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A COMPENSATION-BASED PRICING SCHEME IN MARKETS WITH NON-CONVEXITIES

A compensation-based pricing scheme is a market clearing mechanism that may be applied when a uniform, linear pricing scheme cannot support equilibrium allocations in the auction markets. We analyze extensions of our previously proposed pricing scheme [14] to include various possible representations of bids that reflect some non-convex costs and constraints. We conclude with a discussion on directions for future research.

Keywords: *auction design, electricity market, non-convex bids, minimum profit condition, unit commitment constraints*

1. Introduction

In electricity markets organized as centralized multi-period pool auctions, or power exchange markets, usually there exist non-convexities incorporated through bids, reflecting the operational characteristics of thermal generating units. Such auctions may ensure the feasibility of system operation, together with efficient energy production and maximization of social welfare. However, some difficulties may arise when pricing issues are investigated.

It is now widely recognized that fair market clearing based on non-convex bids cannot be performed using a uniform linear pricing scheme, as the outcomes may be far from optimal for the market participants [2]–[9], [14]. Therefore, in the literature given above, a variety of market-based mechanisms have been proposed to establish appropriate pricing rules that support efficient, centrally imposed schedules. These pricing rules lead to nonlinear prices, which means that trade is cleared based

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mainly on uniform hourly prices. However, some participants (suppliers usually) also receive side payments, or compensation for their profit losses compared to market prices. Such compensation somehow reduces the incongruity between the goals of the market operator and those of self-interested, profit-seeking individual market participants. A discussion of different approaches addressing nonlinear pricing can be found in [14].

In a previous article [14] we proposed a novel pricing scheme for multi-period pool auctions. This scheme, formulated as a MILP optimization problem, enables the determination of uniform market prices minimizing the costs of compensation paid to the market participants (both suppliers and/or customers), who could claim inefficiencies compared to operating under market-clearing prices only. Thanks to such compensation, this method allows each generator and consumer to obtain their maximum profits, while realizing centrally imposed schedules under market prices. In [14] a simplified formulation of such a pricing model was addressed.

In this paper we analyze several applications of an MILP compensation-based pricing scheme for alternative representations of bids, including non-convex costs and constraints, such as start-up, shut-down and fixed costs, minimum-run times and run levels, as well as block bids ("fill or kill"). This allows us to extend the mathematical model for auction pricing provided in [14], so that it is suitable for a broad range of markets with non-convexities. The resulting general optimization model analyzed in this paper can be formulated as a mixed-integer linear programming model. However, as in the case of power exchange with block-bids only linear variables and constraints may be used.

The paper is organized as follows: In the next section we review the compensation-based pricing scheme developed in [14] and discuss the desired properties of non-convex auction pricing. Then, the compensation model is analyzed, followed by a formal representation of different types of non-convex bids. In section 5 we explain how our pricing model can be applied to power exchange with block bids. Finally, we discuss future research in section 6.

2. Properties of a compensation-based pricing scheme

Setting fair market energy prices in auctions with non-convexities is a very challenging problem. There are a number of alternative approaches for pricing in multi-period auctions. However, most research effort has been devoted to designing methods of providing compensation to market participants that is separate from market-clearing price payments [2], [3], [5], [6], [8], [14]. The general idea is to share social costs (compensation costs, or profit-optimality loss) more or less evenly among the market participants. The aim is to treat all the market participants fairly and provide the market operator with more degrees of freedom to appropriately control the strategies of the market participants.

In this paper we refer to [14], where a compensation-based approach for setting fair prices in multi-period pool-based auctions with price-responsive (elastic) demand was developed. The method is based on considering two vectors of competitive market clearing selling prices π^S and purchase prices π^B for energy. The hourly sale and purchase prices are differentiated to share the (nonnegative) costs of the necessary compensation R paid to participants who incur costs due to the necessity of satisfying the network constraints. Such a scheme can be applied as a separate pricing step of the market clearing procedure, after the operational schedules have been determined.

