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SAMPLING INSPECTION PLANS FROM NUMERICAL POINT OF VIEW**

The paper concerns the acceptance sampling plans when the remainder of rejected lots is inspected. Two types of AOQL plans are considered – for inspection by variables and for inspection by variables and attributes (all items from the sample are inspected by variables, the remainder of rejected lots is inspected by attributes). These plans are compared with the corresponding Dodge–Romig AOQL plans for inspection by attributes. An algorithm allowing the calculation of these plans (with the use of software Mathematica) was presented. From the results of numerical investigations it follows that under the same protection of consumer the AOQL plans for inspection by variables are in many situations more economical than the corresponding Dodge–Romig attribute sampling plans.

Keywords: *AOQL, sampling plans, acceptance plan, economical aspects, software Mathematica*

1. Introduction

In [2] sampling plans are considered which minimize the mean number of items inspected per lot of process average quality, assuming that the remainder of rejected lots is inspected

$$I_s = N - (N - n) \cdot L(\overline{p}; n, c) \tag{1}$$

under the condition

$$\max_{0<p<1} \mathrm{AOQ}(p) = p_L \tag{2}$$

(AOQL single sampling plans), or under the condition $L(p_t;n,c) = 0.10$ (LTPD single sampling plans), where N is the number of items in the lot (the given parameter), $\overline{p}$ is

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the process average fraction defective (the given parameter), p_L is the average outgoing quality limit (the given parameter, denoted AOQL), p_t is the lot tolerance fraction defective (the given parameter, denoted LTPD), n is the number of items in the sample ($n < N$), c is the acceptance number (the lot is rejected when the number of defective items in the sample is greater than c), $L(p;\ n,\ c)$ is the operating characteristic (the probability of accepting a submitted lot with fraction defective p), AOQ(p) is the average outgoing quality (the mean fraction defective after inspection when the fraction defective before inspection was p). The average outgoing quality (all defective items found are replaced by good ones) is approximately

$$\mathrm{AOQ}(p) = \left(1 - \frac{n}{N}\right) pL(p;n,c)\,. \tag{3}$$

Therefore condition (2), which protects the consumer against the acceptance of a bad lot, can be rewritten as

$$\max_{0<p<1}\left(1 - \frac{n}{N}\right) pL(p;n,c) = p_L\,. \tag{4}$$

The Dodge-Romig LTPD and AOQL plans can be used under the assumption that each inspected item is classified as either good or defective (acceptance sampling by attributes). The problem to find LTPD and AOQL plans for inspection by variables has been solved in earlier papers, see [4] and [5]. In this paper, we shall report on an algorithm allowing calculation of two types of AOQL plans[1]:

a) For inspection by variables – all items from the sample and all items from the remainder of rejected lot are inspected by variables.

b) For inspection by variables and attributes – all items from the sample are inspected by variables, but the remainder of rejected lots is inspected by attributes only.

Solution to the problem of finding the AOQL plans by variables and AOQL plans by variables and attributes is considerably difficult. We shall use an original method.

2. AOQL plans by variables and comparison with the Dodge-Romig plans

In this paper, it will be assumed that measurements of a single quality characteristic X are independent, identically distributed normal random variables with unknown parameters μ and σ^2. For the quality characteristic X is given either an upper specifi-

[1] For calculation of the LTPD plans by variables and the LTPD plans by variables and attributes and tables of these plans, see [6] and [7].

cation limit U (the item is defective if its measurement exceeds U), or a lower specification limit L (the item is defective if its measurement is smaller than L). It is further assumed that the unknown parameter σ is estimated from the sample standard deviation s (unknown standard deviation plans), no use is made of the average range as an estimator of σ. The inspection procedure is as follows (e.g. [1]):

1. Draw a random sample of n items and compute

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i \,, \quad s = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2} \,. \tag{5}$$

2. Compute $\frac{U-\bar{x}}{s}$ for an upper specification limit, or $\frac{\bar{x}-L}{s}$ for a lower specification limit.

