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AGRICULTURAL SUSTAINABILITY IN A SIMPLE SOCIAL-ECOLOGICAL SYSTEM

There is a growing consciousness of the complexity and universality of interactions between social and ecological systems. Here we investigate a simple social-ecological model of land cultivation. It is shown that even very simple rules of land-use and an ecosystem's dynamics can generate a variety of possible stationary states. In particular, the possibility of the existence of "desirable" stationary states is examined. These desirable states are understood in the sense of sustainable development, that is, profitable for farmers and non-degrading for ecosystems. It turns out that the existence of such states can depend strongly on the parameters that are under a government's control, e.g. subsides, and others. Although real systems can reveal much more complex and counterintuitive behaviour, such a "toy model" can at least give some insight, help to realize the range of possible scenarios and improve our intuition about what might happen in real systems.

Keywords: *socio-ecological interactions, agricultural model*

1. Introduction

There are various and two-way interactions between social systems and ecosystems. Ecosystems, which for a long time were perceived to be unlimited, have become more and more affected by human activity, in a way that even endangers their existence. On the other hand, economic development has become more and more dependent on the natural environment or rather reaching the natural environment's limits has made this dependence more and more clear to humans. We are becoming aware of the role that the natural environment plays, not only as a good in itself, but as well as in

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economic growth. Due to the ongoing degradation of the world's ecosystems, there is an urgent need for sustainable development [6].

Recently, there has been a growing consciousness of both the universality and complexity of interactions between social and ecological systems. In particular, the complexity present in linked social-ecological systems has been investigated [1], [7], [8]. It has become clear that even simple interactions may cause changes which are unpredictable and not proportional to the causal factors. The question of the stability of the state of both ecosystems and social systems is of great practical importance, for such systems have been observed to fall irreversibly into degraded states. Cases of an abrupt shift in the state of an ecosystem to a completely different state, as a response to a small change in external conditions, are examples of this. This may occur in systems that have two (or more) different stable states and returning to the starting state would require a much bigger change in the external conditions that caused the shift (the effect of hysteresis) [2], [4]. The ability of a system to absorb external fluctuations and persist in its current state is called its resilience [3], [8]. Thus the more resilient a system is, the better also for its human participants.

Facing these contemporary problems, we need advanced methods to investigate the conditions that might cause nonlinear responses, especially for cases of irreversible change. Sometimes even a very simple model may provide useful insight into the nature of socio-ecological interactions. In this paper, we investigate a simple integrated model that reveals interesting properties in the context of agriculture and land cultivation.

In the next section, the model will be presented together with differential equations describing its dynamics. In section 3, possible stable states of such a system will be analyzed, while in section 4 its social and ecological properties will be studied. In section 5 some conclusions are given.

2. Description of the model

Let us examine a simple socio-ecological model that represents farmers who choose between intensive cultivation and abandoning their land, depending on the current profit achieved from cultivation. This profit depends on the level of self-regeneration by natural capital, which is decreased by cultivation.

Let us denote the intensively cultivated fraction of the land by I ($0 \le I \le 1$), and the fraction left fallow by $A = 1 - I$. We assume that farmers try to maintain a constant level of production on their farms depending on their land productivity Q and they abandon farming when it becomes unprofitable. The profit R_I from cultivation is the land productivity Q times price p less costs, plus a possible subsidy S_I paid when land is cultivated (hereafter, the subscript I corresponds to intensive cultivation, whereas the subscript A – to abandoning the land, leaving it fallow). Costs increase as natural capital N falls, which may even turn profit into losses:

$$R_I = Qp + S_I - C_I, \tag{1}$$

with the cost function defined as:

$$C_I = c\frac{1+h}{N+h}, \tag{2}$$

where c is a constant and h is a parameter describing how costs increase as the environment degrades. It is assumed to take a small value; when $h \to 0$, costs increase to infinity as the natural capital vanishes.