The general idea is shown in Fig. 1: generators and buyers submit sale and purchase bids to the market operator. The market operator solves an optimization problem that defines the power production schedules of the bidding generators and load schedules of the consumers, so that all the bid-based constraints of the participants are satisfied and the energy balance of demand and supply is accomplished in each period of the scheduling horizon. The main goal of the operator may be to maximize social welfare, which may be defined as the sum of consumer and producer surpluses over all periods of time. Next, the social welfare obtained is distributed between those market participants who submitted competitive bids, based on clearing prices and compensation. The clearing prices are used as the basis for payments from buyers and to sellers, while compensation is paid to some market participants to avoid individual profit-optimality losses faced under the clearing prices due to the non-convexities existing in the market. As mentioned before, the hourly purchase and sale energy prices, as well as minimum levels of possible compensation, are computed using an MILP model.

This MILP compensation-based pricing approach has several important properties that should be satisfied to set efficient prices in auctions with non-convexities.

1) All payments should be based on nondiscriminatory uniform linear prices due to the attractive qualities of such a scheme (clarity and transparency among others). If this is not possible, then some additional non-uniform price based payments (i.e. compensation) should be introduced. It is important that uniform prices should be set at a level that produces the minimum level of compensation based payments. This aspect is well understood by most authors dealing with nonlinear pricing. However, due to the complexity of such problems, only procedures that approximate such a solution are known, see [3] and [6]. Using an MILP compensation-based pricing scheme, the minimum possible level of compensation costs R is obtained, through the minimization of the differences between the purchase and selling prices:

$$\min R = \sum_h D_h(\pi_h^S - \pi_h^B) \tag{1}$$

where D_h is the total amount of energy consumed and produced in hour h. In the case of clearing a market with linear costs and constraints (without non-convexities), the optimal solution of the pricing model gives $R = 0$ and the clearing prices are the same for buyers and sellers, i.e. $\pi^S = \pi^B$ [13].

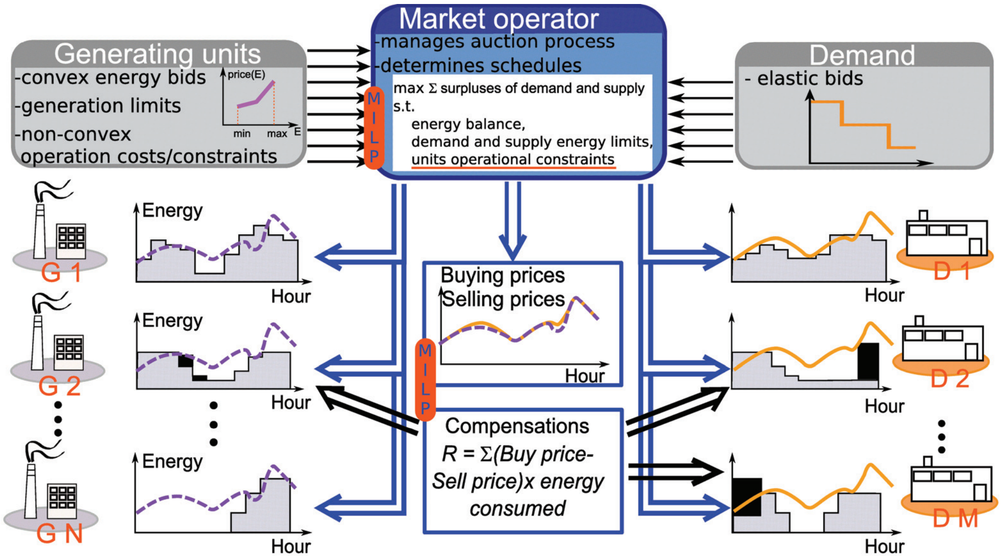

Fig. 1. A pool-based market balancing process under the considered pricing scheme: first efficient schedules are determined, then the prices with compensation

2) The pricing mechanism should be financially neutral, that is all costs must be recovered by charges. This requirement is accomplished in our pricing model using a financial neutrality constraint, requiring that the payments from buyers based on clearing purchase prices minus compensation paid to buyers must be equal to the payments to sellers based on clearing selling prices plus compensation paid to sellers. Some articles, such as [9], do not specify how the non-energy based payments should be collected – the tacit assumption is that the costs of non-linear price payments are charged entirely to consumers by increasing consumer tariffs. Such rules favour the supply side at the expense of the demand side.

