

Grażyna TRZPIOT*, Justyna MAJEWSKA*

ESTIMATION OF VALUE AT RISK: EXTREME VALUE AND ROBUST APPROACHES

The large portfolios of traded assets held by many financial institutions have made the measurement of market risk a necessity. In practice, VaR measures are computed for several holding periods and confidence levels. A key issue in implementing VaR and related risk measures is to obtain accurate estimates for the tails of the conditional profit and loss distribution at the relevant horizons.

VaR forecasts can be heavily affected by a few influential points, especially when long forecast horizons are considered. Robustness can be enhanced by fitting a generalized Pareto distribution to the tails of the distribution of the residual and sampling tail residuals from this density. However, to ensure a sufficiently large breakdown point for the estimator of the generalized Pareto tails, robust estimation is needed (see Dell'Aquila, Ronchetti, 2006). The aim of the paper is to compare selected approaches to computing Value at Risk. We consider classical and robust conditional (GARCH) and unconditional (EVT) semi-nonparametric models where tail events are modeled using the generalized Pareto distribution. We wish to answer the question of whether the robust semi-nonparametric procedure generates more accurate VaRs than the classical approach does.

Keywords: *extreme value theory, Value at Risk, semi-nonparametric method, robust methods of estimation*

1. Introduction

Following the increase in financial uncertainty in the 90's, which resulted in famous financial disasters, there has been intensive research from financial institutions, regulators and academics to develop improved, sophisticated models for the estimation of market risk. The most well known risk measure is Value-at-Risk (VaR). A key issue in implementing VaR and related risk measures is to obtain accurate estimates for the tails of the conditional profit and loss distribution at the relevant horizons. How-

* Department of Demography and Business Statistics, Karol Adamiecki University of Economics, ul. Bogucicka 14, 40-226 Katowice, Poland, e-mail: trzpiot@ae.katowice.pl; justyna.majewska@ae.katowice.pl

ever, traditional value-at-risk models based on the assumption of iid-ness and normality are not able to capture the extremes in emerging markets, where high volatility and the nonlinear behavior of returns are observed. This fact has led to various alternative strategies for VaR prediction.

Extreme value theory (EVT) can be useful in defining supplementary risk measures, because it provides more appropriate distributions to fit extreme events. Unlike VaR methods, no assumptions are made about the nature of the original distribution of all the observations. This theory emphasises the modeling of rare events, mostly events with a low frequency but a high impact. Common practice is to characterize the size and frequency of such extreme events mainly by the extreme value index. The most prominent extreme value methods are constructed using efficient maximum likelihood estimators based on specific parametric models, which are fitted to excesses over large thresholds. Maximum likelihood estimators, however, are often not very robust, which makes them sensitive to a few particular observations. Even in extreme value statistics, where the most extreme data usually receive most attention, this may constitute a serious problem.

Robust methods for extreme values have already been discussed in recent literature. For example, BRAZAUSKAS and SERFLING [2] consider robust estimation in the context of strict Pareto distributions. PENG and WELSH [16] and JUÁREZ and SCHUCANY [11] derived robust estimation methods for the case where the observations follow a generalized extreme value distribution, a generalized Pareto distribution, thin or heavy tailed. Also MANCINI and TROJANI [13] proposed a class of robust semiparametric bootstrap methods to estimate conditional predictive distributions of asset returns in GARCH-type volatility models with applications in VaR calculation.

The objectives of robust statistical analysis and extreme value analysis are apparently contradictory. Whereas extreme data are downweighted when defining robust statistics, these observations receive most attention in an extreme value approach. So within an extreme value framework, some robust algorithms replacing the maximum likelihood part of this methodology can be of use leading to estimates which are less sensitive to a few particular observations.

Robust methods can improve the quality of EVT data analysis by providing information on influential observations, deviating substructures and possible misspecification of a model, while guaranteeing good statistical properties over a whole set of underlying distributions around the assumed one ([4]).

In this study we use two methods: a conditional GARCH(1,1) model – a classical one and with a robust method of estimating volatility and a bootstrapping approach based on a combination of a Generalized Pareto Distribution (GPD) and the empirical distribution of the returns (semi-nonparametric model). These models are used for VaR calculation and compared.