We assume that the productivity Q varies between farmers. For each farmer, it is the realization of a random variable with a logistic distribution around its mean value Q_0. The choice of the logistic distribution is dictated by its similarity in shape to the normal distribution. Thus it seems to be appropriate when modelling natural phenomena. One advantage it has over the normal distribution is that the logistic distribution, contrary to the normal distribution, has a cumulative distribution given in a simple analytical form. The probability density function for the logistic distribution, $f_{\log}$, is of the form:

$$f_{\log}(x) = \frac{\beta\exp(-\beta(x-Q_0))}{\left(1+\exp(-\beta(x-Q_0))\right)^2} \tag{3a}$$

and the cumulative distribution function is:

$$F_{\log}(x) = \frac{1}{1+\exp(-\beta(x-Q_0))}, \tag{3b}$$

where Q_0 is the mean value of productivity and the inverse of the parameter β measures the heterogeneity of the system. When $\beta \to \infty$, the probability of Q^i being different from Q_0 tends to zero. As β decreases, the heterogeneity of Q grows. For all values of β, the distribution is symmetric with respect to Q_0, i.e.,

$$\mathrm{P}(Q^i < Q_0) = \mathrm{P}(Q^i > Q_0) = \frac{1}{2}.$$

The only possible income for farmers who abandon their land is a possible subsidy, S_A:

$$R_A = S_A \tag{4}$$

We assume that land owners choose the state of their lands solely according to the level of achievable profits. Thus the probability that a randomly chosen farmer will switch from abandoning to cultivating the land equals:

$$\begin{aligned}
\mathrm{P}(A \to I) &= \mathrm{P}(R_I > R_A) = \mathrm{P}\left(Qp + S_I - c\frac{1+h}{N+h} > S_A\right) \\
&= \mathrm{P}\left(Q > \frac{S_A - S_I}{p} + \frac{c}{p}\frac{1+h}{N+h}\right) = 1 - \mathrm{P}\left(Q < \frac{S_A - S_I}{p} + \frac{c}{p}\frac{1+h}{N+h}\right) \\
&= 1 - \frac{1}{1+\exp\left(-\beta\left(\frac{S_A - S_I}{p} + \frac{c}{p}\frac{1+h}{N+h} - Q_0\right)\right)} \\
&= \frac{1}{1+\exp\left(-\beta\left(Q_0 + \frac{S_I - S_A}{p} - \frac{c}{p}\frac{1+h}{N+h}\right)\right)}
\end{aligned} \tag{5a}$$

and

$$\mathrm{P}(I \to A) = 1 - \mathrm{P}(A \to I). \tag{5b}$$

The fraction of intensively cultivated land will change in time as:

$$\begin{aligned}
\frac{dI}{dt} &= (1-I)\mathrm{P}(A \to I) - I\mathrm{P}(I \to A) = \mathrm{P}(A \to I) - I \\
&= \frac{1}{1+\exp\left(-\beta\left(Q_0 + \frac{S_I - S_A}{p} - \frac{c}{p}\frac{1+h}{N+h}\right)\right)} - I.
\end{aligned} \tag{6}$$

Natural capital takes values within the range $0 \le N \le K$, where K is the carrying capacity of the environment, which is defined as the environment's maximal load. We assume that natural capital renews itself logistically with growth factor r and is lowered by intensive agriculture at rate k:

$$\frac{dN}{dt} = rN\left(1 - \frac{N}{K}\right) - kI. \tag{7}$$

Taking into account equations (6) and (7), the model is described by the following set of equations:

$$\frac{dI}{dt} = \frac{1}{1+\exp\left(-\beta\left(Q_0 + \frac{S_I - S_A}{p} - \frac{c}{p}\frac{1+h}{N+h}\right)\right)} - I$$
$$\frac{dN}{dt} = rN\left(1-\frac{N}{K}\right) - kI. \tag{8}$$

In order to simplify these equations, we assume $K = 1$ (which is equivalent to substituting rK by r and N/K by N) and dimensionless notation is introduced:

1) $I_0 = \frac{r}{4k}$,

2) $\gamma = \frac{\beta c}{p}$,

3) $\rho = S_I - S_A + \rho_0 = \frac{S_I}{c} - \frac{S_A}{c} + \frac{Q_0 p}{c}$,

4) $f(N) = \rho - \frac{1+h}{N+h}$ – the expected scaled difference between the profits from cultivating and abandoning the land,

5) $f_0(N) = \rho_0 - \frac{1+h}{N+h}$ – the expected scaled bare difference between the profits from cultivating and abandoning the land, excluding subsides.