3) All market participants must be treated without discrimination, that is to say that no agent would accept losses in profits based on market prices and bids. This is a controversial point, as most authors use confiscatory non-linear prices that allow reducing the price-based payment a supplier should receive [3], [4], [7]–[9]. We state that all agents should obtain their maximum profits by realizing the operational schedule, as this is the only way to reduce the incongruence between individual goals and the operator's goals. In a market environment, profits can never be too high from the point of view of a firm. This condition is also raised in [2], [5], [6].

Concluding this section, the considered MILP pricing scheme satisfies various properties desirable to the operator of a non-convex market. Furthermore, it can be stated as an optimization problem, with (1) as the (minimized) objective function and the following constraints:

- the financial neutrality constraint ensuring that consumer payments based on purchase prices balance the price based generator payments and compensation costs,
- hourly purchase and selling price constraints ($\pi^B >= \pi^S$) ensuring that purchase prices are not less than selling prices in any hour,
- auxiliary nonnegative deficit and surplus price constraints determined for every hourly offer,
- price-dependent compensation constraints linking compensation payments to energy prices, on the basis of the optimal and actual profits made by each of the participants, determined for each participant at any time,
- price-dependent optimal hourly profits determination made by each of the generator, defined on the basis of submitted offers for each generator at each hour.

For details of the complete MILP optimization model, the interested reader is referred to [14]. The last two groups of constraints are analyzed more closely in the next two sections, showing the possibility of adapting them to a wide range of auctions with non-convexities.

3. Compensation and maximum profit constraints

The compensation-based pricing mechanism under consideration was designed to set hourly linear sale and purchase prices, which may be differentiated to share the (nonnegative) costs of compensation paid to participants who lose due to the need to satisfy the network constraints. The aim is to offset financial losses or foregone opportunities for profit, providing profit optimality to all market participants. The compensation paid in addition to the market price-based payments make operational schedules profit maximizing for all consumers and generating units.

A consumer's profits may be defined as the difference between the bid-based benefit function of demand and the price-based payments. Similarly, a generating unit's profits may be defined as the difference between the price-based payments and the bid-based cost function of supply. Henceforth, we will only refer to generating units, since the analysis of the demand side is straightforward as the purchase bids do not incorporate any nonlinear constraints or characteristics. Under uniform linear energy prices, a generator's hourly revenue (profit) ϕ can be formally calculated based on the market clearing price π, amount of energy p to be purchased and the linear, bid-based cost $C(p)$ for producing:

$$\phi = \pi p - C(p)\,. \tag{2}$$

Given the price π, a generator should choose the optimal level of utilization maximizing the profit function (2). When the optimal level of utilization, denoted here by $\hat{p}$, yields less profit than the operational level p^* determined by the market operator, then there is a need for hourly compensation R supporting the economic utilization solution:

$$R = \phi(\hat{p}, \pi) - \phi(p^*, \pi)\,. \tag{3}$$

Notice that the price dependent function $\phi(\hat{p}, \pi)$ is equivalent to the dual function of the bid-based costs $C(p)$. This dual function $P(\pi)$ defines a generator's optimal profit function dependent on price [11]. In Fig. 2 we give an example hourly profit function that is the dual: a) to a stepwise monotonically increasing bid curve; b) to a piecewise, linear bid curve.