3. Accept the lot if

$$\frac{U-\bar{x}}{s} \geq k \quad \text{or} \quad \frac{\bar{x}-L}{s} \geq k \,. \tag{6}$$

The problem is to determine the sample size n and the critical value k. There are different solutions to this problem. In the present paper we shall look for the acceptance plan *(n,k)* minimizing the mean inspection cost per lot of process average quality C_{ms} under the condition (4). Inspection cost per lot, assuming that the remainder of rejected lots is inspected by attributes *(the inspection by variables and attributes),* is nc_m^* with probability $L(p; n, k)$ and $[nc_m^* + (N-n)c_s^*]$ with probability $[1-L(p; n, k)]$, where c_s^* is the cost of inspection of one item by attributes, and c_m^* is the cost of inspection of one item by variables. The mean inspection cost per lot of process average quality is therefore

$$C_{ms} = nc_m^* + (N-n)\, c_s^* \cdot [1 - L(\bar{p}; n, k)] \,. \tag{7}$$

Let us denote

$$c_m = \frac{c_m^*}{c_s^*} \,. \tag{8}$$

Now, we shall look for the acceptance plan (n, k) minimizing

$$I_{ms} = nc_m + (N-n)\,[1 - L(\bar{p}; n, k)] \tag{9}$$

instead of C_{ms} (both functions C_{ms} and I_{ms} have a minimum for the same acceptance plan, $C_{ms} = I_{ms}\, c_s^*$) under the condition

$$\max_{0<p<1}\left(1 - \frac{n}{N}\right) pL(p; n, k) = p_L \,. \tag{10}$$

For these AOQL plans for inspection by variables and attributes (the type (b)) *the new parameter* c_m was defined, see (8). This parameter must be statistically estimated in each real situation. Usually, there is

$$c_m > 1\,. \tag{11}$$

Putting formally c_m = 1 into (9) (I_{ms} in this case is denoted by I_m) we obtain

$$I_m = N - (N-n)\cdot L(\overline{p};n,k)\,, \tag{12}$$

i.e., the mean number of items inspected per lot of process average quality, assuming that both the sample and the remainder of rejected lots are inspected by variables. Consequently we shall study *the AOQL plans for inspection by variables* (the type(a)) as a special case of *the AOQL plans by variables and attributes* for c_m = 1. From (12) it is evident that for the determination of AOQL plans by variables it is not necessary to estimate c_m (c_m = 1 is not a real value of this parameter).

Summary: For the given parameters N, $\overline{p}$, p_L and c_m we must determine the acceptance plan (n, k) for inspection by variables and attributes, minimizing I_{ms} in (9) under the condition (10).

First, we shall deal with the solution of equation (10). The operating characteristic, using the normal distribution as an approximation of the non-central t distribution (see [3]), is

$$L(p;n,k) = \Phi\left(\frac{u_{1-p}-k}{A}\right), \tag{13}$$

where

$$A = \sqrt{\frac{1}{n} + \frac{k^2}{2(n-1)}}\,. \tag{14}$$

The function Φ in (13) is a standard normal distribution function and u_{1-p} is a quantile of order 1 – p, i.e., $\Phi(u) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{u} \exp(-x^2/2)\,dx$, $u_{1-p} = \Phi^{-1}(1-p)$ (the unique root of the equation $\Phi(u)$ = 1 – p). The approximation (13) holds both for an upper specification limit U and for a lower specification limit L. The equation (10), using (13), has an (approximately) equivalent form

$$\max_{0<p<1} p\cdot\Phi\left(\frac{u_{1-p}-k}{A}\right) = \frac{p_L}{1-\frac{n}{N}}\,. \tag{15}$$

Let us denote

$$G(p;n,k) = p\Phi\left(\frac{u_{1-p}-k}{A}\right), \quad M(n,k) = \max_{0<p<1} G(p;n,k)\,. \tag{16}$$

Let n, N, p_L be given parameters (for the given n we shall write $M_n(k)$ instead of $M(n, k)$). At first we shall look for the critical value k for which (15) holds, i.e.,

$$M_n(k) = p_L / \left(1 - \tfrac{n}{N}\right). \tag{17}$$

Theorem 1. Let n, N, p_L be given parameters, $p_L < \frac{1}{4} - \frac{7}{4N}$. If

$$n \in \langle 7, (1-4p_L)N \rangle , \tag{18}$$

then each solution k of equation (17) is nonnegative, i.e., $k \geq 0$.

