OPERATIONS RESEARCH AND DECISIONS

No. 4 2020
DOI: 10.37190/ord200406

MOVING AVERAGE CONTROL CHARTS FOR BURR X AND INVERSE GAUSSIAN DISTRIBUTIONS

AMBREEN SHAFQAT[1], MUHAMMAD ASLAM[2*], MOHAMMED ALBASSAM[2]

[1]Department of Statistics and Financial Mathematics, School of Science,
Nanjing University of Science and Technology, Nanjing, Jiangsu, P.R. China

[2]Department of Statistics, Faculty of Science, King Abdulaziz University, Jeddah, Saudi Arabia

The Burr X and inverse Gaussian (IG) distributions have been considered to design an attribute control chart for time truncated life test with the moving average (*MA*) scheme *w*. The presentation of the *MA* control chart has been estimated in terms of average run length (*ARL*) by using the Monte Carlo simulation. The *ARL* is determined for different values of sample sizes, *MA* statistics size, parameters' values, and specified average run length. The performance of this new *MA* attribute control chart has been compared with the usual time truncated control chart for Burr X and IG distributions. The performance of a new control chart is better than that of the existing control chart.

Keywords: *control chart, Burr X distribution, inverse Gaussian distribution, average run length*

Acronyms

MA – moving average
IG – inverse Gaussian
ARL – average run length
AT – average time
EWMA – exponentially weighted moving average
LCL – lower control limit
UCL – upper control limit
SPC – statistical process control

1. Introduction

Nowadays, many technical methods for monitoring the industrial process are widely available. One of them is a statistical method, which supplies many useful ways for

*Corresponding author, email address: aslam_ravian@hotmail.com
Received 27 October 2019, accepted 2 December 2020

industrial process control. The quality control chart, introduced by Shewhart in 1924, is the most popular and useful method, which can be used to detect the presence of assigned resources in the manufacturing industry. The main purpose of control charts is created on the assumption of the different distributions of the quality specifically is normal or approximately normal.

The control chart is an important and useful statistical method that helps practitioners to monitor and overcome the statistics of one, two, or more variables, when the quality of the process is characterised by a certain value of this or that variable. A process in-control when no assignable causes are presented in the process, and out-of-control when it has some assignable causes, like machinery error, etc.

The control chart procedure is quite easy to perform in every type of process. However, the control charts are mostly used in the manufacturing area of industry for preserving the quality of the final products or process. Process monitoring and control are accomplished at the quality of the products and process variations, and average levels. The researcher provides full discussion about the statistical process control (SPC) techniques in the manufacturing industry [1].

Apart from that, the number of applications implemented into areas outside of predictable production system has been increasing in recent years. Control charts can be classified into two types, one is variables control charts, and the other is control charts for attributes. The control charts for variables are used for quality appearances, which are computable like temperature, length, width, viscosity, and ductile strength, etc. We use variable control charts, such as R-chart, X-bar chart, and S-chart, whereas the control charts for attributes are operated for those quality features which are non-computable. The count data control charts that we usually use are *p*-chart, *np*-chart, *u*-chart, and *c*-chart.

Montgomery [2] states that the control charts for attributes are generally very imperative in non-industrial superiority progress struggles, in which the target quality appearances are difficult to calculate on a statistical scale. The detection of defective items in the industrial process is mutual for refining the quality of the manufacturing products. Kramer and Schmid [3] discussed the basic process parameters and the behaviour of the Shewhart chart about the rest of the Shewhart chart and design autoregressive models that supposed the assessment of the autoregressive coefficients. Finally, they matched the results of *ARL* for independent variables with *ARL* of the control chart with predictable parameters and with the *ARL* of the outline for identified parameters.

A review and short-term summary [4] of the slog on the variables control chart to observe the procedure mean and diffusion from 1993 to 2008 is stated in [4]. Gadre and Rattihalli [5] use a multi-attribute *p*-control chart to identify a modification in the values of distribution factors. According to them, the sizes of the proposed change in each parameter must be well-defined in advance. Huang et al. [6] developed an average time to signal-unbiased control chart with known parameter and make a comparison of the traditional and proposed method. The results show this control chart is more sensitive to system deterioration.