One can also notice that natural capital and the fraction of intensively cultivated land cannot be negative, i.e. once they reach zero they cannot decrease any further. Moreover, as we assumed above, only existing natural capital may cause its renewal, that is: $(dN/dt)\big|_{N=0} = 0$. Considering the fraction of cultivated land, $(dI/dt)\big|_{I=0} \geq 0$.

Thus the system of Eqs. (8) holds when N and I are positive. As for I, one can see that the right hand sides of (8) will be nonnegative for $I = 0$. Thus, in dimensionless units our model's dynamics finally is given by:

$$\begin{cases} \frac{dN}{dt} = k\left(4I_0 N(1-N) - I\right) \quad \text{for} \quad N > 0 \quad \text{and} \quad \frac{dN}{dt} = 0 \quad \text{for} \quad N = 0 \\ \frac{dI}{dt} = \frac{1}{1+\exp\left(-\gamma\left(\rho - \frac{1+h}{N+h}\right)\right)} - I \quad \text{for} \quad I \geq 0. \end{cases} \tag{9}$$

Using the following function:

$$Y(x)=\begin{cases}1 & \text{if} \quad x>0\\ 0 & \text{if} \quad x\le 0\end{cases}$$

we may rewrite the dynamics of our model as:

$$\begin{cases}\dfrac{dN}{dt}=Y\big(Y(N)+Y(u_N)\big)u_N\\ \dfrac{dI}{dt}=u_I\end{cases} \tag{10a}$$

where

$$\begin{cases}u_N=k\big(4I_0N(1-N)-I\big)\\ u_I=\dfrac{1}{1+\exp\left(-\gamma\left(\rho-\dfrac{1+h}{N+h}\right)\right)}-I.\end{cases} \tag{10b}$$

The above set of two coupled equations completely describes the dynamics of our socio-economic-ecological model.

3. Analysis of steady states

Let us analyze the fixed points of the system described by equation (10). The condition for equilibrium is:

$$\frac{dN}{dt}=\frac{dI}{dt}=0. \tag{11}$$

If an equilibrium is to persist for a long time, then it has to be a stable one. That is, after any small deviation, the system must return to this state. Any other states will evolve in time to eventually reach one of the steady states.

Inserting condition (11) into (10), one obtains the following two conditions for equilibrium states:

$$\begin{cases}\dfrac{dI}{dt}=0 \quad \Leftrightarrow \quad I=\dfrac{1}{1+\exp\left(-\gamma\left(\rho-\dfrac{1+h}{N+h}\right)\right)}\\ \dfrac{dN}{dt}=0 \quad \Leftrightarrow \quad I=4I_0N(1-N) \quad \text{or} \quad N=0\end{cases}. \tag{12}$$

For $N^* = 0$, there exists an equilibrium with:

$$I^* = \frac{1}{1+\exp\left(-\gamma\left(\rho-1-\frac{1}{h}\right)\right)}$$

(hereafter equilibrium states, fulfilling condition (11), will be denoted with an asterisk). This solution exists for all possible choices of the parameters' values and corresponds to "breakdown" of the system. For $h \to 0$ (as mentioned above, this corresponds to infinitely increasing costs with vanishing natural capital), the breakdown of the ecological component of the system is associated with the simultaneous breakdown of the economic system: $I^* = N^* = 0$.

Another family of equilibrium states is given by the solution of the equation:

$$4I_0 N(1-N) = \frac{1}{1+\exp\left(-\gamma(\rho)-\frac{1+h}{N+h}\right)}. \tag{13}$$

This equation cannot be solved analytically; therefore we will perform a graphical analysis of the number and character of its solutions.