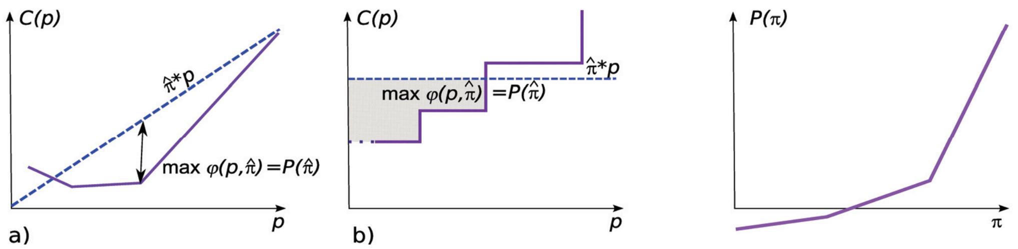

Fig. 2. Optimal profit determination for a given price $\hat{\pi}$ and cost function $C(p)$ resulting from: (a) piecewise linear bid; (b) stepwise bid. The optimal profit function $P(\pi)$ is the dual function to $C(p)$ and has the same, piecewise linear form in both cases

Generally, for any well-behaved, continuous function of bid-based costs $C(p)$, it is easy to derive its dual function $P(\pi)$ [11] that describes the linear dependencies between a generator's optimal profit and the energy price. Then (3) can be presented as:

$$R = P(\pi) - \phi(p^*, \pi) \tag{4}$$

and the prices and compensation payments can be computed using a simple LP compensation minimizing model [13]. The hourly compensation R paid to the agent in a given hour naturally augments its revenue function to guarantee the optimal profit maximizing level of utilization for this agent. We obtain

$$\tilde{\phi} = \phi(p^*, \pi) + R = P(\pi)\,. \tag{5}$$

In a more general case, decision models require not only continuous supply variables p, but also binary commitment variables v. This happens when generators are allowed to specify non-convex, bid-based costs $S(v)$ connected with commitment variables. Bid-based commitment costs may include fixed costs, as well as start-up and shut-down costs [5], [9]. In such a case, the total revenue is the sum of hourly revenues minus the costs connected with commitment:

$$\Phi = \sum_{h \in H} P(\pi_h) v_h - S(v) . \tag{6}$$

In (6) a utilized generator always gets an optimal profit $P(\pi_h)$ from (5), which is not necessarily positive (see Fig. 2). The market price could be lower than the variable costs, but for a generator with commitment constraints it could still be profitable to stay online. Given a vector of hourly prices π, a generator should use the optimal commitment schedule, maximizing (6). When the optimal commitment schedule, denoted here by the vector $\hat{v}$, yields less profit than the operational commitment schedule v^* determined by the market operator, then the need for commitment compensation R^c appears:

$$R^c = \Phi(\hat{v}, \pi) - \Phi(v^*, \pi) . \tag{7}$$

The compensation R^c may act as a nonnegative reward, given to a generator that conforms to the pool operational schedule.

To determine R^c and the prices under such an optimization model, it is necessary to derive the price dependent function $\Phi(\hat{v}, \pi)$. One such way is to use forward dynamic programming to determine a generator's optimal profit [14]. The Bellman inductive function $\omega_{h+1}(v_{h+1})$ denoting the optimum profits from hour 0 to h + 1 for an operating generator that is in state v_{h+1} at hour h + 1 is given by:

$$\omega_{h+1}(v_{h+1}) = P(\pi_{h+1}) v_{h+1} + \max_{v_h \in \{0/1\}} \{\omega_h(v_h) - S(v_h, v_{h+1})\} . \tag{8}$$

The total maximum profit $\Phi(\hat{v}, \pi)$ is obtained in one of the states "on" or "off" in the last hour |H| (Fig. 3).

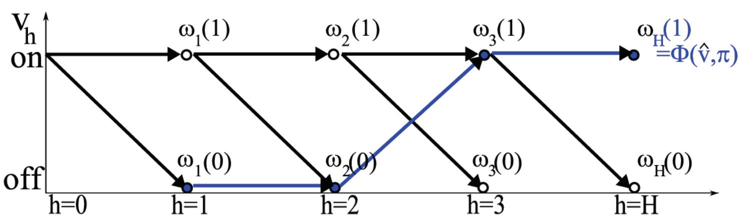

Fig. 3. Idea of the method of determining a unit's optimal total profit based on dynamic programming

Then, we can rewrite (7) as:

$$R^c \geq \omega_{|H|}(v_{|H|}) - \Phi(v^*, \pi) \quad \text{for} \quad v_{|H|} \in \{0/1\}. \tag{9}$$

Proper representation of the inductive profit functions ω requires the introduction of binary variables into the pricing model. For details of complete MILP prices and the formulation of a compensation optimization model, the interested reader is referred to [14].