A lot of control charts are already designed for the moving average (*MA*) charts. This control chart is very simple and easy to smear. It is constructed on a simple average of diverse sizes. Additionally, the *MA* control charts are more delicate to identify small shifts in the procedure as equated to the Shewhart control chart [7]. Michael [8] discussed an easier, efficient and alternative approach of the *p*-chart. He presented the corresponding simulation and mathematical calculations results for the *ARL* which show that the performance of the new approach is superior to that of the standard approach. Wong et al. [9] designed a simple procedure for the *MA* control chart and the combined *MA*–Shewhart arrangement for an easy application. Patil et al. [10] make an economic design of *MA* control charts for ceased as well as for continued production. Recently, Patil and Shirke [11] have developed an economic cost function for *MA* chart under non-normal quality characteristics.

The *MA* control charts have been widely recycled for checking the manufacturing development in the industry because they custom the information which they achieved from the whole structure of points, and Shewhart charts only use present-day information. In [12], the scholar worked on hybrid exponentially weighted *MA* control chart. Aslam et al. [13] work on a double *MA* control chart for the exponential distribution. The study [14] developed a control chart for Com-Poisson distribution by consuming several state sampling. Azam et al. [15] worked on a design of X-bar chart for Burr distribution under repetitive sampling. Shafqat et al. [16] introduced the X-bar control chart for Rayleigh distribution, using the repetitive sampling scheme. A time truncated *MA* control chart for the Weibull distribution has been designed by Alghamdi et al. [17]. Haq et al. [18] worked on improved fast initial response features for exponentially weighted *MA* and cumulative sum control charts, while Khoo and Yap [19] proposed a *MA* control chart for combined monitoring of variance and mean in the procedure.

For measuring, the presentation association of all types of control charts is evaluated by the design of the average run length. It is the usual number of samples, which is composed before an out-of-control signal. Apart from that, the average run length value is a high concentration in the improvement of all control charts outline. Li et al. [20] proposed a study on average run length and the average time to a pointer. Molnau et al. [21] examine *ARL* for multivariate EWMA control charts. Crowder [22] studied average run length for an EWMA chart and Chananet et al. [23] operated on *ARL*, using a Markov chain. GMA (group moving average) control charts of different sizes are developed [24] for process monitoring. GMA control chart plans have good command on *ARL* properties over the range of location shift and Sparks [25] studies weighted *MA* chart for an efficient plan of monitoring specific location shift. Wetherill and Brown [26] provided some different and beneficial discussion on the *ARL* of *MA* charts, showing that the formula for computing the *ARL* of an *MA* chart given in Wetherill and Brown [26] is not good but it may provide an upper bound for the *ARL*.

MA control charts are used for checking if the quality of interest follows the normal distribution or not. Due to the increment of product reliability such as mobile, it is not possible to give a long time for the test. In this situation, the *MA* control chart for the lifetime-truncated test is useful to observe the manufacturing process. Based on the contribution of different authors about the *MA* charts, there is no effort on the proposal of *MA* control charts for Burr X and inverse Gaussian (IG) distributions. The designed *MA* control chart works with diverse sample sizes for the in-control procedure to observe the modifications in scale parameters of Burr X and IG distributions. The proposed *MA* control chart with various sample sizes for the in-control chart method is used to display the change in the scale parameter of the Burr X and IG distributions. The results in this paper are calculated based on the scale parameter of the distributions and sample sizes of the construction process.

The structure of this paper is as follows: Section 2 describes the technique of the proposed control chart. Section 3 explores the *MA* control chart based on Burr X and IG distributions. The presentation of the proposed control chart expending simulation is given in Section 4, and a comparison is included in Section 5. Towards the end of Section 6, conclusion and prospect proposals are formulated.

2. Technique of *MA* control chart

This selection will monitor the *MA* control chart design as mentioned in [17]. The moving average value of size w at time i for the number of failures D_i' is calculated as

$$MA_i = \frac{D_i + D_{i-1} + D_{i-2} + \ldots + D_{i-w+1}}{w} \tag{1}$$

Here, D_i distribution is used equally as a binomial distribution with mean np_0 and variance $np_0(1-p_0)$. We can compute the mean and variance of MA_i statistic as follows:

$$E\left(MA_i\right) = E\left(D_i\right) = np_0 \tag{2}$$

$$\begin{aligned}\text{Variance}\left(MA_i\right) &= \frac{1}{w^2}\sum_{j-i-(w+1)}^{i} \text{Var}\left(D_i\right)\\ &= \frac{1}{w^2}\sum_{j-i-(w+1)}^{i} np_0\left(1-p_0\right) = \frac{np_0\left(1-p_0\right)}{w}\end{aligned} \tag{3}$$

The proposed *MA* control chart is identified as follows:

Step 1. Choose a sample size n from the production process at the i subgroup and start the process. Calculate the number of failures D_i in the termination time. Calculate the MA_i by using equation (1).

