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Dorota KUCHTA*

GENERALIZATION OF THE CRITICAL CHAIN METHOD SUPPORTING THE MANAGEMENT OF PROJECTS WITH A HIGH DEGREE OF UNCERTAINTY AND IMPERFECT INFORMATION

In the critical chain method the fundamental notion is the project buffer, and its length is based on task estimation risk. This estimation is almost never unequivocal. If it is not correct, the whole method may turn out to be ineffective. Different experts may have different opinions about this risk. The critical chain method allows to take into account the opinion of only one expert, which may seriously falsify the image of the project situation. This paper proposes a generalization of the critical chain method allowing the use of the opinions of several experts – both while planning a project and while controlling it. Thanks to such an approach, in each phase of project planning and control we are aware of the opinions of various experts as to the correctness of the deadline which was agreed upon with the customer, as to the chances of meeting this deadline and as to the necessity of strengthening project control or introducing changes into the project.

Keywords: *time management, risk management, project, critical chain*

1. Introduction

GOLDRATT [2] proposed the critical chain method as a method of planning and controlling the time for realizing a project, aimed especially at such projects which are affected by a high degree of uncertainty, lack of knowledge and variability. Like any other method, this one is based on several assumptions. They are linked to knowledge about the planned duration time of the individual project tasks. These duration times are considered by Goldratt to be random variables with known distributions, deter-

* Institute of Organisation and Management, Wrocław University of Technology, ul. Smoluchowskiego 25, 50-372 Wrocław, Department of Management, Gen. Tadeusz Kościuszko Military School of Higher Education, ul. Czajkowskiego 109, 51-150 Wrocław, e-mail: Dorota.Kuchta@pwr.wroc.pl

mined by an expert. In many cases, this knowledge may be far less unequivocal than Goldratt assumes and may depend to a large degree on the expert being asked. In this paper we will propose a generalization of the critical chain method allowing us to take into account the opinions of many experts, which will enable much better estimation of the risk of not finishing a project in time – both in the planning and in the realisation phase of the project. The efficient risk management of projects is based on identifying and understanding various risk types – without this it is not possible to either avoid risk or consciously accept its consequences. In the identification phase, during the process of recognizing the risks facing the project, the role of experts, their experience, knowledge and intuition are crucial. Since a single expert, like any other human being, usually has limited capabilities of „foreseeing" the future, the more experts (within the limits of practicability and common sense, of course), the more efficient will be the identification and management of project risk.

The paper is structured as follows. In Section 2 we present the idea of the original critical chain method, proposed by GOLDRATT [2]. In Section 3 we propose a generalization of this method and in Section 4 some numerical examples are discussed.

2. The critical chain method in planning and controlling project realization

We assume that a project is composed of N tasks, Z_i, $i = 1, ..., N$. To be realized, they need one renewable resource type (e.g. people[1]). Between each pair of tasks (Z_j, Z_k), such that $j < k$, we may have the precedence relationship *END_START*, meaning that the start of task Z_k cannot take place before the end of Z_j. Moreover, for each $i = 1, ..., N$ we assume that we know the probability distribution of the – unknown during the project planning phase – duration time of task Z_i, denoted by the random variable $D(i)$ ($i = 1, ..., N$). We assume that this distribution has known characteristics: $F(i)$ denotes its distribution function and $m(i)$ its mean value or median (depending on what the expert gives). On the basis of this distribution, the following magnitudes are defined:

$N\_D(i)$ – the normal length or the normal duration of task Z_i, believed in the planning phase to correspond to more or less normal circumstances when realizing a task, equal to $m(i)$ – i.e. the mean or median, as appropriate.

$R\_T(i)$ – time reserve, understood as such a period of time for which the value $F(i)(N\_D(i)+ R\_T(i))$ is high, usually 80–90% is assumed.

[1] The assumption about a single resource type (meaning that all resource units are mutually replaceable) is in fact not a restrictive assumption. The critical chain method allows several resource types, our assumption only simplifies the presentation.

Apart from this, we have to know *N_R*(*i*) – the number of resource units required to realise task Z_i.

As far as the characteristics of the whole project (and not of individual tasks) are concerned, we know the number of resource units available throughout the period of project realisation, denoted by *L_R*.