4. Specifications of pool based auction bids

To cope with specific market requirements, some market designers have established alternative rules that can be imposed on market participants with some degree of arbitrariness and equity. Usually, pool auctions enable generating units to adapt their bidding strategies to capacity constraints and production characteristics: up and down ramp constraints, minimum start-up and minimum shut-down commitment constraints, fixed costs, etc. [1].

Our pricing model can be applied to a number of energy auctions dealing with non-convex bids, as long as they are commitment dependent, as we state in (6) or (8). To be more specific, we show how to model $S(v_h, v_{h+1})$ in (8) to address specific state and inter-state dependent costs:

1) Start-up costs. As we have already shown in [14], start-up costs S^u can be included in (8) as:

$$S(v_h, v_{h+1}) = S^u (1 - v_h) v_{h+1} \,. \tag{10}$$

An illustration is given in Fig. 4.

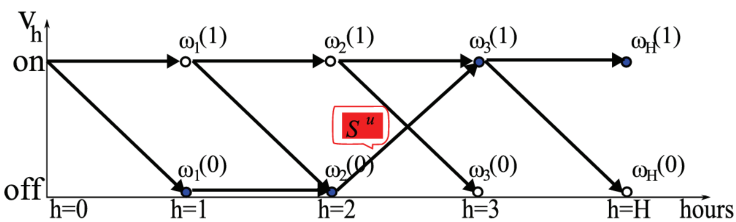

Fig. 4. Inclusion of start-up costs in DP schema

2) Shut-down costs. These costs are ignored in most papers, as the inclusion of shut-down along with start-up characteristics brings no significant contribution, except

from complicating the formulation. Shut-down costs S^d can be modeled analogously to start-up costs:

$$S(v_h, v_{h+1}) = S^d v_h (1 - v_{h+1}) . \tag{11}$$

An illustration is given in Fig. 5.

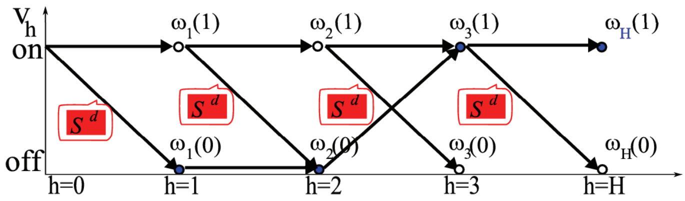

Fig. 5. Inclusion of shut down costs in DP schema

3) Fixed costs. Fixed costs that appear in operational hours are the simplest bid components that bring non-convexities into the market. Bids with fixed-costs have been studied, for example, in [9] and in [5]. In our model, fixed costs S^f can be simply represented by:

$$S(v_h, v_{h+1}) = S^f v_{h+1} . \tag{12}$$

An illustration is given in Fig. 6.