Step 2. The process is declared to be out of control when $MA_i > UCL$ or $MA_i < LCL$, and the process declares to be in control when $LCL \le MA_i \le UCL$

$$UCL = np_0 + k\sqrt{\frac{np_0\left(1-p_0\right)}{w}} \tag{4}$$

$$LCL = np_0 - k\sqrt{\frac{np_0\left(1-p_0\right)}{w}} \tag{5}$$

The value of k should be calculated by supposing the mark value of in-control ARL_0, which is control constant here. The control limits here may be flexibly designed, according to the mark value of in-control ARL_0 and stated time test. In-control and out-of--control probability can be as

$$P_{in}^{0} = \sum_{d=LCL+1}^{UCL} \binom{n}{k} p_0^d \left(1-p_0\right)^{n-d} \tag{6}$$

Note that, if $LCL < 0$, then $d = 0$.

$$P_{in}^{1} = \sum_{d=LCL+1}^{UCL} \binom{n}{k} p_1^d \left(1-p_1\right)^{n-d} \tag{7}$$

If the $LCL = 0$, then d must be zero. The ARL is used for checking the performance of the control chart. We consider the process to be in control when ARL_0 is given in before the signal and process is out of control when ARL_1 values are detected after a signal.

The probability distribution function of the Burr X distribution where β is the scale parameter and α is the shape parameter

$$f\left(t;\, \alpha, \beta\right) = 2\alpha\beta^2 t \exp\left(-\left(\beta t\right)^2\right)\left(1-\exp\left(-\left(\beta t\right)^2\right)\right)^{\alpha-1}, \quad t>0,\, \alpha>0,\, \beta>0 \tag{8}$$

The cumulative function of Burr X distribution is

$$F\left(t;\, \alpha, \beta\right) = \left(1-\exp\left(-\left(\beta t\right)^2\right)\right)^{\alpha}, \quad t>0 \tag{9}$$

Suppose t_q denotes the true percentile value and t_{q0} denotes the specified percentile value. The proposed control chart is designed for monitoring the percentile ratio which is between true and specified percentile (described detail after equation (13)) as a shift by observing the number of failed products by the specified time t_0.

The probability function of the Burr X distribution is

$$q = \left(1 - \exp\left(-\left(\beta t\right)^2\right)\right)^{\alpha} \tag{10}$$

The value of p here is the probability of a fail item in any group before truncation time t_0. If we specify t_0 in terms of a multiple of the in-control process percentile through $t_0 = \delta_q t_{q0}$ for a constant δ_q, the population parameter β in equation (9) is given as follows:

$$t_q = \beta^{-1}\sqrt{-\ln\left(1 - q^{1/\alpha}\right)}, \quad 0 < p < 1 \tag{11}$$

and after transforming we have

$$\beta = \frac{\sqrt{-\ln\left(1 - q^{1/\alpha}\right)}}{t_q} \quad 0 < p < 1$$

Now, substituting the β value from equation (11) into equation (9) we obtain:

$$P = F\left(t_q\right) = \left(1 - \exp\left(-\left(\frac{\sqrt{-\ln\left(1 - q^{1/\alpha}\right)}}{t_q} t_{q0}\right)^2\right)\right)^{\alpha} \tag{12}$$

So, now for in-control (IC) and out-of-control (OOC) processes we can transform equation (12) as a function of true and specified percentiles t_q/t_{q0}, and then we arrive at

$$P = F\left(\frac{t_q}{t_{q0}}\right) = \left(1 - \exp\frac{\ln\left(1 - q^{1/\alpha}\right)}{\left(\frac{t_q}{t_{q0}}\right)^2}\right)^{\alpha} \tag{13}$$