The paper is organized as follows: in Section 2 we present the data sets and preliminary analysis, in Section 3 – basic information about VaR estimation, in Section 4 we present extreme value theory and the semi-nonparametric method for estimating VaR proposed by C. Brooks, A.D. Clare, J.W. Dale Molle and G. Persaud, while some facts regarding robust methods of estimation are described in Section 5. Section 6 displays and analyses the results and finally, Section 7 concludes.

2. Preliminary data analysis

In this study we examined the behavior of chosen indices of four national stock exchanges from EU states, namely DAX (Germany), FTSE100 (UK), BUX (Hungary), WIG20 (Poland) and one Canadian S&PTSX Composite. The data span the period of February 16th 2001 to June 29th 2007 (observations from 30 June 2007 to 20 June 2008 are reserved for out-of-sample testing). For all these indices, we compute daily log-returns.

Table 1 provides summary statistics as well as the Jarque–Bera value.

Table 1. Descriptive statistics of the daily returns, $\log(x_t/x_{t-1})$, from five stock markets

	DAX	FTSE100	BUX	WIG20	S&PTSX
Mean	-0.00001	0.00005	0.000862	0.00053	0.000208
Standard Deviation	0.015814	0.01105	0.013327	0.01445	0.009999
Skewness	-0.170991*	-0.17628*	-0.174125*	0.04870*	-0.820490*
Kurtosis	3.796985*	3.860970*	1.302525*	1.05130*	7.73350*
Normality test statistic ^a	1103.301*	993.3196*	118.2762*	72.4430*	4821.548*

* represents significance at the 5% level (two-tailed test),

^a Jarque–Bera test.

Table 1 shows that all five returns series show strong evidence of skewness – only the WIG20 returns are positively skewed, while the returns on the others are negatively skewed. This indicates that the asymmetric tail extends more towards negative values than positive ones. Based on the sample kurtosis estimates, it may be argued that the return distributions in all the markets are leptokurtic. In particular, the Canadian S&PTSX series has the highest coefficient of excess kurtosis. The Jarque–Bera test statistic consequently rejects normality for all the log-return series. The fat-tailed nature of the five series provides strong motivation for the estimation methodologies employed in this paper.

3. Estimating Value at Risk

VaR has become a standard for measuring and assessing risk. It is defined as a quantile of the distribution of returns (or losses) for an asset or portfolio. It can also be defined as the predicted worst-case loss at a specific confidence level over a certain period of time.

Formally, let $x_t = \log(p_t / p_{t-1})$ be the return at time t , where p_t is the price of an asset at time t . We denote the $(1-q)\%$ quantile estimate at time t for a one-period-ahead return as $\text{VaR}(q)$, so that

$$P(x_t < \text{VaR}_t(q)) = q.$$

More formally, VaR is calculated according to the following equation

$$\text{VaR}_t = F^{-1}(q)\sigma_t,$$

where $F^{-1}(q)$ is the corresponding quantile of the assumed distribution and σ_t is the forecast of the, usually conditional, standard deviation at time $t - 1$.

We compare four different models for estimating returns one period ahead. These models are:

- VaR with GARCH-N (the error term is assumed to be normally distributed) where the initial estimate of σ_0^2 is given by the sample variance (parametric, conditional method),
- VaR with GARCH-N (the error term is assumed to be normally distributed) where the initial estimate of σ_0^2 is given by a robust estimator (it is essential to have a very robust initial estimate, which is very stable and ensures reasonable accuracy of the first estimate),
- classical EVT model (semi-nonparametric, unconditional method),
- semi-nonparametric, unconditional method based on EVT with a robust method of estimation.

4. Some key facts in extreme value theory and VaR with EVT

One of the most important goals of financial risk management is the accurate calculation of the probabilities of large potential losses due to extreme events, such as stock market crashes, currency crises or trading scandals. It is well known that traditional parametric and nonparametric methods for estimating distributions and densities

often give very poor fits to the tails. In recent years, there have been a number of extreme value studies in the finance literature which showed that EVT is a powerful and yet fairly robust framework to study the tail behavior of a distribution.