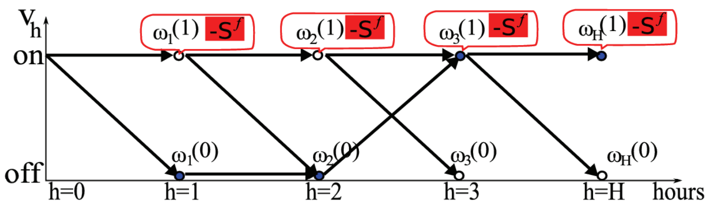

Fig. 6. Inclusion of fixed costs in DP schema

Example. Let us consider the example introduced in [5], based on the case studied in [9]. There are two units each with two generators, each generator having different variable generation costs for producing up to 100 MW each. These costs are \$ 65/MWh and \$ 110/MWh respectively for unit A, \$ 40/MWh and \$ 90/MWh for unit B. Furthermore, unit B has fixed commitment costs equal to \$ 6000/h. Problems with pricing appear for a load that is more than 100 MW and less than 300 MW. With such loads the more expensive generator of unit B is utilized, while unit A is off, or partially utilized. Thus, there are no fair uniform prices that support such an operational schedule. Based on the example studied in [5], we report a comparison of the compensation payments obtained by applying three pricing models – the restricted model “*U r*” proposed in [9], the model used by New York’s Independent System Operator “U d” and the convex hull model “*U h*” proposed in [5]. The first model sometimes gives lower and sometimes lower compensation payments than the second model, but the last method always gives the lowest compensation payments – see Fig. 7. Applying this one-period example to our pricing model gives exactly the same results as using the third model, the lowest compensation costs and consumer payments. However, the model proposed in [5] requires the use of the Lagrangian Relaxation method for the more general multi-period case, while our model can be solved exactly by using a commercial solver (Cplex). Furthermore, none of the previous methods deal with elastic demand.

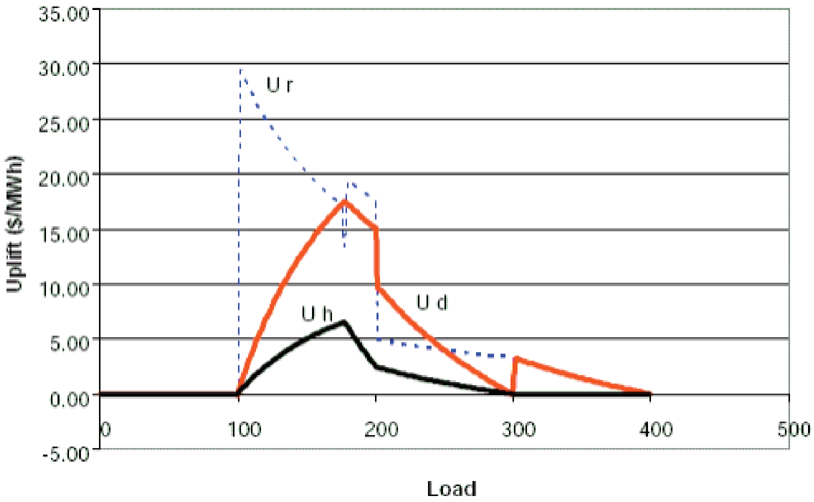

Fig. 7. Comparison of example uplift (compensation) costs.
U r – restricted model [10], *U d* – model of New York ISO, *U h* – convex hull model [5].
The results of our model are the same as those of *U h*.
Source [5]

4) Capacity constraints. Constraints on the minimum generation capacity also bring non-convexities into the auction, but they only affect the hourly revenue function (2), and together with that the price dependent profit function $P(\pi)$. In a market with only capacity constraints, a simple LP pricing model without commitment compensation may be applied.

5) Start-up and shut-down time characteristics. The inclusion of a start-up time T^u and shut-down time T^d involves the formulation of the recursive function $\omega_{h+1}(v_{h+1})$ given by (8), see Fig. 8.

The transition to a given commitment state v_{h+1} can be performed without delay only from the same commitment state, whereas the transition from the other state requires T^u or T^d hours, depending on the state v_{h+1}. Then, instead of by (8), the recursive function ω determining the optimal profit at hour $h+1$ is:

$$\omega_{h+1}(v_{h+1}) = P(\pi_{h+1})v_{h+1} + \max_{v_h \in \{0/1\}}\{\omega_h(v_h),$$

$$\omega_{h-T^u} v_{h+1} - T^d v_{h+1}(1-v_{h+1}) - S^u v_{h+1} - S^d(1-v_{h+1})\} \tag{13}$$

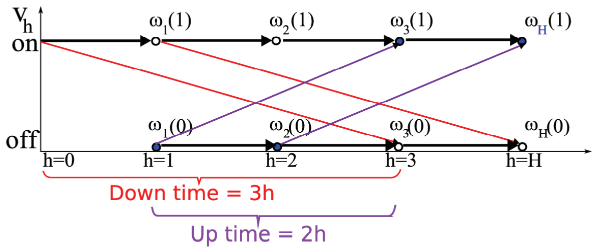

Fig. 8. Inclusion of start up and shut down time in the DP schema

The formulation in (13) gives a general idea of how ω is determined when the time characteristics are considered. To model all dependencies, one should investigate both the beginning and end of a planning horizon, where start-ups and shutdowns are not considered. This would bring no significant contribution, but would lead to more complicated notation.