The requirement of knowing the distribution *D*(*i*), or at least selected characteristics, is rather strong. In the case of projects associated with high uncertainty and variability it may be difficult to estimate the probability distribution of the task durations. Of course, we are not talking about estimates of probability distributions based on historically recorded frequencies, as the tasks under consideration are usually being performed for the first time (under the given circumstances), but about subjective probability, based on expert opinion and experience. Such estimates will depend strongly on the assessor. Any one expert may be wrong, or incapable of foreseeing the future correctly. Anyway, in the classical critical chain method it is assumed that the probability distribution of the duration time of each task is unequivocally determined. The method proposed in the next section is based on a criticism of this assumption. However, for the moment we return to the presentation of the original method.

The critical chain method then finds, under the assumption that *task i* will take *N_D*(*i*) units of time, such a schedule in which the scheduled project completion time (*P_C*) is as short as possible, but the precedence relations are satisfied and the total demand for resources does not exceed *L_R*. The determination of such a schedule requires the use of an appropriate algorithm. It may also happen that there is more than one solution and then a choice has to be made on the basis of other criteria. We also have to be conscious that the determination of such a schedule is a hard problem from a computational point of view and may require the use of approximate algorithms, which do not guarantee delivering an optimal solution. This problem is discussed by HERROELEN and LEUS in [3]. We assume here that an appropriate algorithm has been determined and chosen in an unequivocal way. Then we determine such a sequence of tasks $(Z_{l_1}, ..., Z_{l_k})$ which determines the planned realization time of the whole project, i.e. such a sequence whose tasks have been scheduled one after another (sometimes because of the imposed precedence relations, sometimes because of a lack of the required resources) and the sum $\sum_{s=1}^{k} N\_D(i_s)$ is equal to *P_C*[2]. Again, there may be other such sequences and then the method has to be slightly modified [5].

[2] For simplicity, we assume that such a sequence exists. It may happen that, because of precedence relations, limits on available resources or other constraints, that such a sequence „without breaks" will not exist, because in some periods no tasks can be performed. In this case, we will have $\sum_{s=1}^{k} N\_D(i_s) < P\_C$ and the method will have to be modified slightly.

However, we assume here that there is just one such sequence. It is called the critical chain. Then the critical chain method determines the length of the project buffer, denoted by L_B. There are several ways of doing this calculation [4], but in each case L_B must be considerably lower than the sum of the time reserves of the critical chain tasks, i.e. less than $\sum_{s=1}^{k} R\_T(i_s)$. E.g. we can use the formula

$$L\_B = \frac{1}{2}\sum_{s=1}^{k} R\_T(i_s)\,. \tag{1}$$

Then the scheduled project completion time (P_C) is corrected (since P_C has been determined on the basis of either the median or mean values of the unknown duration times of the critical chain tasks and it would not be wise to propose this to the client as the completion time for the project as a whole, because it is affected by high risk). The corrected estimate takes the following form:

$$P\_C = \sum_{s=1}^{k} N\_D(i_s) + L\_B. \tag{2}$$

Formula (2) represents the essential idea of the critical chain method. L_B is, independently of the formula we choose to calculate it, considerably smaller than the sum of the time reserves of the critical chain tasks. Thus, when scheduling a project, we do not use the sum of the reserves for the critical chain tasks $\sum_{s=1}^{k} R\_T(i_s)$, which is usually done in the classical approach to project planning[3]. At the same time, the part of those reserves which is taken into account (thus L_B) is managed jointly and openly. Such an approach, as shown by the praxis [5], considerably diminishes the risk of exceeding the deadline for the project agreed upon together with the customer, P_C from (2).

In the planning phase, the critical chain method also comprises other buffer types (feeding buffers and resource buffers), but the aim of the above considerations was to present just the main idea of project scheduling according to the critical chain method and we will limit ourselves to this. This idea will be now illustrated with an example.

Example 1.

Let us consider the following project, in which $L\_R = 2$, $N = 6$, $N\_R(i) = 1$ for $i = 1, ..., N$. In Table 1 we present the precedence relations between pairs of task (the relations are of the *END_START* type):

[3] What is more, this is done in a hidden way, because in the classical approach the tasks are simply assumed to have duration equal to $N\_D(i) + R\_T(i)$, while the two components of the sum are not distinguished.