In extreme-value theory the extreme value index γ is used to characterize the tail behavior of a distribution. This real-valued parameter helps to indicate the size and frequency of certain extreme events under a given probability distribution – the larger γ , the heavier the tail.

The tail of a distribution can be modeled using the conditional excess distribution function, which describes the conditional distribution of exceedences (or excesses) over a given threshold level. A theorem by BALKEMA and de HAAN (1975) shows that for a sufficiently high threshold the distribution function of the excess may be approximated by the generalized Pareto distribution (GPD). The distribution of excesses over a high threshold converges to one of three different extreme value distributions: the exponential, Pareto, and Uniform distributions, for $\gamma = 0$, $\gamma > 0$ and $\gamma < 0$ respectively.

Let $(X_n, n \in \mathbb{Z})$ be a family of random variables representing daily observations of the log-returns on stock prices. We make the crude assumption that these returns are independent and identically distributed (iid) from an unknown underlying distribution F . In addition, let x_L and x_U represent lower and upper thresholds of the tails, such that x_t exceeds x_L if and only if $x_t < x_L < 0$ and x_t exceeds x_U if and only if $x_t > x_U > 0$ for $t = 1, 2, \dots, n$.

The parameters of GDP are typically estimated using the method of maximum-likelihood. The log-likelihood function for estimating σ_U and γ at an upper tail threshold U is

$$l(\gamma_U, \sigma_U, k) = -k \ln(\sigma_U) - (1 + \frac{1}{\gamma_U}) \sum_{i=1}^k \ln(1 + \frac{\gamma_U y_i}{\sigma_U}) \quad \text{for } \gamma \neq 0,$$

where

k is the number of exceedences in a sample of n observations,

σ_U – scale parameter,

γ_U – maximum likelihood tail index estimator,

y_i – the exceedance of the threshold for the i -th observation¹.

The crucial step in estimating the parameters of the GPD is the determination of a threshold value. While MCNEIL and FREY [14], MCNEIL (1997) and MCNEIL (1999) use the “mean-excess-plot” as a tool for choosing the optimal threshold level, Gavin (2000) uses a threshold level according to a somewhat arbitrarily chosen confidence level of 90%. In NEFTCI [15] the threshold level is 1.645 times the unconditional variance of the data, which means 5% of the observations would be above the upper

¹ For a lower threshold L , the log-likelihood function can be derived similarly.

threshold if the data were normally distributed. In this paper, we follow a slightly different approach. We first estimate the GDP parameters corresponding to various threshold levels, representing respectively 1%, 2%, 3%, ... till 15% of the extreme observations. Then we plot a graph of the estimated parameters and choose the threshold level at which the estimate stabilizes. This is a non-parametric way of choosing the optimal threshold level which is useful when the mean-excess-plot or the assumption of normality of the returns distribution fail.

The selected number of exceedances, maximum likelihood estimates (MLE) of the parameters of the GDP and threshold level are presented in table 2.

Table 2. Number of extremes, maximum likelihood parameters of GDP and the threshold levels

	DAX	FTSE100	BUX	WIG20	S&PTX
Lower tail					
k	32	54	41	45	19
$\hat{\sigma}$	0.0038	0.0128	0.0020	0.0053	0.0329
$\hat{\gamma}$	0.2830	0.0013	0.1771	0.8042	0.2660
T_L	0.0479	0.0372	0.0281	0.0347	0.0293
Upper tail					
k	50	52	53	59	40
$\hat{\sigma}$	0.0109	0.0078	0.0032	0.0042	0.0057
$\hat{\gamma}$	0.1014	0.5693	0.1198	0.6809	0.1804
T_U	0.0415	0.0348	0.0303	0.0352	0.0247

The empirical results are clear-cut and allow one to determine the type of general Pareto distribution: for all five indexes the asymptotic distribution belongs to the domain of attraction of the Pareto distribution.