Furthermore, the pricing scheme considered can be easily extended to include constraints imposed by the transmission network. In [15] we explain how such a mechanism can determine compensation and hourly selling and purchase prices π^S_{nh}, π^B_{nh} at a network node n. Differences in purchase and selling prices at the nodes enable covering the costs of the necessary compensation paid to the participants whose optimal unconstrained schedules are incompatible with the operational schedules. Differences in nodal selling prices enable covering the congestion costs.

5. Power exchange with non-convex bids

Pool-based auctions do not exhaust all the possibilities of non-convex market. Some market rules other than centralized unit commitment frameworks can also help the participants to satisfy their non-convex operational constraints. For example, as in the Amsterdam Power Exchange, market participants are allowed to use block bids to specify their requirements for a minimum income (maximum payments) [10].

A block bid is a bid for which participants offer to buy or sell the same quantity of energy Q, for a given period of T consecutive hours at a minimum unit price b for sales (maximum price for purchases). An illustration is given in Fig. 9.

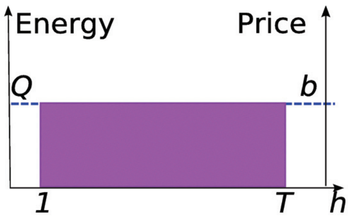

Fig. 9. An example of a block bid

The approach of using block bids to take non-convexities into account in an auction enables simplifying the interpretation of R^c compensation given in (7). Compensation R^c is paid only for block bids (other bids can only receive dispatch compensation) when non-competitive pricing exists. The market operator decides whether a block bid is accepted or rejected. This is indicated using a binary variable z. The revenue from a block bid is the sum of hourly revenues in the given time period T:

$$\Phi(z) = z \sum_{h \in T} (\pi_h - b) Q \,. \tag{14}$$

Given the price π, the generator should attempt to achieve the optimal acceptance decision $\hat{z}$, maximizing (14). When the optimal decision $\hat{z}$ yields lower profit than the operational decision z^*, then theneed for compensation R^c appears:

$$R^c \geq \Phi(1 - z*) - \Phi(z*) \,. \tag{15}$$

In the case of an exchange market with a minimum profit condition, the prices and compensation payments can be computed using a simple LP compensation minimizing model.

It should be noted that the conditions for assigning compensation to block bids differ from those postulated in [10]. Here, all block bids, both accepted and rejected, obtain the optimal profits under market prices. In [10] the authors suggest that rejected bids should get confiscatory prices d to inform the bidder of the minimum amount that the bid would need to be changed by to be accepted. If such a market rule is desired, then (15) should be rewritten as:

$$R^c \geq \Phi(1 - z*) - \Phi(z*) - d(1 - z*) \,. \tag{16}$$

As the nonnegative price *d* does not play a role in redistributing the surplus among the market participants, it should be included in the objective function defining the compensation to be minimized (1).

Table 1. Example of a two period block bid.
Source [10]

Bid	Period 1		Period 2	
	Price	Quantity	Price	Quantity
1	70	100		
2	–5	125*	–5	125*
3	–10	60		
4	–30	100*	–30	100*
5	–40	70	–40	160
6			90	150

The operational schedule maximizing social welfare is given in Table 2.

