utility model of delay discounting*, Psychological Review, 2009, 116, 602–619.
[18] KONTEK, K., *Decision utility theory: Back to von Neumann, Morgenstern, and Markowitz*, Working Paper, 2010 [online]: http://ssrn.com/abstract=1718424.
[19] KUCHTA D., *Fuzzy capital budgeting*, Fuzzy Sets and Systems, 2000, 111, 367–385.
[20] KWAKERNAAK H.K., *Fuzzy random variables I*, Information Sciences, 1978, 15, 1–29.
[21] KWAKERNAAK H.K., *Fuzzy random variables II*, Information Sciences, 1979, 17, 253–278.
[22] LESAGE C., *Discounted cash-flows analysis. An interactive fuzzy arithmetic approach*, European Journal of Economic and Social Systems, 2001, 15 (2), 49–68.
[23] MARKOWITZ H.S.M., *Portfolio selection*, Journal of Finance, 1952, 7, 77–91.
[24] MISES L. VON, *The ultimate foundation of economic science. An essay on method*, D. van Nostrand Company, Inc., Princeton, 1962.
[25] PIASECKI K., *Decyzje i wiarygodne prognozy*, Zeszyty Naukowe, s. I. Prace doktorskie i habilitacyjne, z. 106, Akademia Ekonomiczna w Poznaniu, Poznań, 1990.
[26] PIASECKI K., *O sposobie poszukiwania dobrej metody inwestowania na giełdzie*, [in]: J. Hozer (Ed.), *Metody ilościowe w ekonomii*, Uniwersytet Szczeciński, Studia i Prace Wydziału Nauk Ekonomicznych i Zarządzania nr 11, Szczecin, 2009, 387–398.
[27] PIASECKI K., *Behavioural present value*, Behavioral and Experimental Finance eJournal, 2011 (4), [online].
[28] PIASECKI K., ZIOMEK R., *Zbiory intuicyjne w prognozowaniu rynku finansowego*, Zeszyty Naukowe Uniwersytetu Szczecińskiego, Finanse, Rynki Finansowe, Ubezpieczenia, 2010, 28, 45–60.
[29] SHEEN J.N., *Fuzzy financial profitability analyses of demand side management alternatives from participant perspective*, Information Sciences, 2005, 169, 329–364.
[30] TOMASIK E., *Rozkłady prawdopodobieństwa stop zwrotu indeksów i akcji notowanych na GPW w Warszawie*, [in:] W. Ostasiewicz (Ed.), *Statystka aktuarialna – teoria i praktyka*, Wydawnictwo Uniwersytetu Ekonomicznego we Wrocławiu, Wrocław, 2008, 24–38.
[31] TOMASIK E., *Homogeneity hypothesis for tail index of return rate distributions*, [in:] P. Chrzan, T. Czernik (Eds.), *Innowacje w finansach i ubezpieczeniach. Metody matematyczne, ekonometryczne i informatyczne*, Wydawnictwo Akademii Ekonomicznej w Katowicach, Katowice, 2009, 147–159.
[32] TOMASIK E., PIASECKI K., *Return rates of WIG20 index in the situation of extreme tide turning on the Warsaw Stock Exchange*, Capital Markets: Asset Pricing & Valuation eJournal, 2011, 4 (4), [online].
[33] TOMASIK E., WINKLER-DREWS T., *Rozkłady stabilny, uogólniony Pareto oraz t-Studenta w modelowaniu dziennych stóp zwrotu indeksu WIG20*, [in:] P. Chrzan, T. Czernik (Eds.), *Innowacje w finan-*

sach i ubezpieczeniach. Metody matematyczne, ekonometryczne i informatyczne, Wydawnictwo Akademii Ekonomicznej w Katowicach, Katowice, 2009, 101–110.
[34] TSAO C.-T., *Assessing the probabilistic fuzzy net present value for a capita. Investment choice using fuzzy arithmeti*c, J. Chin. Ins. Industrial Engineers, 2005, 22 (2), 106–118.
[35] WARD T.L., *Discounted fuzzy cash flow analysis*, Proceedings of 1985 Fall Industrial Engineering Conference, Seattle, 1985, 476–481.
[36] WINKLER-DREWS T., *Zarządzanie ryzykiem zmiany ceny*, PWE, Warszawa, 2009.
[37] ZAUBERMAN G., KYU KIM B., MALKOC S., BETTMAN J.R., *Discounting time and time discounting: Subjective time perception and intertemporal preferences*, Journal of Marketing Research, 2009, 46, 543–556.