The highest value of the estimated tail index for the left tail belongs to the WIG20 index and this is an indication of the high risk associated with this market. For the FTSE100 index, this estimator is only 0.0013, which indicates that the negative returns distribution is not so heavy-tailed. For the BUX, DAX, WIG20 and S&PTX indexes, the right tail index is less than the left tail index, so we can conclude that risk and reward are not equally likely in these markets. On the other hand, the FTSE100 has a higher estimate of the right tail index than of the left tail index. Therefore, high positive returns are more likely than similar losses in this market.

The EVT can be utilized to estimate the upper and lower VaR threshold levels. The critical step in calculating VaR is the estimation of the threshold point that will define what variation in x_t is to be considered "extreme." Denote (following BROOKS et al. [3]) $T_U = T_U(1 - q)$ and $T_L = T_L(q)$ to be the q -th and $(1 - q)$ -th percentiles ($0 < q < 1$)².

² x_U has to be selected so that the threshold needed for value at risk calculations, namely T_U , is much farther into the tail, so that $T_U > x_U$.

The estimator of the tail probability is (Bali [1])

$$1 - F^{ML}(x_u + y_t) \cong \frac{k}{n} \left(1 + \frac{\gamma^{ML} y_t}{\sigma_U}\right)^{-1/\gamma^{ML}},$$

where σ_U and γ_U are estimated using the method of maximum likelihood.

The VaR for the upper tail is obtained by denoting the VaR threshold probability using the percentile $q = F(x_U + y_t)$:

$$T_U = \mu + \frac{\sigma_U^{ML}}{\gamma^{ML}} \left[\left(\frac{qn}{k} \right)^{-\gamma^{ML}} - 1 \right].$$

Table 2 shows results for both lower and upper threshold values³.

For the upper tail, the threshold is further into the tail for the DAX Index (set at 0.04148) followed by the WIG20, FTSE100, BUX, and S&PTX. A different result is obtained for the lower tail: the threshold is further into the tail for the DAX Index (set at 0.04785) followed by the FTSE100, WIG20, S&PTX and BUX.

The results show that the extreme tails yield threshold points, T_L and T_U , that are up to 61% higher for the DAX Index than the thresholds, T_L and T_U , one obtains for the S&PTX Index.

4.1. A semi-nonparametric methodology for estimating VaR

Extant approaches using EVT focus only on the tails and do not consider observations in the center of the distribution. In this paper we use an EVT semi-nonparametric approach, which makes use of information from both the tails and the center of the distribution, but treats each separately.

In the empirical study, VaR is estimated for 1 day investment horizons by simulating the conditional densities of price changes, using the bootstrapping methodology ([5]). The simulation study is conducted for the generalized Pareto model⁴ by bootstrapping from both the two fitted tails and from the empirical distribution function derived from the log returns ([3]).

In the case of the generalized Pareto model, the path for future prices is simulated as follows:

³ In these calculations q is set to be 0.01.

⁴ The reason why the generalized Pareto distribution is used for the tails rather than simply using the empirical distribution throughout is that the number of observations in the tails may be insufficient to obtain accurate results without using an appropriate fitted distribution.

- draw samples, with replacement, of the x_t from the empirical distribution $F(x_t)$
 - if $x_t < T_L$, then draw from the generalized Pareto distribution fitted to the lower tail, however
 - if $x_t > T_U$, then draw from the generalized Pareto distribution fitted to the upper tail, or
 - if x_t falls in the middle of the empirical distribution, i.e. $T_L < x_t < T_U$, then x_t is retained.

The number of draws of x_t is equal to the length of the investment horizon.

HSIEH [9] assumed that prices are log-normally distributed, i.e. that the lowest (highest) of the log ratios of the simulated prices over the holding period, i.e. $x_t = \ln(P_t/P_0)$, is normally distributed. However, in this paper we do not impose this restriction, but instead the distribution of x_t is transformed into a standard normal distribution by matching the moments of the distribution of the simulated values of x_t to one of a set of possible distributions known as the Johnson family of distributions ([10]) or ([12]). Matching moments to the family of Johnson distributions requires the specification of a transformation that maps the distribution of x_t to a distribution that has a standard normal distribution. In this case, matching moments implies finding a distribution whose first four moments are known, i.e. one that has the same mean, standard deviation, skewness and kurtosis as the distribution of the samples of x_t .