Let us denote the function $I(N)$ resulting from the condition for the stationarity of I as $I_I(N)$:

$$I_I(N) = \frac{1}{1+\exp\left(-\gamma\left(\rho-\frac{1+h}{N+h}\right)\right)}$$

and the one resulting from the condition for the stationarity of N as $I_N(N)$:

$$I_N(N) = 4I_0 N(1-N).$$

Thus, graphically, the equilibria are determined by the intersection points of $I_I(N)$ and $I_N(N)$. The plot of the function $I_N(N)$ is a parabola with a maximum at $N = 1/2$, $I = I_0$. For $N < 1/2$ it describes the solution N_- of the condition $dN/dt = 0$ and for N >1/2 – the solution N_+. As for the function $I_I(N)$, it is an increasing function of N:

$$\frac{dI_I(N)}{dN} = \frac{\gamma(1+h)}{4(N+h)^2\cosh^2\left(\frac{1}{2}\gamma\left(\rho-\frac{1+h}{N+h}\right)\right)} > 0,$$

and takes values within the range (0, 1): $0 < I_I(N) < 1$; its plot has one inflection point.

Therefore, typically there may be zero (figure 1), two (figures 2, 3) or four (figures 4, 5) intersections of the curves $I_I(N)$ and and $I_N(N)$. The rightmost intersection point may satisfy $N \geq 1/2$ (figures 2, 4) or all the intersection points may satisfy $N < 1/2$ (figures 3, 5). At the bifurcation points, where two neighbouring intersection points join and form a point of oscillation, there are an odd number of intersection points. Note that in all cases there exists a trivial fixed point $I_I = 0$.

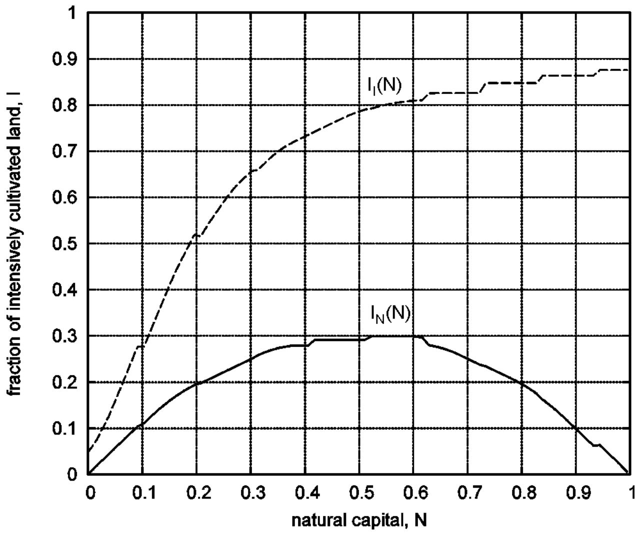

Fig. 1. Graphical solution of the equation $I_I(N) = I_N(N)$ for $I_0 = 0.3$, $\gamma = 1$, $\rho = 3$, $h = 0.2$

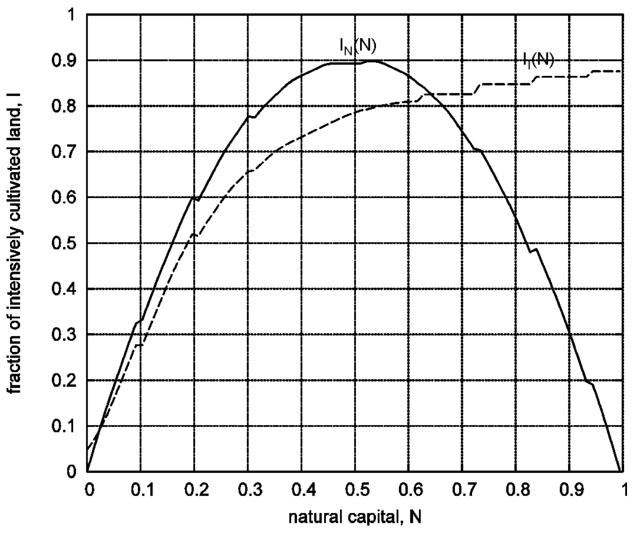

Fig. 2. Graphical solution of the equation $I_I(N) = I_N(N)$ for $I_0 = 0.9$, $\gamma = 1$, $\rho = 3$, $h = 0.2$