Proof. If $k < 0$, then $L(\frac{1}{2}) = \Phi(-k/A) > \frac{1}{2}$ and $M_n(k) > \frac{1}{4}$, but the right hand side of (17) is for $n \in \langle 7, (1-4p_L)N \rangle$ less or equal to $\frac{1}{4}$. □

Remark 1. The assumption (18) is not limiting one from practical point of view. From numerical investigations it follows that for most of the given parameters N, $\overline{p}$, p_L and c_m the assumption (18) is valid. If assumption (18) is not valid (very small lots), AOQL plans for inspection by variables and attributes are not considered for economical reasons.

Let us denote[2]

$$K_n = \{k \geq 0; M_n(k) \geq p_L\} . \tag{19}$$

Theorem 2. Let p_L be the given parameter, $n \in \langle 7, (1-4p_L)N \rangle$. If for n,

$$\Phi \left(-(n-1)\sqrt{\tfrac{2}{n}}\right) \leq p_L , \tag{20}$$

holds, then the function $M_n(k)$ is decreasing in K_n.

Proof. See [8]. □

Remark 2. For usually chosen p_L the assumption (20) holds. The left hand side of (20) is decreasing function of n and for $n = 7$ the left hand side of (20) is approximately 0.0007 (minimum value of AOQL in [2] is $p_L = 0.001$).

From Theorem 2 it follows that each solution of equation (17) is unique. Since an explicit formula for k does not exist, we have to solve (17) numerically. We use Newton's method[3], therefore we must determine $M_n(k)$ and the derivative $M_n'(k)$. According to (16) one has

$$M_n(k) = p_M \Phi\left(\frac{u_{1-p_M} - k}{A}\right), \tag{21}$$

[2] If $k \notin K_n$, then k is not a solution of equation (17).

[3] Numerical investigations show that the function $M_n(k)$ is also convex in K_n (if we choose start value $k_0 = 0$, then Newton's method is always convergent).

where $p_M \in (0,1)$ is the value of p, for which the function $G(p;\, n,\, k)$ in (16) has a maximum. Evidently, it holds that $G(0;\, n,\, k) = G(1;\, n,\, k) = 0$ and $G(p;\, n,\, k) > 0$ for $p \in (0,\, 1)$. Since the function $G(p;\, n,\, k)$ is continuous for $p \in \langle 0,\, 1\rangle$, the value p_M exists. We determine the value p_M as a solution of the equation $G'(p) = 0$, i.e.,

$$\Phi\left(\frac{u_{1-p}-k}{A}\right)-\frac{p}{A}\exp\left[-\frac{1}{2A^2}[(1-A^2)\,u_{1-p}^2-2ku_{1-p}+k^2]\right]=0\,. \tag{22}$$

Theorem 3. Let n be the given parameter, $n\in\langle 7,(1-4p_L)N\rangle$, $k_r=(n-1)\sqrt{\frac{2}{n}}$. If $k = k_r$, then $p_M=\Phi\left(-\frac{k_r}{2}\right)$ is a solution of equation (22).

Proof. For $k_r=(n-1)\sqrt{\frac{2}{n}}$ one obtains $A = 1$. Since $u_{1-p_M}=\frac{k_r}{2}$, it is evident that $p_M=\Phi\left(-\frac{k_r}{2}\right)$ satisfies equation (22). □

Theorem 4. Let n be the given parameter, $n\in\langle 7,(1-4p_L)N\rangle$, $k_r=(n-1)\sqrt{\frac{2}{n}}$. If $k\in\langle 0,\infty\ \rangle-\{k_r\}$, then solution p_M of equation (22) is between p_a and p_r, where

$$p_a=\Phi\left(\frac{-k-A\sqrt{k^2-2(1-A^2)\ln A}}{1-A^2}\right),\quad p_r=\Phi\left(-\frac{k}{1+A}\right). \tag{23}$$