For IC, when the ratio of the true percentile and specified percentile is equal to one, it is written as

$$P_0 = F\left(\frac{t_q}{t_{q0}} = 1\right) = \left(1 - \exp \ln\left(1 - q^{1/\alpha}\right)\right)^{\alpha} \tag{14}$$

For OOC, when the ratio between the true percentile and specified percentile is equal to shift f, it is written as

$$P_1 = F\left(\frac{t_q}{t_{q0}} = f\right) = \left(1 - \exp \frac{\ln\left(1 - q^{1/\alpha}\right)}{f^2}\right)^{\alpha} \tag{15}$$

IG distribution is normally used for modelling the lifetime and it is also useful for holding the long-tailed data. The procedure of the IG distribution is a stochastic process. Now, we describe IG process as is done in detail in [27]

$$f\left(t;\, \mu, \lambda\right) = \sqrt{\frac{\lambda}{2\pi t^3}} \exp\left(-\frac{\lambda\left(t-\mu\right)^2}{2t\mu^2}\right), \quad 0 < t < \infty$$

where λ is a scale parameter and mean is a μ. The cumulative density function of IG distribution can be written as

$$F\left(t;\, \mu, \lambda\right) = \Phi\left(\sqrt{\frac{\lambda}{t}}\left(\frac{t}{\mu} - 1\right)\right) + \left(\exp \frac{2\lambda}{\mu}\right) \Phi\left(-\sqrt{\frac{\lambda}{t}}\left(\frac{t}{\mu} + 1\right)\right) \tag{16}$$

where Φ shows the cumulative distribution function of a standard normal distribution.

The OC function is given by

$$L\left(p\right) = \sum_{i=LCL+1}^{UCL} \binom{n}{i} p^i \left(1-p\right)^{n-i}$$

where p is the probability that an item fails by time t_0, that is $p = F(t_0)$, so we will consider $t_0 = a\mu_0$ for a constant a.

Then, the cumulative distribution function can also be written as

$$P = F\left(t_0\right) = \Phi\left(\sqrt{\frac{\lambda}{a\frac{\mu}{\mu_0}}}\left(\frac{1}{\frac{\mu}{\mu_0}} - 1\right)\right) + \left(\exp 2\lambda\right) \Phi\left(\frac{\lambda}{a\frac{\mu}{\mu_0}}\left(\frac{a}{\frac{\mu}{\mu_0}} + 1\right)\right) \tag{17}$$

when the ratio is equal to 1, then equation (10) is written as

$$P_0 = F(t_0) = \exp 2\lambda\Phi\left(\sqrt{\frac{\lambda}{a}}(a+1)\right) \tag{18}$$

when $\mu/\mu_0 = f = 1, 0.9, 0.8, 0.7, \ldots$ the in-control process P can be written as

$$P_1 = F(t_0) = \Phi\left(\sqrt{\frac{\lambda}{a(f)}}\left(\frac{1}{f}-1\right)\right) + (\exp 2\lambda)\,\Phi\left(\sqrt{\frac{\lambda}{a(f)}}\left(\frac{a}{f}+1\right)\right) \tag{19}$$

3. Presentation of the proposed control chart by expending simulation

The most famous scheme in quality control for the calculation of control chart is a simulation that is useable when the theoretical approach is difficult to tool, and also when it provides approximate solutions in the mathematical problems by solving the statistical sampling problems in the computer. Schaffer and Kim [28] studied a large number of statistical sampling experiments. Numerous authors, including Shafqat et al. [29], Aslam [30], Sullivan and Woodall [31], and Fu et al. [32], worked on Monte Carlo simulation. In this paper, we introduce the *ARL* values for in-control and out-of-control chart by using the equations:

$$ARL_0 = \frac{1}{1-P_{in}^0} \tag{20}$$

$$ARL_1 = \frac{1}{1-P_{in}^1} \tag{21}$$

Table 1. The Burr X *ARLs* of the proposed *MA* chart with $w = 3$ for $ARL_0 = 370$ and $\alpha = 1.856$

n	20	30	40	50
q	0.6327	0.6342	0.6368	0.9285
k	4.207398	4.6342	7.033455	6.047895
Shift(f)	*ARL*			
1.0	370.9603	370.5056	370.4220	370.422
0.9	206.404	342.65	189.768	206.03
0.8	14.78	148.50	11.09	60.918
0.7	11.98	10.16	1.0001	16.56
0.6	1.01	1.23	1.00	4.96
0.5	1.00	1.00	1.00	1.86
0.4	1.00	1.00	1.00	1.09
0.3	1.00	1.00	1.00	1.00
0.2	1.00	1.00	1.00	1.00
0.1	1.00	1.00	1.00	1.00