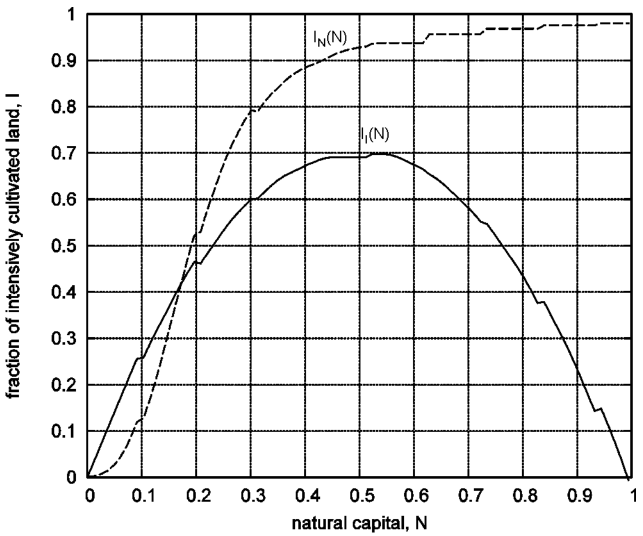

Fig. 3. Graphical solution of the equation $I_I(N) = I_N(N)$ for $I_0 = 0.7$, $\gamma = 2$, $\rho = 3$, $h = 0.2$

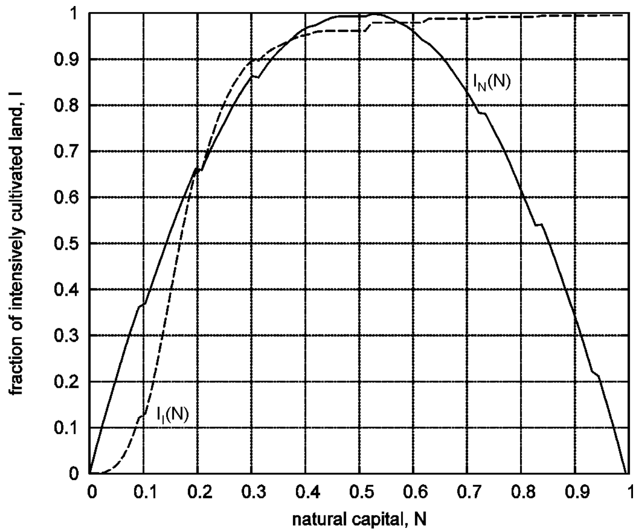

Fig. 4. Graphical solution of the equation $I_I(N) = I_N(N)$ for $I_0 = 1.0$, $\gamma = 2.5$, $\rho = 3.2$, $h = 0.2$

If an equilibrium state is to be steady, small perturbations must be followed by a return to the equilibrium point. The curves in figures 1–5 partition the phase space (N, I) according to the signs of the two components of the velocity vector of the system (cf. figure 6)

$$\bar{v} \equiv (v_N, v_I)^T = \left(\frac{dN}{dT}, \frac{dI}{dt}\right)^T .$$

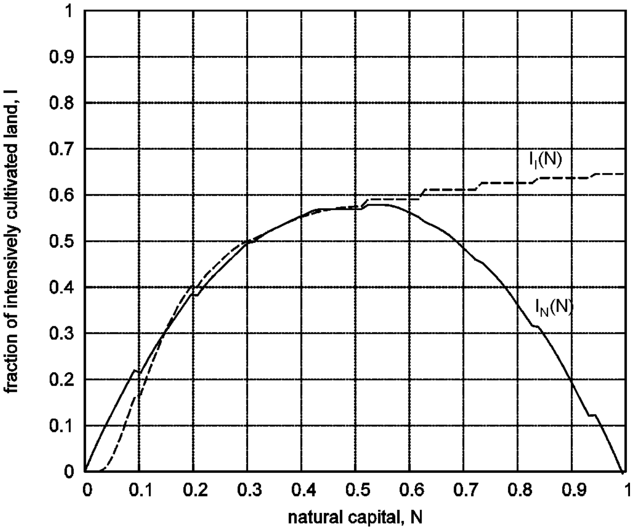

Fig. 5. Graphical solution of the equation $I_I(N) = I_N(N)$ for $I_0 = 0.58$, $\gamma = 0.28$, $\rho = 3.2$, $h = 0.01$