Proof. See [8]. □

Instead of p_M we shall look for $x_M=u_{1-p_M}$ $(p_M=\Phi(-x_M))$ as a solution of the equation $G'(x) = 0$, i.e.,

$$\Phi\left(\frac{x-k}{A}\right)-\frac{\Phi(-x)}{A}\cdot\exp\left[-\frac{1}{2A^2}[(1-A^2)x^2-2kx+k^2]\right]=0\,. \tag{24}$$

The equation (24) must be solved once more numerically. From Figure 1, it is evident that numerical solution of the equation $G'(x) = 0$ depends on good first approximation x_0. Under assumptions of Theorem 4, solution x_M of equation (24) is between x_r and x_a, where

$$x_r=\frac{k}{1+A},\quad x_a=\frac{k+A\sqrt{k^2-2(1-A^2)\ln A}}{1-A^2}\,. \tag{25}$$

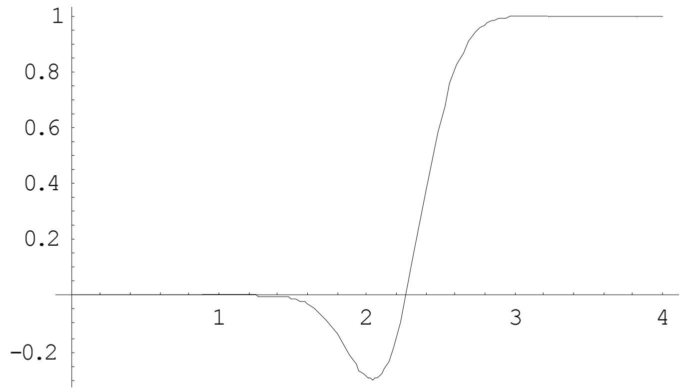

Fig. 1. The function $G'(x)$ for $n = 60$ and $k = 2.2$

Using (25) we choose for x_0 the following point (numerical investigations show that this point is a good start value for solution of equation (24))

$$x_0 = \frac{(100+n)x_r + nx_a}{2n+100}. \tag{26}$$

If we find x_M for which (24) holds, then we determine $M_n(k)$ from the formula

$$M_n(k) = \Phi(-x_M)\,\Phi\left(\frac{x_M - k}{A}\right) \tag{27}$$

and the derivative $M'_n(k)$ from the formula[4]

$$M'_n(k) = -\frac{\Phi(-x_M)}{A^3\sqrt{2\pi}} \cdot \left[\frac{1}{n} + \frac{kx_M}{2(n-1)}\right] \exp\left[-\frac{1}{2A^2}(x_M - k)^2\right]. \tag{28}$$

Determination of the acceptance plans (n, k) for which (17) holds is in comparison with the solution of the equation $L(p_t; n, c) = 0.10$ in a previous paper (see [4]) considerably more difficult. From these plans we must choose the acceptance plan (n, k) minimizing $I_{ms} = nc_m + (N-n)\,\alpha$, where

$$\alpha = 1 - L(\overline{p}; n, k) = \Phi\left(\frac{k - u_{1-\overline{p}}}{A}\right) \tag{29}$$

is producer's risk (the probability of rejecting a lot of process average quality). We shall solve this problem once more numerically.

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[4] We obtain $M'_n(k)$ from (21) using the fact that (20) holds for x_M.

For the comparison of AOQL plans by variables and AOQL plans by variables and attributes with the Dodge-Romig plans from an economical point of view we use parameters E and e defined by relations

$$E = \frac{I_m}{I_s} 100 , \quad e = \frac{I_{ms}}{I_s} 100 . \tag{30}$$

If c_m is statistically estimated and $Ec_m < 100$, then the AOQL plans for inspection by variables are more economical than the corresponding Dodge–Romig AOQL plans. The AOQL plans for inspection by variables and attributes are more economical than the corresponding Dodge–Romig plans when $e < 100$ (see [5]).

3. Numerical solution

For calculation of the AOQL plans by variables and attributes we shall use software Mathematica, see [9].