Table 2. Optimal schedule for two period block bid.
Source [10]

Bid	Period 1		Period 2	
	Price	Quantity	Price	Quantity
1	70	100		
2	–5	0	–5	0
3	–10	0		
4	–30	100*	–30	100*
5	–40	0	–40	50
6			90	150

Table 3. Comparison of the prices obtained under the model described in [10] and the one discussed in this paper

	Model [10] – solution		Model [10] – alternative solution		Model discussed in this paper	
	Purchase price	Selling price	Purchase price	Selling price	Purchase price	Selling price
Period 1	10+4.4445	10	10+10	10	20	10
Period 2	40+3.7	40	40	40	40	40

Block bid 4 is accepted, while the cheapest block bid, 2, is rejected, as it is not feasible in period 1 (the demand is less than the quantity offered by Bid 2). On the other hand, Bid 3 with price 10 is rejected as it offers too small a quantity of energy. The payments determined under the multi-part prices proposed in [10] yield exactly the same results as our pricing scheme, i.e. consumer payments are equal to 8000 and compensation costs are equal to 1000 (see Table 3). We reproduce two equivalent solutions to the example given in [10] (the second one gives the same prices as ours), but generally there is a continuum of potential prices that may clear the market. Nevertheless, they will always give the same total payments and compensation.

The compensation paid to Bid 4 is 1000 for both approaches (under the assumption that the disincentive price is sent to Bid 2). The solution and degree of freedom in pricing are the same for both approaches. However, it is not explained in [10] how the proposed model could be extended to consider, for example, start-up costs. Our pricing scheme, as shown in this simple example, has no limitations to including further constraints.

6. Summary and future research

We have analyzed a pricing model that computes the minimum compensation required to provide the participants of non-convex markets with fair revenues. We have described extensions of the basic model to cope with various non-linear costs and constraints, as well as several forms of bids. Using simple examples found in the very recent and relevant literature, we have illustrated the desirable properties of our compensation based pricing scheme in comparison to other methods. We have given some guidelines on how to use the considered pricing model in various pool-like multiperiod markets.

Within the framework of centralized auctions, there may be many exact or near-optimal solutions. For example, if Lagrangean relaxation is used, it may result in similar commitments or MIP near-optimal solutions [5], [12]. However, small variations in near-optimal commitments could yield significantly different payoffs to the market participants. This issue is comprehensively studied in [12].

The pricing model considered here may serve as a mechanism for prices that can support operational solutions and is applied as a separate step of the market clearing procedure, after solving the unit commitment problem. Examination of the economic properties of this pricing model should be addressed further using a number of tests, but the preliminary analysis of some case studies has produced promising results with a highly reduced profit volatility.

The future line of research should also include an analysis of incentive compatibility. The main need for complex auctions is to design fair pricing rules that would give incentives to the marginal participants to reveal their true costs. This requires broadening the theory on strategic behavior for complex auctions.

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Mechanizm rozliczeń oparty na rekompensatach do zastosowania na rynkach z ofertami nieliniowymi

Rozważany w pracy model rozliczania obrotu oparty na rekompensatach jest odpowiednim narzędziem wyceny na rynkach giełdowych w sytuacji, gdy nie można wyznaczyć jednolitych cen równowagi. Sytuacja taka ma miejsce np. na bilansującym rynku energii elektrycznej, gdy w procesie bilansowania rynku są uwzględniane indywidualne ograniczenia i nieliniowe koszty jednostek wytwórczych. W pracy rozważono model wyceny opracowany przez autorów w [14], analizując jego właściwości istotne z perspektywy projektowania mechanizmów rynkowych (indywidualna racjonalność, neutralność finansowa, efektywność). W głównej części pracy przedstawiono szersze zastosowania modelu do różnych typów ofert (schodkowe, przedziałami liniowe) oraz wprowadzanych ograniczeń (czasy rozruchu i odstawienia, koszty rozruchu i odstawienia, koszty stałe). Ważnym obszarem zastosowań mechanizmu mogą być również giełdy energii dopuszczające składanie ofert blokowych. Pokazano schematy modelowania zadania rozliczania obrotu w postaci zadania MILP dla różnych typów rynków. Poszczególne mozliwośći zastosowania zilustrowano na przykładach. W podsumowaniu zaprezentowano dalsze kierunki rozwoju badań nad modelami wyceny na rynkach z ograniczeniami.

Słowa kluczowe: *projektowanie aukcji, rynek energii elektrycznej, oferty nieliniowe, ograniczenia jednostek cieplnych*