Table 2. The Burr X distribution of the proposed *MA* control chart with $w = 5$ for $ARL_0 = 370$ and $\alpha = 1.856$

n	20	30	40	50
q	0.8345	0.9347	0.96455	0.99449
k	6.448308	7.863895	8.626822	8.916839
Shift	*ARL*			
1.0	369.9402	369.5499	370.2416	370.0577
0.9	198.67	103.82	211.78	193.95
0.8	52.08	14.92	110.54	78.36
0.7	26.12	2.78	57.63	10.79
0.6	5.90	1.00	10.89	2.09
0.5	1.09	1.00	3.89	1.00
0.4	1.00	1.00	1.00	1.00
0.3	1.00	1.00	1.00	1.00
0.2	1.00	1.00	1.00	1.00
0.1	1.00	1.00	1.00	1.00

The value of k and time-truncated constants will be generated according to the sample size n and moving average statistics size w such as in equation (20). The ARL_1 will be calculated by using equation (21), according to different values of the shift f. The *ARLs* are calculated by using the R-language program.

Table 3. The Burr X distribution of the proposed *MA* control chart with $w = 3$ for $ARL_0 = 370$ and $\alpha = 2$

n	20	30	40	50
q	0.6837	0.6355	0.6368	0.6502
k	4.791058	5.245382	5.186646	5.0849
Shift	*ARL*			
1.0	370.70	370.1183	370.2953	370.0380
0.9	341.73	209.87	309.34	231.18
0.8	109.78	78.65	101.06	10.98
0.7	13.89	20.54	10.05	02.03
0.6	4.06	9.02	1.43	1.00
0.5	1.08	1.00	1.00	1.00
0.4	1.00	1.00	1.00	1.00
0.3	1.00	1.00	1.00	1.00
0.2	1.00	1.00	1.00	1.00
0.1	1.00	1.00	1.00	1.00

The above *MA* control chart scheme has been planned for various processes. We use *MA* sizes for the control chart $w = 3$, and 5 with different values of shape and scales parameters of the Burr X and IG distributions, while the subgroup sizes of n equal to 20, 30, 40, and 50 are considered. The performance of *ARLs* of the *MA* control chart is computed for different shift levels from 1.0 to 0.1. The target in-control levels of $ARL_0 = 370$ is specified in Tables 1–-6. In Table 7, we show both distributions existing and proposed chart at $ARL_0 = 290$ with different values of scale and shape parameters.

Table 4. The Burr X distribution of the proposed *MA* control chart with $w = 5$ for $ARL_0 = 370$ and $\alpha = 2$

n	20	30	40	50
q	0.5116	0.5483	0.6563	0.6654
k	6.679675	6.786885	6.759283	6.65274
Shift	*ARL*			
1.0	369.4383	370.5378	369.8119	370.4504
0.9	291.08	119.76	276.54	175.54
0.8	89.45	54.34	78.16	155.47
0.7	35.12	6.01	24.84	91.78
0.6	4.78	1.09	8.29	43.06
0.5	1.01	1.00	3.13	18.18
0.4	1.00	1.00	1.51	1.00
0.3	1.00	1.00	1.05	1.00
0.2	1.00	1.00	1.00	1.00
0.1	1.00	1.00	1.00	1.00

Table 5. The IG proposed *MA* control chart with $w = 3$ for $ARL_0 = 370$ and $\lambda = 2.9$

n	20	30	40	50
a	0.9654	0.8740	0.7793	0.7007
k	5.108475	5.15429	4.965404	5.062807
Shift	*ARL*			
1.0	370.1332	370.1992	370.5447	370.0973
0.9	290.98	342.65	308.66	288.78
0.8	7.29	108.57	3.06	7.90
0.7	1.69	10.16	1.06	2.01
0.6	1.00	1.23	1.00	1.00
0.5	1.00	1.00	1.00	1.00
0.4	1.00	1.00	1.00	1.00
0.3	1.00	1.00	1.00	1.00
0.2	1.00	1.00	1.00	1.00
0.1	1.00	1.00	1.00	1.00

From Tables 1–6, we note that the values of *ARLs* are decreasing as the sample size n is increasing to 20, 30, 40, 50. With fixed values of both distributions' scale parameters, the *ARLs* decreases when w values increase. Also, *ARLs* values increase when values of shape parameters increase.