Table 1. Project tasks from Example 1 and precedence relations between them

PREDECESSOR	SUCCESSOR
Z_1	None
Z_2	Z_1
Z_3	Z_1
Z_4	Z_2
Z_5	Z_3
Z_6	Z_4, Z_5

We also assume that for all these tasks the „normal" duration times have been given, as well as the time reserves, based on the duration time distributions assumed by the expert for the individual tasks: $N\_D(i) = 2$, $R\_T(i) = 1$ for $i = 1, ..., 6$.

The critical chain for the project has been identified. It is composed of the tasks with indices: $i_1 = 1$, $i_2 = 2$, $i_3 = 3$, $i_4 = 5$, $i_5 = 6$. Thus we have (from (1) and (2)):

$$L\_B = 5/2,\ P\_C = 10 + 5/2 = 12.5$$

The location of the critical chain and the project buffer in the example considered is illustrated by Fig. 1[4]:

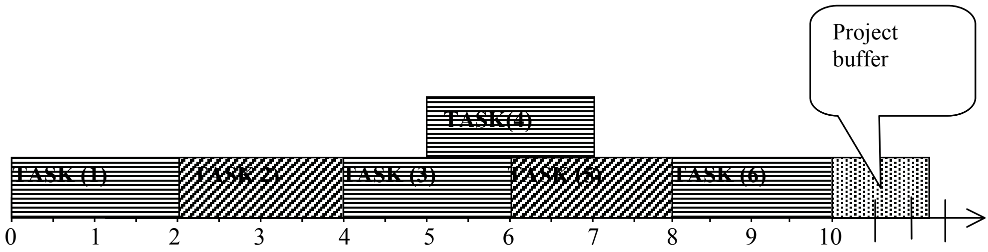

Fig. 1. The schedule with project buffer for Example 1

Figure 1 presents the project schedule for Example 1, constructed according to the critical chain method. We must not forget, however [3], [7], that in projects managed according to the critical chain method, the schedule does not play the same role as in projects managed using traditional methods. The project team should not keep to the start and end times of the individual tasks specified in the schedule. They should simply carry out the tasks as soon as possible (of course satisfying the quality requirements), and the schedule only plays the role of a reference point. Now we will say a few words about the control of project realisation control.

[4] The exact location of task Z_4, not belonging to the critical chain, is a consequence of other features of the critical chain schedule, not discussed here.

According to the critical chain method, in the phase of project realisation, it is above all the usage of the buffer that is controlled, without much attention being paid to the schedule itself. It is only when the usage of the buffer has become too great that the project tasks are controlled, but only those belonging to the critical chain. In Example 1 most tasks have this property, but in real world projects the proportions are often the other way round, which means a considerable reduction in the work load during project control with respect to traditional approaches, where all the tasks are controlled.

Let us suppose that t is a moment during project realisation. At selected control points t the following magnitude is calculated[5], which reflects the usage of the project buffer:

$$U\_B(t) = \frac{\max\left(\sum_{i \in ZK(t), i=i_1,\ldots,i_k} (\langle D(i) \rangle - N\_D(i)), 0 \right)}{L\_B}$$

where $ZK(t)$ is the set of indices of the tasks Z_i that have been finished before t, and $\langle D(i) \rangle$ denotes the actual duration of the already completed tasks. The difference $(\langle D(i) \rangle - N\_D(i))$ shows whether completion of the task Z_i made use of the common buffer (this is the case when this difference is positive), or maybe on the contrary, has even added part of its "normal" duration to the buffer, to be at the disposal of other tasks (the case of a negative difference). The authors of the critical chain method propose the following action rule: if $U\_B(t) \le 1/3$, nothing is to be done, such limited use of the buffer (even if it is positive) is considered to be perfectly normal, if $1/3 \le U\_B(t) \le 2/3$ then those tasks from the critical chain which are not finished should undergo a certain level of control, but the situation is still not considered to be worrying. If however, $U\_B(t) \ge 2/3$, the project, to be more exact the tasks in the critical chain, have to be looked at and analyzed carefully, because there is a high risk of not being able to achieve the deadline P_C. Of course, we might also choose other limits on buffer usage, defining whether the situation is to be considered serious or not. This approach will be illustrated by means of an example:

Example 2. (continuation of Example 1):

Let us suppose that at moment $t = 4$ the project situation is as follows: $ZK(4) = \{1\}$, $\langle D(1) \rangle = 4$. Thus we have:

$$U\_B(4) = \frac{(4-2)}{2.5} = 0.8$$

[5] This is a simplified procedure, a more exact description can be found in (Kuchta 2009).