For all five indexes, the distributions of the x_t values were found to match the unbounded Johnson distribution. Therefore, the estimated 5th percentile of the simulated distribution of x_t fitted to the Johnson system is based on the following transformation:

$$x_{t,5}^* = \sinh\left(\frac{-1,645 - a}{b}\right)d + c$$

where $x_{t,5}^*$ is the 5th percentile of the Johnson distribution derived from the samples of x_t , and a , b , c and d are estimated parameters, whose values are determined by the first four sample moments of the simulated distribution of x_t .

The distribution of Q/P_0 , where Q is the maximum loss, will depend on the distribution of P_t/P_0 . So the first step is to find the 5th percentile of the distribution of x_t ,

$$\frac{x_t - \mu_t}{\sigma_t} \cong x_{t,5}^*,$$

where μ_t is the expected value of the simulated distribution of x_t and σ_t is usually the standard deviation of the simulated distribution of x_t . Expressions for Q/P_0 and P_t/P_0 can be found by exponentiating both sides of the expressions from the above equation, i.e.

$$\frac{P_t}{P_0} = \exp((x_{t,5}^* \cdot \sigma_t) + \mu_t) \quad \text{and} \quad \frac{Q}{P_0} = 1 - \exp((\pm x_{t,5}^* \cdot \sigma_t) + \mu_t).$$

5. Robust methods of estimation: some theoretical results

The primary goal of robust statistics is to provide tools not only to assess the robustness properties of classical procedures, but also to produce new estimators and tests that are robust to deviations from the model. From a data-analytic point of view, robust statistical procedures will find the structure which fits the majority of the data in the best way, identify deviating points (outliers) and substructures for further treatment, and in unbalanced situations identify and warn about highly influential data points. HUBER [8] considers a stricter concept of a robust estimate. It should satisfy the following two properties:

- the estimate should be still reliable and reasonably efficient under small deviations from the assumed parametric model
- replacing a small fraction of observations in the sample by outliers should produce a small change in the estimate.

Statisticians have already developed various sorts of robust statistical methods of estimation. In the literature, particular attention has been paid to estimators with: a high breakdown point (near 50%, but for realistic applications 20% is satisfactory), the property of affine equivariance, computationally fast algorithms to compute them. Standard properties, such as bias and precision, are also of interest.

For further consideration, let a parametric model given by a distribution function F_θ with density f_θ . Some of the most popular robust estimators are M-estimators, first introduced by HUBER [7]. M-estimators represent a very flexible and general class of estimators, which have played an important role in the development of robust statistics and in the construction of robust procedures. However, this idea is much more general and is an important building block in many different fields.

We consider here the general class of M -estimators which are defined implicitly as the solution in θ of

$$\sum_{i=1}^n \psi(x_i; \theta) = 0$$

with ψ being a function satisfying mild conditions (HUBER [7]).

General results in robust statistics imply that an estimator with

- a bounded asymptotic bias in a neighbourhood of the considered model can be constructed by choosing a *bounded* function ψ (in x);
- a high *asymptotic efficiency* can be achieved by choosing a ψ function which is *similar* to the score function $s(x, \theta)$ in the range where most of the observations lie.

In general, the ψ function defining the M-estimator is itself a function of the score function, i.e. $\psi(x; \theta) = K[s(x; \theta)]$. For example, if

$$\psi(x; \theta) = s(x; \theta),$$

we have the ML estimator, and if

$$\psi(x; \theta) = H_c(A[s(x; \theta) - a]),$$

where

$$H_c(z) = \min\left(1, \frac{c}{\|z\|}\right)$$

are Huber weights dependent on the tuning constant c , A is a pxp matrix and a is a $px1$ vector which are determined implicitly by

$$E[A[s(x; \theta) - a][s(x; \theta) - a]^T A^T] = I,$$

$$E[A[s(x; \theta) - a]] = 0$$

we have the optimal B-robust estimator for the standardized case (see [6]). This estimator is the most efficient estimator in the class of M-estimators with a bounded influence function.