Example. Let N =1000, p_L = 0.0025, $\bar{p}$ = 0.001 and c_m = 1.8 (the cost of inspection of one item by variables is higher by 80% than the cost of inspection of one item by attributes). We shall look for the AOQL plan for inspection by variables and attributes. Furthermore we shall compare this plan and the corresponding Dodge-Romig AOQL plan for inspection by attributes.

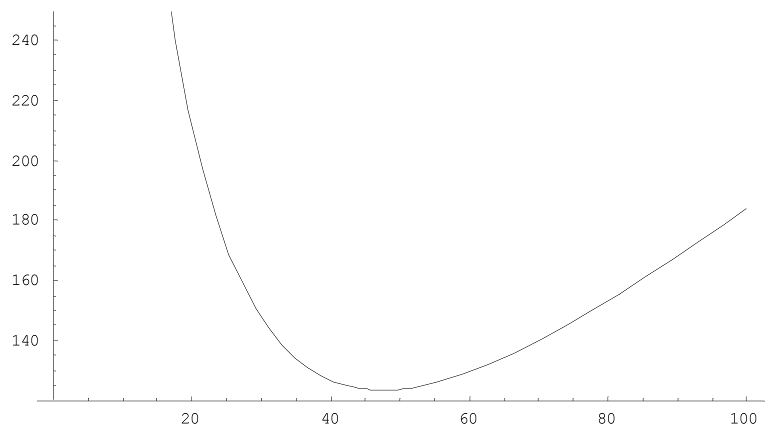
According to (14), (24), (25) and (26) we have

```
In[1]:= << Statistics`NormalDistribution`

In[2]:= ndist = NormalDistribution[0, 1]

In[3]:= cm = 1.8

In[4]:= pL = 0.0025

In[5]:= pbar = 0.001

In[6]:= nbig = 1000

In[7]:= A[n_, k_] := Sqrt[1/n + k^2/(2n - 2)]

In[8]:= G'[x_, n_, k_] := CDF[ndist, (x - k)/A[n, k]] - CDF[ndist,
                          -x]* Exp[-((1 - A[n, k]^2) x^2 - 2k x +
                          k^2)/ (2A[n, k]^2)]/A[n, k]
```

```
In[9]:= xr[n_, k_] := k/(1 + A[n, k])

In[10]:= xa[n_, k_] := (k + A[n, k]*Sqrt[k^2 - 2(1 - A[n, k]^2)*
                    Log[A[n, k]]])/(1 - A[n, k]^2)

In[11]:= x0[n_, k_] := ((100 + n)*xr[n, k] + n*xa[n, k])/(2n + 100)

In[12]:= FR[n_, k_] := FindRoot[G'[x, n, k] == 0, {x, x0[n, k]}]

In[13]:= xM[n_, k_] := x /. FR[n, k]
```

Using Newton's method (see (27) and (28)) with start point o = 1.6 and (29) we have

```
In[14]:= c[n_, k_] := -(CDF[ndist, -xM[n, k]]*CDF[ndist, (xM[n, k]
                    -k)/A[n, k]] - pL/(1 - n/nbig))/ (-
                    CDF[ndist, -xM[n,k]]*(1/n + k xM[n, k]/ (2n
                    - 2))*Exp[-(xM[n, k] - k)^2/(2A[n, k]^2)]/
                    (A[n, k]^3*Sqrt[2Pi]))
In[15]:= o = 1.6

In[16]:= fRecAux[n_,i_]:= fRecAux[n,i]=fRecAux[n,i-1]+c[n,
           fRecAux[n,i-1]]; fRecAux[n_,0]=o

In[17]:= k[n_]:=fRecAux[n,7]

In[18]:= a[n_] := CDF[ndist, (k[n] - Quantile[ndist, 1 - pbar])
                /Sqrt[1/n + k[n]^2/(2n - 2)]]

In[19]:= Ims[n_] := n cm + (nbig - n)*a[n]

In[20]:= Plot[Ims[n], {n, 10, 100}]
```



```
In[21]:= Table[{n, k[n], Ims[n]}, {n, 40, 50, 1}]

In[22]:= TableForm[%]

Out[22]//TableForm=
           40 2.56734 126.755
 41 2.56613 125.879
 42 2.56501 125.157
 43 2.56397 124.579
 44 2.56302 124.135
 45 2.56214 123.815
 46 2.56133 123.61
 47 2.56058 123.515
 48 2.55988 123.52
 49 2.55923 123.621
 50 2.55863 123.81
```