This means that task Z_1 by itself (as the other ones have not been finished) has used the project buffer to a degree of 0.8. The situation is thus rather serious and it is necessary to carefully control the realisation of the following tasks. Let us assume now that the control of project realisation and motivation of the project team have been strengthened and at moment $t = 6$ we have: $ZK(6) = \{1,2,3,5,\}$, $\langle D(1)\rangle = 4$, $\langle D(2)\rangle = 0.5$, $\langle D(3)\rangle = 0.5$, $\langle D(5)\rangle = 1$. Then we obtain:

$$U\_B(6) = \frac{\max((4-2)+(0.5-2)+(0.5-2)+(1-2),0)}{2.5} = 0\,.$$

This means that the project team has made up the delays and that control of the realisation of individual tasks, those from the critical chain, is not needed – we only have to the check the values $WB(t)$ for $t > 6$.

The critical chain method is adapted to projects with a high degree of variability and uncertainty. For this reason, the estimation of the distributions of the random values $D(i)$, and consequently of $N\_D(i)$ and $R\_T(i)$ ($i = 1, ..., N$), may cause many problems. When estimating these values, it will always be necessary to rely on expert opinions, but very often the opinions of various persons will be different and in order to estimate the planned completion time of a project, as well as the risk of exceeding it, in a thorough and correct way, it may be necessary to take into account the opinions of many experts at the same time. This will be made possible by the approach proposed in the following chapter.

3. The generalized critical chain method in project planning and realisation control

A new proposal for applying the critical chain method involves making use of the opinions of many experts with regard to the duration times of individual tasks, $D(i)$. According to *expert l* the duration time of task Z_i will be a random variable $D_l(i)$, $l = 1, ..., w$ (we assume that there are w experts, where w should be as big as possible from the practical point of view, but at least 5). Thus, as in the classical approach to the critical chain, we assume knowledge of the distribution of the duration times of tasks. However, we do not require identical opinions, but take into account the different opinions of several experts. The probability distribution given by the l-th expert has a density function $f_l(i)$, distribution function $F_l(i)$, mean value (or possibly median) $m_l(i)$ and variance $\sigma_l^2(i)$.

Then we can determine the equivalents of the values $N\_D(i)$ and $R\_T(i)$ from the classical approach. Thus we define $N\_D_l(i)$ as $m_l(i)$, and $R\_C_l(i)$ as such a number that $F_l(i)$ $(N\_D_l(i)+R\_T_l(i))=$ 90% (of course, the limit probability value, 90%, can be changed and even made dependent on the experts' opinions). Making use of the opinions of many experts, we get an variable project buffer length, defined as e.g. $L\_B_l=\frac{1}{2}\sum_{s=1}^{k}R\_T_l(i_s)$, l = 1, ..., w. The planned schedule and the planned project duration will also be variable. As far as the existence of different schedules is concerned, this does not cause many problems, because the schedule – as mentioned in the previous chapter – only plays the role of a reference point in the critical chain method. There is thus no problem about having more schedules, each expert can use his own in the control process. However, as far as the planned project duration is concerned, it would – in the case where more experts are involved – be a set of values P_C_l, l = 1, ..., w, defined in the following way:

$$P\_C_l=\sum_{s=1}^{k}N\_D_l(i)+L\_B_l \tag{3}$$

and determining which project completion time should be given to the customer according to each of the experts. Of course, such information is unsatisfying for the customer – he should be given one specific value P_C^*, which should be determined somehow on the basis of the more complex information given in (3).