In this study, robust estimators are used for the conditional variance GARCH-N (more precisely, estimation of σ_0^2) and the parameter $\theta = [\sigma, \gamma]^T$ of the Generalized Pareto Distribution. In order to obtain robust estimates for these parameters, we adopt the optimal B-robust estimator.

6. Empirical analysis

In the present study we carried out a comparative study of the predictive ability of VaR estimates obtained from the following techniques: semi-nonparametric bootstrapping – the EVT (unconditional) model and the parametric model – GARCH(1,1)⁵, using two approaches in each case (classical and robust).

In-Sample Performance

Daily VaR measures at the 99% probability level are estimated for the period of February 16th 2001 to June 29th 2007 for the DAX, FTSE100, BUX, WIG20 and S&PTX Composite Indexes. The results are presented in table 3.

⁵ Parameter estimates for the models selected were obtained by the method of maximum likelihood and the log-likelihood function of the data was constructed by assuming that the conditional distribution of innovations is Gaussian.

Table 3. Value at Risk (99% probability level) calculated by semi-nonparametric bootstrapping using (unconditional) EVT models – with both classical and robust techniques of estimation, and the robust conditional and unconditional GARCH(N)

Index \ Model	Parametric models		Semi-nonparametric bootstrapping	
	GARCH(N) con. classic	GARCH(N) con. robust	EVT – classical	EVT – robust
DAX	0.0445	0.0397	0.1335	0.117
FTSE100	0.0159	0.0089	0.0275	0.0201
BUX	0.0292	0.0215	0.0959	0.0467
WIG20	0.0522	0.0312	0.6781	0.593
S&PTX	0.0168	0.0178	0.0507	0.0394

The VaRs based on the semi-parametric bootstrapping model are always higher than for the parametric methods of calculation. Comparing the VaRs calculated directly from the two parametric models, there appears to be a tendency for the GARCH(1,1) with robust estimation to generate slightly smaller VaRs than those with standard estimators.

In general, conditional models yield lower estimates for VaR. This occurs because conditional models take into account the current conditions in the economy expressed by the current clustered volatility. On the other hand, unconditional models capture extreme events that have appeared at certain times in the price history.

Out-of-Sample Performance

In order to evaluate the models, we generate out-of-sample VaR forecasts for the five equity indices for the period of 30th June 2007 to 20th June 2008⁶. For all the models and equity indices, we use a rolling sample of 250 observations with the same number of VaR forecasts⁷. We generated one-day VaR forecasts with a 99% confidence level and compared the actual daily profits and losses of the five indexes with their daily estimates of value at risk.

The measure of a model’s performance was chosen to be the number of times the VaR “underpredicts” realized losses over the out-of-sample period. The percentages of days that the VaRs were exceeded by the actual trading losses are given in table 4 for each model.

The nominal probability of a violation is set at the 1% level (i.e. the percentage of exceedences should be close to the nominal 1% value). An accurate and appropriate model is obtained when the expected violation ratio is equal to 1%.

⁶ The Basel Committee requires the use of a 1-trading year “back-test” sample of returns, in order to evaluate the suitability of the model.

⁷ The parameters of the models are re-estimated every trading day.

Table 4. Out of sample tests – realized percentage of daily VaR violations

Index \ Model	Parametric models		Semi-nonparametric bootstrapping	
	GARCH(N) con. classic	GARCH(N) con. robust	EVT – classical	EVT – robust
DAX	3.8	1.1	0.4	0.0
FTSE100	0.7	2.5	0.0	0.0
BUX	2.3	1.9	1.2	1.1
WIG20	6.8	2.4	1.9	0.8
S&PTX	3.4	3.6	1.4	0.7

Analyzing the results from table 4, we can conclude that the procedures based on the bootstrap perform better, with very few or no exceedences. The percentage of exceedences is close to 1% for these procedures. Considering the WIG20 for example, the parametric models lead to too low a VaR and slightly over-estimate the VaRs for the DAX index. The percentage of exceedences is close to 7% for the classical GARCH(1,1) model in the case of the WIG20 and 3% for the S&PTX. It is observed that the semi-nonparametric procedure based on robust estimators generated more accurate VaRs than any other method.