Table 6. The IG proposed *MA* control chart
with $w = 5$ for $ARL_0 = 370$ and $\lambda = 2.9$

n	20	30	40	50
a	2.1504	2.1878	2.1015	2.6969
k	6.41575	7.389155	6.924204	6.883389
Shift	*ARL*			
1.0	370.4597	370.3311	370.9367	370.3617
0.9	265.96	105.55	242.65	202.45
0.8	105.01	60.19	148.57	19.07
0.7	11.26	16.62	10.16	2.01
0.6	4.69	4.96	1.23	1.00
0.5	1.04	1.86	1.00	1.00
0.4	1.00	1.09	1.00	1.00
0.3	1.00	1.00	1.00	1.00
0.2	1.00	1.00	1.00	1.00
0.1	1.00	1.00	1.00	1.00

4. Model study

Here we present a performance of the planned *MA* control chart by expanding simulated data. The data for *MA* chart is made from the IG distribution with $\lambda = 2$, and $w = 2$. The first 20 observations of size 20 data generated by using the in-control process with the in-control parameter $\mu_0 = 5$, and the second 20 observations are created by using the shifted process with the parameter $\mu_1 = 0.5\mu_0$. The number of defectives or failures D's are achieved by pleasing the $t_0 = 4.5$. The calculated values of D_i and MA_i are mentioned below.

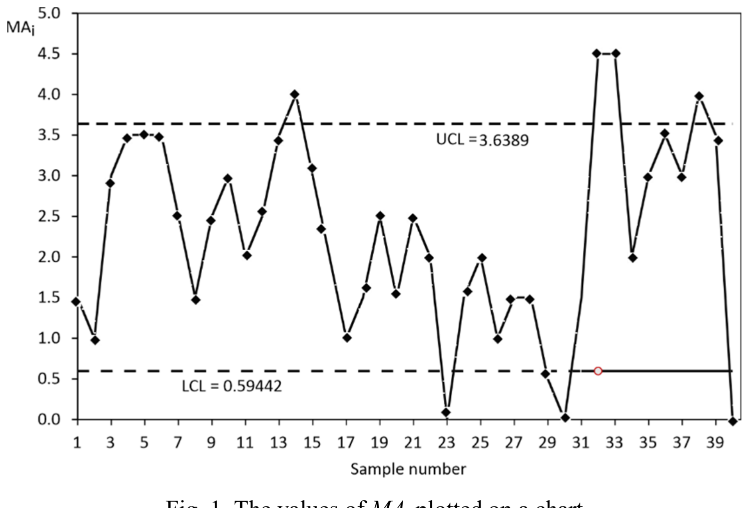

Fig. 1. The values of MA_i plotted on a chart

In Figure 1, the moving average values are plotted and we can see that the first shift detect the out-of-control signal at 14th sample. But the shifted values of the proposed control chart detect the out-of-control point at lower limits first time at 23rd sample. The proposed chart shows that total 7 points are out of control throughout the process.

5. Comparison

The *ARLs* values of *MA* proposed control chart with $w = 5$ and existing time truncated control chart when $ARL_0 = 290$ for both distributions (scale parameters of Burr X and IG distributions) are given in Table 7.

Table 7. The values of defective items and *MA*

D_i	MA_i	D_i	MA_i	D_i	MA_i	D_i	MA_i
3	1.5	2	2	1	2.5	0	1.5
0	1	2	2.5	4	2	3	4.5
2	3	3	3.5	0	0	6	4.5
4	3.5	4	4	0	1.5	3	2
3	3.5	4	3	3	2	1	3
4	3.5	2	2	1	1	5	3.5
3	2.5	2	1	1	1.5	2	3
2	1.5	0	1.5	2	1.5	4	4
1	2.5	3	2.5	1	0.5	4	3.5
4	3	2	1.5	0	0	3	NA

Table 8. A comparison of the proposed *MA* chart for IG and Burr X distributions having $w = 5$, $ARL_0 = 290$ with the existing chart [27]