The AOQL plan for inspection by variables and attributes is (minimum of the function I_{ms} is $I_{ms} = 123.515$)

$$n_1 = 47, \quad k = 2.56058.$$

The corresponding AOQL plan for inspection by attributes can be found in [2]. For given parameters N, p_L and $\overline{p}$ we have

$$n_2 = 130, \quad c = 0.$$

For the comparison of these two plans from an economical point of view we use parameter e (see (30)). The Mathematica gives

```
In[23]:= n1 = 47

In[24]:= k = 2.56058

In[25]:= Ims = 123.515

In[26]:= n2 = 130

In[27]:= c = 0

In[28]:= e = 100*Ims/(nbig - (nbig - n2)Sum[Binomial[nbig*pbar, i]*
             Binomial[nbig - nbig*pbar, n2 - i]/Binomial[nbig, n2],
             {i, 0, c}])

Out[28]:= 50.8083
```

Since e = 50.8083%, using the AOQL plan for inspection by variables and attributes (47, 2.56058) there can be expected approximately 49% saving of the inspection cost in comparison with the corresponding Dodge–Romig plan (130, 0).

Further we compare the operating characteristics of these plans (see (13))

```
In[29]:= L1[p_] := CDF[ndist, (N[Quantile[ndist, 1 - p], 16] - k)/
              Sqrt[1/n1 + k^2/(2*n1 - 2)]]
```

```
In[30]:= L2[p_] :=Sum[Binomial[nbig*p,i]* Binomial[nbig-nbig*p,n2-i]/
               Binomial[nbig,n2],{i,0,c}]
```

```
In[31]:= Table[{p, N[L1[p], 5], N[L2[p], 5]}, {p, 0.001, 0.031,
0.002}]
```

```
In[32]:= TableForm[%]
```

```
Out[32]//TableForm=
```

0.001	0.959165	0.87
0.003	0.730845	0.658207
0.005	0.51999	0.497674
0.007	0.36707	0.376067
0.009	0.260801	0.284003
0.011	0.187205	0.214346
0.013	0.135854	0.161675
0.015	0.0996376	0.121872
0.017	0.0738028	0.0918112
0.019	0.0551687	0.0691225
0.021	0.0415875	0.0520083
0.023	0.0315927	0.039107
0.025	0.0241711	0.0293876
0.027	0.0186145	0.0220699
0.029	0.0144223	0.0165638
0.031	0.0112372	0.0124235

For example, we get $L_1(\overline{p})$ = $L_1(0.001)$ = 0.959165, i.e., the producer's risk for the AOQL plan for inspection by variables and attributes is therefore approximately

$$\alpha = 1 - L_1(\overline{p}) = 0.04.$$

The producer's risk for the corresponding Dodge–Romig plan is

$$\alpha = 1 - L_2(\overline{p}) = 1 - 0.87 = 0.13.$$

Finally, let us present the graphic comparison of the operating characteristics of these plans (see Figure 2):

```
In[33]:= oc1 = Plot[L1[p], {p, 0, 0.03}, AspectRatio -> 1.3,
                AxesLabel -> {"p", "L(p)"}]

In[34]:= oc2 = Plot[L2[p], {p, 0, 0.03}, AspectRatio -> 1.3,
                AxesLabel -> {"p", "L(p)"}]

In[35]:= Show[oc1, oc2]
```