If we apply this approach to project planning, realization control will be somewhat modified. At any control point t we determine the following values (l = 1, ..., w):

$$U\_B_l(t)=\frac{\max\left(\sum_{i\in ZK(t),i=i_1,\dots,i_k}(\langle D(i)\rangle-N\_D_l(i)),0\right)}{\max\left(P\_C^*-\sum_{i\in ZK(t),i=i_1,\dots,i_k}N\_D_l^i,0\right)} \tag{4}$$

In the denominator of formula (4) we have a modified project buffer – it takes into account the „normal" duration times given by each of the experts, but it does not directly take into account the planned duration of the project estimated by each of the experts. The latter has been replaced by the project duration announced to the customer, estimated on the basis of the opinions of all the experts. This modified project buffer is the buffer left according to the l-th expert based on the decision about the value of P_C^*. This modified project buffer can be equal to, smaller or greater than L_B_l.

Then at control point t we can determine the following sets:

– $\mathrm{A}_1(t)=\{l:U\_B_l(t)\le p_1^l\}$

– $A_2(t)=\{l: p_1^l \le U\_B_l(t) \le p_2^l\}$

– $A_3(t)=\{l: U\_B_l(t) \ge p_2^l\}$,

where p_1^l, p_2^l, $0 \le p_1^l \le p_2^l \le 1$ are selected (by the respective experts) values of the relative buffer usage, at which the situation should start to be considered as rather worrying or serious, respectively. Of course, we can adopt the values from the classical critical chain method, ($p_1^l = 1/3$, $p_2^l = 2/3$), but each expert can also assume his own values.

Then it is necessary to calculate the power of the set defined above with respect to the number of experts *w*. Let us denote these ratios by $S(A_i(t))$, *t* = 1, 2, 3. Naturally, $\sum_{i=1}^{3} S(A_i(t)) = 1$. On the basis of the values of $S(A_i(t))$, we can determine the degree to which the project is in danger with respect to the deadline. If $S(A_3(t))$ is large – again, we should decide by ourselves what this means to us, maybe bigger than 1/2 – it means that this proportion of experts thinks that the situation is serious, then we might assume it is serious from an objective point of view. If $S(A_3(t))$ is small, e.g. smaller than 0.1, then only a few experts think that the situation is serious. If at the same time $S(A_1(t))$ is very large, e.g. greater than 0.8, we may assume that the situation is not serious, although – and we must not forget this in our risk analysis- according to some experts it may be. In this way we listen to the voices of several experts, which is especially important in the case of projects associated with a high degree of uncertainty and imperfect knowledge.

During project realisation, at each control point *t*, for each *l* = 1, ..., *w* we might additionally calculate $PR_l^t = P(RP\_C_l^t \le P\_C_l)$, where $RP\_C_l^t$ denotes the random variable representing the actual, but yet unknown project completion time at control point *t* according to the opinion of the *l*-th expert, *l* = 1, ..., *w*. The actual project completion time at moment *t* is a random variable, which may have a different distribution according to various experts. What is more, this distribution may change as the project progresses, thus it may be different at each control moment. Thus the value $PR_l^t = P(RP\_C_l^t \le P\_C_l)$ will express the opinion of the *l*-th expert at control point *t* as to the probability of finishing the project within the time estimated by this very expert. At the start of project realisation (thus at moment *t* = 0) the distribution $RP\_C_l^0$, for the *l*-th expert, *l* = 1, ..., *w*, will be the distribution of the sum $\sum_{s=1}^{k} D_l(i)$ and at control point *t* it will be the distribution of the sum $\sum_{i \in ZK(t), i=i_1,\dots,i_k} \langle D(i) \rangle + \sum_{i \notin ZK(t), i=i_1,\dots,i_k} D_l(i)$, whose first component is a constant. We can also calculate the values $PR_l^{t,*} = P(RP\_C_l^t \le P\_C^*)$, where P_C^* is the deadline announced to the customer.

It is worthwhile tracking the values PR_l^t and $PR_l^{t,*}$ at the control points (using the classical critical chain method this is not done, even for the uniquely defined probability distributions of the duration times of tasks assumed there), because they deliver important information as to the chances of finishing the project in time, estimated from several points of view. Thus they allow better management of the risk of exceeding the project deadline. Depending on the values PR_l^t calculated at individual control points, which estimate the probability of completing the project by the planned completion time given by the l-th expert P_C_l, and also depending on the probabilities $PR_l^{t,*}$, referring to the deadline P_C^* announced to the customer, we may have to take the decision to renegotiate with the customer regarding the deadline P_C^*. The earlier we are conscious of the necessity of such renegotiations, the better.