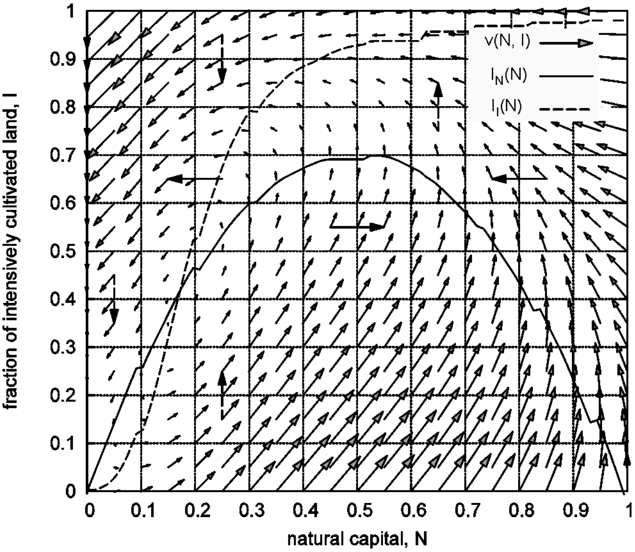

Fig. 6. Velocity field $\overline{v}(N, I)$ for the parameters as in Fig 4; $k = 1$; the vectors are scaled by a factor of 0.08. The black solid and dashed arrows show the signs of the components of the velocity vector in the areas delimited by the respective curves

Let us notice that below the curve $I_N(N)$ we have $v_N > 0$ and $v_N < 0$ and above it while below the curve $I_I(N)$ we have $v_I > 0$ and $v_I < 0$ above it.

Determining the signs of these components often allows us to quickly determine the character of an equilibrium point simply from the plot of the velocity field $\overline{v}(N,I)$. Such a crude analysis suggests that equilibrium points with $N > 1/2$ (on the decreasing part of the curve $I_I(N)$) will be attractive ones (thus stable) and for $N < 1/2$ (on the increasing part of the curve $I_I(N)$) – repelling ones (thus unstable), apart from the trivial equilibrium, which is stable. However, the situation is not that simple. A more precise analysis requires expanding the velocity vector around the equilibrium point $\overline{x}^* = (N^*, I^*)^T$. Summarizing this, for $N > 1/2$, equilibria will be indeed always stable, while for $N < 1/2$ not all equilibrium points will be stable.

4. Ecological and economic properties of the system

From ecological and economic points of view, the best functioning systems seem to be characterized by the following conditions:

- the system should be in a stable state, so that small variations in N and I do not drive it to another stable state;
- small variations in the parameters should not push the system far from its initial state;
- the value of natural capital should be as large as possible;
- profits from cultivation – possibly without taking subsidies into regard – should be as large as possible, and, by necessity, positive.

For the first three requirements to be satisfied, it is enough that $N^* \geq 1/2$, as such states are attractive (that is, small deviations in the state of the system will lead to it returning to the starting point) and are attractive for all values of the parameters (that is, changing the parameters will not change the stable character of the system's state) (see [9]).