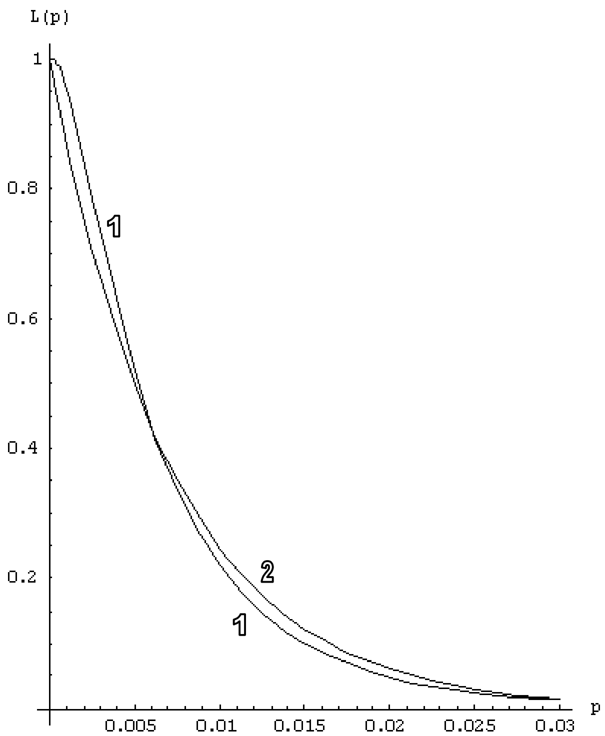


Fig. 2. OC curves for the AOQL sampling plans:
1 – for inspection by variables and attributes (47, 2.56058),
2 – for inspection by attributes (130, 0)

Conclusion

From these results it follows that the AOQL plan for inspection by variables and attributes is more economical than the corresponding Dodge-Romig AOQL attribute sampling plan (49% saving of the inspection cost). Furthermore the OC curve for the AOQL plan by variables and attributes is better than corresponding OC curve for the AOQL plan by attributes, see Figure 2 (for example, the producer's risk for the AOQL plan by variables and attributes $\alpha = 0.04$ is less than that for the corresponding Dodge–Romig plan $\alpha = 0.13$).

References

[1] COWDEN D. J., *Statistical Methods of Quality Control*, Prentice-Hall, Englewood Cliffs, New Jersey 1957.

[2] DODGE H. F., ROMIG H. G., *Sampling Inspection Tables*, John Wiley, New York 1959.

[3] JOHNSON N. L., WELCH B. L., *Applications of the non-central t-distribution*, Biometrika, 1940, 31, 362–389.

[4] KLŮFA J., *Acceptance sampling by variables when the remainder of rejected lots is inspected*, Statistical Papers, 1994, 35, 337–349.

[5] KLŮFA J., *Dodge–Romig AOQL single sampling plans for inspection by variables*, Statistical Papers, 1997, 38, 111–119.

[6] KLŮFA J., *Tables of sampling inspection plans by variables*, Ekopress, Prague 2000.

[7] KLŮFA J., PETRÁŠKOVÁ V., *Sampling Acceptance Plans for Practical Purposes,* Operations Research and Decision 3–4, 2002, 85–92.

[8] LIKEŠ J., KLŮFA J., JAROŠOVÁ E., *Statistical Methods in Reliability and Quality Control*, University of Economics, Prague 1991.

[9] WOLFRAM S ., *Mathematica*, Addison–Wesley, 1991.

Statystyczne plany odbiorcze z numerycznego punktu widzenia

Artykuł dotyczy statystycznej kontroli odbiorczej z wykorzystaniem schematów próbkowania dla pozostałości po odrzuceniu wadliwych partii. Rozważane są dwa typy planów odbiorczych AOQL: według oceny liczbowej właściwości oraz według zadanych charakterystyk (wszystkie elementy próbki są weryfikowane z użyciem właściwości liczbowych, reszta – po odrzuceniu wadliwych partii – według zadanych charakterystyk). Przedstawione plany odbiorcze porównano z odpowiadającymi im planami odbiorczymi Dodge–Romiga według zadanych charakterystyk. Algorytm numeryczny dotyczący przedstawionych planów został dołączony do artykułu (zaimplementowany w programie Mathematica). Analiza właściwości numerycznych zaproponowanych rozwiązań pozwala stwierdzić, że przy tym samym poziomie ochrony konsumenta plany odbiorcze według oceny właściwości liczbowych są bardziej ekonomiczne od planów Dodge–Romiga dotyczących zadanych charakterystyk.

Słowa kluczowe: *AOQL, schematy próbkowania, plan odbiorczy, aspekty ekonomiczne, program Mathematica*