Shift	Burr X		IG	
	Proposed chart	Existing chart	Proposed chart	Existing chart
1.0	274.28	284.28	290	290.98
0.9	120.20	206.00	5.23	6.29
0.8	5.03	60.19	1.00	2.69
0.7	1.00	16.62	1.00	1.00
0.6	1.00	4.96	1.00	1.00
0.5	1.00	1.86	1.00	1.00
0.4	1.00	1.09	1.00	1.00
0.3	1.00	1.00	1.00	1.00
0.2	1.00	1.00	1.00	1.00
0.1	1.00	1.00	1.00	1.00

The *ARL* values of the proposed charts are smaller as compared to the existing charts and it shows the efficiency of the proposed charts. The Burr X distribution proposed chart a shift is also detected earlier as compared to the existing chart but detects out-of-control shift later as compared to the IG distribution in the proposed chart. So, the IG control chart is more efficient than the proposed Burr X control chart, and other existing charts, too.

6. Conclusion

A new *MA* control chart is proposed for the Burr X and IG distributions under a time truncated life test. The coefficients of control chart and test time constants are resolved for different producers of both distributions parameters settings. The presentation of *MA* control chart is described according to the *ARLs* values, by using the different shift values. We made a comparison of the new *MA* control chart and the existing time truncated control chart [27], showing the better efficiency of the new control chart than the existing control chart. So, we can say that the new *MA* control chart will help the engineers to check the quality of products with short time and price more than the existing time truncated control chart. It has been indicated that the proposed *MA* control chart will be a more efficient toolkit for quality checking in the industry. The *MA* control chart can be extended as future research to other non-normal distributions.

Acknowledgements

The authors express their deep gratitude to the editor and reviewers for their valuable suggestions that helped to improve the quality of the paper.

References

[1] BISSELL D., MONTGOMERY D.C., *Introduction to statistical quality control*, Stat., 2006, DOI: 10.2307/2988304.
[2] DOUGLAS M.C., *Introduction to Statistical Quality Control*, Wiley, 2009.
[3] KRAMER H., SCHMID W., *The influence of parameter estimation on the ARL of Shewhart type charts for time series*, Stat. Pap., 2000, DOI: 10.1007/bf02926102.
[4] PRAJAPATI D.R., MAHAPATRA P.B., *Control charts for variables to monitor the process mean and dispersion. A literature review*, Int. J. Product. Qual. Manage., 2009, DOI: 10.1504/IJPQM.2009.024223.
[5] GADRE M.P., RATTIHALLI R.N., *Some group inspection based multi-attribute control charts to identify process deterioration*, Econ. Qual. Control., 2010, DOI: 10.1515/eqc.2005.191.
[6] HUANG S., YANG J., XIE M., *A Study of control chart for monitoring exponentially distributed characteristics based on type-II censored samples*, Qual. Reliab. Eng. Int., 2017, DOI: 10.1002/qre.2122.
[7] MONTGOMERY D.C., *Introduction to statistical quality control*, 6th Ed., 2009, DOI: 10.1017/CBO9781107415324.004.
[8] KHOO M.B.C., *A moving average control chart for monitoring the fraction non-conforming*, Qual. Rel. Eng. Int., 2004, DOI: 10.1002/qre.576.
[9] WONG H.B., GAN F.F., CHANG T.C., *Designs of moving average control chart*, J. Stat. Comput. Simul., 2004, DOI: 10.1080/0094965031000105890.

[10] PATIL S.H., RATTIHALLI R.N., *Economic design of moving average control chart for continued and ceased production process*, Econ. Qual. Control., 2010, DOI: 10.1515/eqc.2009.129.

[11] PATIL S.H., SHIRKE D.T., *Economic design of moving average control chart for non-normal data using variable sampling intervals*, J. Ind. Prod. Eng., 2015, DOI: 10.1080/21681015.2015.1023854.

[12] AZAM M., ASLAM M., JUN C.H., *Designing of a hybrid exponentially weighted moving average control chart using repetitive sampling*, Int. J. Adv. Manuf. Technol., 2015, DOI: 10.1007/s00170-014-6585-x.

[13] AHMAD L., ASLAM M., KHAN N., JUN C.H., *Double moving average control chart for exponentially distributed life using EWMA*, AIP Conf. Proc., 2017, DOI: 10.1063/1.5012222.