4. Computational example

This example, used in order to illustrate the proposed approach, is a continuation of Example 1 in the sense that we adopt from it the number of tasks and the content of Table 1. As far as task durations are concerned, we assume that $w = 10$, and the l-th expert ($l = 1, ..., 10$) thinks that the duration time of task Z_i ($i = 1, ..., 6$) has a triangular distribution with mean $m_l(i) = 2$ and a density function which is symmetric with respect to the mean, such that the set of arguments where it takes positive values is the interval $[2-0.1l, 2+0.1l]$ ($l = 1, ..., 10$). Thus the density function f_l of the duration time of individual tasks according to the opinion of the l-th expert is a symmetric triangular function taking the value 0 for arguments outside the interval $[2-0.1l, 2+0.1l]$ and the value $1/0.1l$ for the argument 2 ($l = 1, ..., 10$).

For triangular distributions with a symmetric density function taking 0 outside the interval $[2-0.1l, 2+0.1l]$, we have: $m_l(i) = 2$, $\sigma_l^2(i) = \dfrac{0.01l^2}{6}$ ($i = 1, ..., 6$).

We assume: $N\_D_l(i) = 2$ ($i = 1, ..., 6$, $l = 1, ..., 10$). If the experts take 90% for the limit value defining the time reserve (thus the condition $F_l(i)(N\_D_l(i) + R\_T_l(i)) = 90\%$ is fulfilled), we have (from the properties of the triangular distribution density):

$$R\_T_l(i) = 0.1l(1-\sqrt{0.2}).$$

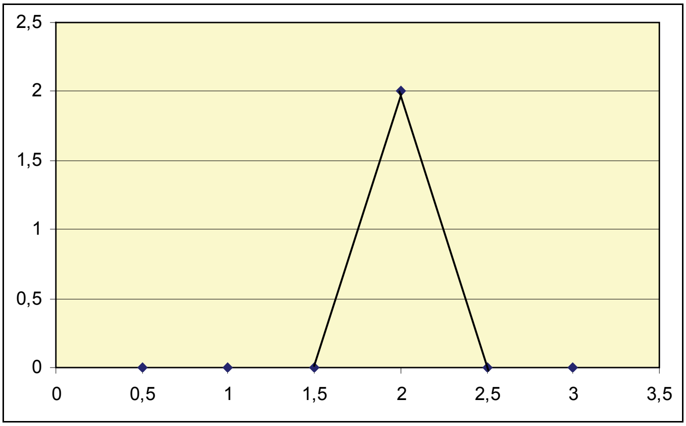

Fig. 2. The density function of the duration time of individual tasks of the project from Example 3 according to the 5th expert ($f_5(i)$, $i = 1, ..., 6$)

As in Example 1, the critical chain is composed of the tasks with indices $i_1 = 1$, $i_2 = 2$, $i_3 = 3$, $i_4 = 5$, $i_5 = 6$ (Fig. 1). We have ($l = 1, ..., 10$):

$$L\_B_l = \frac{0.5}{2} \cdot (1 - \sqrt{2}) l \,,$$

$$P\_C_\lambda = 10 + \frac{0.5}{2} \cdot (1 - \sqrt{2}) l \,.$$

Table 2 gives the values of the buffer and the planned realisation time of the project according to the individual experts.

Table 2. The values of the buffer and the planned realisation time of the project according to the individual experts ($l = 1, ..., 10$) for the given example

l	L_B_l	P_C_l
1	0.14	10.14
2	0.28	10.28
3	0.41	10.41
4	0.55	10.55
5	0.69	10.69
6	0.83	10.83
7	0.97	10.97
8	1.11	11.11
9	1.24	11.24
10	1.38	11.38

Of course, fractional values will usually only be of limited importance (of course, this depends on what the basic scheduling time unit is – week, month or year), but the opinions of different experts allow us to state that the customer should be given a deadline of about 10–11 time units, but according to Experts 8, 9, 10 this time should be longer than 11 time units. Let us suppose that we have chosen $P\_C^* = 10.5$.