7. Conclusion

The purpose of this paper has been to carry out a comparative study of the predictive ability of VaR estimates from various estimation techniques for five different stock indexes from Germany, UK, Hungary, Poland and Canada.

Particular attention has been given to methodology based on Extreme Value Theory and robust methods of estimation. The main question was how well robust methods of estimation used in EVT – models perform in modelling the tails of distributions and in estimating and forecasting VaR measures.

We used an unconditional EVT bootstrapping approach based on a combination of the generalized Pareto distribution and the empirical distribution of returns, as well as a parametric model – GARCH(1,1) – using both classical and robust methods of estimation.

Our main finding is that out-of-sample tests of the calculated VaRs show that the proportion of exceedences produced by the semi-nonparametric approach using Extreme Value Theory and based on a robust method, which separately models the tail and central regions, are considerably closer to the nominal probability of violations than competing approaches which fit a single model to the whole distribution.

Under the European Union's CAD II, investment firms and banks in Europe are now permitted to use their own internally developed VaR models to calculate the re-

quired capital to cover losses in their trading positions. Given that lots of models are now in widespread use, it is crucial to compare different approaches to computing value at risk. Thus, as EVT is of increasing importance in risk management, robust methods can improve the quality of EVT data analysis.

Literature

- [1] BALI T.G., *An extreme value approach to estimating volatility and value at risk*, Journal of Business 2003, Vol. 76, No. 1, 83–108.
- [2] BRAZAUSKAS V., SERFING R., *Robust estimation of tail parameters for two-parameter and exponential models via generalized quantile statistics*, Extremes, 2000, Vol. No. 3, 231–249.
- [3] BROOKS C., CLARE A.D., DALE MOLLE J.W., PERSAND G., *A comparison of extreme value theory approaches for determining value at risk*, Journal of Empirical Finance, 2005, Vol. 12, 339–352.
- [4] DELL'AQUILA R., EMBRECHTS P., *Extremes and robustness: a contradiction?* Springer, Berlin–Heidelberg–New York, 2006.
- [5] EFRON B., *The jackknife, the bootstrap and other resampling plans*, Society of Industrial and Applied Mathematics CBMS-NSF Monographs, 1982, Vol. 38.
- [6] HAMPEL F., RONCHETTI E., ROUSSEEUW P., STAHEL W., *Robust Statistics: The Approach Based on Influence Functions*, Wiley, New York, 1986.
- [7] HUBER P., *Robust estimation of a location parameter*, Annals of Mathematical Statistics, 1964, Vol. 35, No. 1, 73–101.
- [8] HUBER P.J., *Robust Statistics*, Wiley, New York, 1981.
- [9] HSIEH D.A., *Implications of nonlinear dynamics for financial risk management*, Journal of Financial and Quantitative Analysis, 1993, Vol. 28, No. 1, 41–64.
- [10] JOHNSON N.L., *Systems of frequency curves generated by methods of translations*, Biometrika, 1949, Vol. 36, 149–176.
- [11] JUÁREZ S., SCHUCANY W., *Robust and efficient estimation for the generalized Pareto distribution*, Extremes, 2004, Vol. 7, No. 32, 231–257.
- [12] KENDALL M.G., STUART A., ORD J.K., *Kendall's Advanced Theory of Statistics*, Oxford University Press, New York, 1987.
- [13] MANCINI L., TROJANI F., *Robust Value at Risk Prediction*, Swiss Finance Institute Research Paper Series, 2007, No. 31, pp.
- [14] MCNEIL A.J., FREY R., *Estimation of tail-related risk measures for heteroscedastic financial time series: An extreme value approach*, Journal of Empirical Finance, 2000, Vol. 7, 271–300.
- [15] NEFTCI S.N., *Value at Risk Calculations, Extreme Events, and Tail Estimation*, The Journal of Derivatives, 2000, Vol. 7, 23–37.
- [16] PENG L., WELSH A., *Robust estimation of the generalized Pareto distribution*, Extremes, 2001, Vol. 4, No. 1, 53–65.