The expected profit from intensive cultivation is given by:

$$R_I(N^*) - R_A(N^*) = cf(N^*) = \frac{p}{\beta}\ln\frac{I^*}{1-I^*} = \frac{p}{\beta}\ln\frac{4I_0N^*(1-N^*)}{1-4I_0N^*(1-N^*)} \tag{14a}$$

(for the definition of the function f see point 5 in Section 2), or, including subsidies:

$$\begin{aligned} R_I(N^*) - R_A(N^*) - (S_I - S_A) &= cf_0(N^*) = \frac{p}{\beta}\ln\frac{I^*}{1-I^*} - (S_I - S_A) \\ &= \frac{p}{\beta}\ln\frac{4I_0N^*(1-N^*)}{1-4I_0N^*(1-N^*)} - (S_I - S_A). \end{aligned} \tag{14b}$$

Now we want to obtain a steady state with $N^* \geq 1/2$, and N^*, (14a) and (14b) as large as possible. For sure, we will have to somehow compromise here.

Fixed points with $N^* \geq 1/2$ will exist if $I_I(N = 1/2) < I_N(N = 1/2)$. As we know that $I_N(1/2)\ I_0 = r/4k$, this has to be satisfied:

$$\frac{1}{1+\exp\left(-\gamma\left(\rho-\frac{1+h}{\frac{1}{2}+h}\right)\right)} \leq I_0. \tag{15}$$

For $I_0 \geq 1$, this is obviously always satisfied; otherwise, for $I_0 < 1$, the following must hold:

$$\rho \leq \frac{1}{\gamma}\ln\frac{I_0}{1-I_0}+\frac{1+h}{\frac{1}{2}+h}. \tag{16}$$

For (14a) and (14b) to be satisfied, it is necessary that:

$$I^* = 4I_0N^*(1-N^*) > \frac{1}{2} \tag{17a}$$

and

$$I^* = 4I_0N^*(1-N^*) > \frac{1}{1+\exp\left(-\frac{\gamma}{p}(S_I-S_A)\right)} \tag{17b}$$

respectively. Moreover, the greater I^*, the better (in terms of maximizing profits), since the values of the profit functions defined in (14a, b) are increasing in I^*. Let us finally establish conditions on the parameter values for the profits defined in (14a, b) to be positive. As the derivatives of the profit functions with respect to N^* are positive for $N^* \geq 1/2$, thus these functions are increasing in this range and $f(1/2) > 0$ for (14a) (or $f_0\,(1/2) > 0$ for (14b)) is a sufficient condition for profits to be positive. Substituting $N = 1/2$ into (12a), one gets the conditions:

$$\rho > \frac{1+h}{\frac{1}{2}+h}$$

or

$$\rho > \frac{1+h}{\frac{1}{2}+h} + S_I - S_A,$$

for (14a) or (14b) to be satisfied, respectively.
Together with condition (16), this gives:

$$0 < \rho - \frac{1+h}{\frac{1}{2}+h} \le \frac{1}{\gamma}\ln\frac{I_0}{1-I_0} \tag{18a}$$

or

$$S_I - S_A < \rho - \frac{1+h}{\frac{1}{2}+h} \le \frac{1}{\gamma}\ln\frac{I_0}{1-I_0} \tag{18b}$$

for (14a) or (14b) to be satisfied, respectively.

Either one of conditions (18) ensures the existence of a stable and profitable (in either sense) system. Not fulfilling these conditions does not exclude such a possibility, but the detailed analysis of such a case requires numerical analysis.

5. Summary and conclusions

Properties of a simple socio-economic system have been investigated, in particular the properties of equilibrium states. It has revealed a great variety of possible outcomes, depending on the external parameters. Several fixed points may exist, and some of them may be stable. Moreover, some stable fixed points represent a profitable system with a relatively high level of natural capital, provided that the parameters fulfil some specified conditions. Since there also exists a stable state corresponding to the total collapse of both the economic and ecological subsystems, it can be seen how, changing external parameters, such as prices or subsidies, unwelcome state can be avoided.

Agricultural sustainability is a growing challenge in global politics. The dynamic complexity of agricultural systems, underlying bifurcations and the possibility of multiple stability domains amplifies this challenge by making the system behaviour counterintuitive. It is well documented that in complex systems decision making is often distorted and learning hampered [5]. Simple models, sometimes called “toy models”, such as the one described in this paper, do not provide simple answers to these problems. However, they at least give some insight, helping to realize the range of possible scenarios and improve our intuition about what might happen in real systems.

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