[14] ASLAM M., AHMAD L., JUN C.H., ARIF O.H., *A control chart for COM-Poisson distribution using multiple dependent state sampling*, Qual. Reliab. Eng. Int., 2016, DOI: 10.1002/qre.1965.

[15] AZAM M., AHMAD L., ASLAM M., *Design of X-bar chart for Burr distribution under the repetitive sampling*, Sci. Int., 2016, 28 (4), 3265–3271.

[16] SHAFQAT A., HUANG Z., ASLAM M., *Design of X-bar control chart based on inverse Rayleigh distribution under repetitive group sampling*, Ain Shams Eng. J., 2020, DOI: 10.1016/j.asej.2020.06.001.

[17] ALGHAMDI S.A.D., ASLAM M., KHAN K., JUN C.H., *A time truncated moving average chart for the Weibull distribution*, IEEE Access., 2017, DOI: 10.1109/ACCESS.2017.2697040.

[18] HAQ A., BROWN J., MOLTCHANOVA E., *New exponentially weighted moving average control charts for monitoring process mean and process dispersion*, Qual. Reliab. Eng. Int., 2015, DOI: 10.1002/qre.1646.

[19] KHOO M.B.C., YAP P.W., *Joint monitoring of process mean and variability with a single moving average control chart*, Qual. Eng., 2005, DOI: 10.1081/QEN-200028689.

[20] LI Z., ZOU C., GONG Z., WANG Z., *The computation of average run length and average time to signal. An overview*, J. Stat. Comput. Simul., 2014, DOI: 10.1080/00949655.2013.766737.

[21] MOLNAU W.E., RUNGER G.C., MONTGOMERY D.C., SKINNER K.R., LOREDO E.N., PRABHU S.S., *A program for ARL calculation for multivariate EWMA charts*, J. Qual. Technol., 2001, DOI: 10.1080/00224065.2001.11980109.

[22] CROWDER S.V., *A simple method for studying run-length distributions of exponentially weighted moving average charts*, Technometrics, 1987, DOI: 10.1080/00401706.1987.10488267.

[23] CHANANET C., SUKPARUNGSEE S., AREEPONG Y., *The ARL of EWMA Chart for monitoring ZINB model using Markov chain approach*, Int. J. Appl. Phys. Math., 2014, DOI: 10.7763/ijapm.2014.v4.290.

[24] SPARKS R.S., *A group of moving averages control plan for signaling varying location shifts*, Qual. Eng., 2003, DOI: 10.1081/QEN-120018385.

[25] SPARKS R., *Weighted moving averages: An efficient plan for monitoring specific location shifts*, Int. J. Prod. Res., 2004, DOI: 10.1080/0020754042000197720.

[26] WETHERILL G.B., BROWN D.W., *Statistical process control for the process industries*, Front. Stat. Qual. Control, 1992, DOI: 10.1007/978-3-662-11789-7_17.

[27] SHAFQAT A., HUSSAIN J., AL-NASSER A.D., ASLAM M., *Attribute control chart for some popular distributions*, Commun. Stat.-Theory Meth., 2018, DOI: 10.1080/03610926.2017.1335414.

[28] SCHAFFER J.R., KIM M.J., *Number of replications required in control chart Monte Carlo simulation studies*, Commun. Stat. Simul. Comput., 2007, DOI: 10.1080/03610910701539963.

[29] SHAFQAT A., HUANG Z., ASLAM M., NAWAZ M.S., *A nonparametric repetitive sampling DEWMA control chart based on linear prediction*, IEEE Access., 2020, DOI: 10.1109/ACCESS.2020.2989132.

[30] ASLAM M., BANTAN R.A.R., KHAN N., *Design of SN2-NEWMA control chart for monitoring process having indeterminate production data*, Processes, 2019, DOI: 10.3390/pr7100742.

[31] SULLIVAN J.H., WOODALL W.H., *A comparison of multivariate control charts for individual observations*, J. Qual. Technol., 1996, DOI: 10.1080/00224065.1996.11979698.

[32] FU J.C., SPIRING F.A., XIE H., *On the average run lengths of quality control schemes using a Markov chain approach*, Stat. Probab. Lett., 2002, DOI: 10.1016/S0167-7152(01)00183-3.