The distribution of the variables $RP\_C_l^t$ will be approximated (on the basis of the central limit theorem) by the normal distributions with the corresponding means and variances. Thus at moment 0 (the project start) $RP\_C_l^0$ will approximately have a normal distribution with mean $m_l(i)$ equal to 10 and variance equal to $5\,\sigma_l^2(i) = \frac{0.05l^2}{6}$.

Let us now consider the realization of the project as given in Example 2. The relative use of the buffer at both control points considered ($t = 4$ and $t = 6$) will be equal according to all the experts. This is because in our example all the experts agree as to the means $m_l(i)$, and thus as to $N\_D(i)$, $i = 1, ..., 6$, so the modified buffer in the denominator of (4) is identical according to all the experts and equal to 0.5. At moment $t = 4$ the relative use of the buffer is equal to 4, and at moment $t = 6$ – to 0. This means that all the experts evaluate the situation at $t = 4$ as bad, and at $t = 6$ as good. Thus we have $S(A_3(4)) = 1$, $S(A_1(6)) = 1$. If the experts did not agree with respect to the means, they might also disagree as to the evaluation of the situation at each control point.

Table 3. Values PR_l^4, $l = 1, ..., 10$ for the situation in the given example

l	PR_l^4
1	0.00
2	0.00
3	0.00
4	0.00
5	0.00
6	0.00
7	0.02
8	0.06
9	0.11
10	0.18

At moment $t = 4$, $RP\_C_\lambda^4$ will have approximately a normal distribution with mean 12 ($= \langle D(1) \rangle + 4 \cdot 2$) and variance $4\sigma_l^2(i) = \frac{0.04l^2}{6}$, $l = 1, ..., 10$ (task Z_1 was finished after 4 time units, its duration is not a random variable any more, but the duration times of the other tasks are still random variables and we assume that the experts still have the same opinion regarding their distribution. The probability of finishing the project within a period of 10.5 units (i.e. $PR_l^{4,*}$, $l = 1, ..., 10$**)** estimated by the individual experts at the control point $t = 4$, would be then equal practically to zero (only in the cases of Experts 8, 9 and 10 would it be greater than 0.01). On the other hand, the probabilities of completing the project by the deadlines given by individual experts (i.e. $PR_l^4, l = 1, ..., 10$)) would be a bit higher in the case of those experts who estimated the project completion time to be greater than 10.5, which might be an indication that renegotiation of the deadline should be considered.

On the other hand, if ⟨D(1)⟩ was equal to 3, at the control point $t = 4$ we would have the following values for $PR_l^{4,*}$ and $PR_l^4, l = 1, ..., 10$.

Table 4. Values $PR_l^{4,*}$ and $PR_l^4, l = 1, ..., 10$ for the situation in the given example under the assumption ⟨D(1)⟩ = 3

L	$PR_l^{4,*}$	PR_l^4
1	0.00	0.00
2	0.00	0.00
3	0.02	0.00
4	0.06	0.04
5	0.11	0.14
6	0.15	0.26
7	0.19	0.38
8	0.22	0.48
9	0.25	0.56
10	0.27	0.62

Clear differences can be seen here. First of all, the evaluation of the situation by individual experts differs. Secondly, so does their opinion about the chances of completing the project by the deadlines P_C^* and P_C_l. These estimates give a clear indication that it is necessary to renegotiate the deadline or to introduce radical changes in the way the project is carried out.

Such analysis, performed at each control point, allows more efficient time management of a project and provides an early warning system for possible problems – especially in the case of projects involving innovation with a high degree of uncertainty, where we have to rely on experts' opinions, because there are very little data helping us to estimate the duration of a project and the risk associated with its realisation.

5. Summary

This paper proposes the direct use of the opinions of many experts in the critical chain method. Thanks to this, the critical chain method will support the management of project time and risk more effectively. Further research is needed to test the proposed approach using practical examples and also to create a system supporting dialog with experts and the generation of probability distributions for the duration times of individual tasks. It would also be interesting to try to combine the proposed approach with other recent modifications of the critical chain or critical path methods, known from the literature [8].

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