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APPLICATION OF AN EVOLUTIONARY ALGORITHM TO SIMULATION OF THE CO_2 EMISSION PERMITS MARKET WITH PURCHASE PRICES

This article describes the problem of the CO_2 emission permits market and introduces several important changes to the standard model, in particular a new goal function, transactions with price negotiations between regions and - as a consequence of introducing prices for permits – the possibility of investigating the influence of purchase/sale prices on the market. An additional novelty is the method of simulating such a market, which is based on a specialized evolutionary algorithm (EA).

Keywords: *emission permits trading, Kyoto Protocol, evolutionary algorithm*

1. Introduction

Long-term observations of climate change indicate that global climate warming is becoming a real threat for human civilization. Many researchers claim that emission of CO_2 and other greenhouse gases is responsible for this. Thus, great efforts are being made to reduce these emissions. An accepted method to make this burden lighter is to implement a system of tradable emission permits. It is commonly claimed that this is an efficient strategy for achieving environmental goals. Countries[1] participating in an emission permits system have limitations imposed on their emissions. If the limitations are too low for some countries, they can do two things. They can buy permits from other countries, or reduce their emissions by applying new technologies to produce more of their energy needs without the combustion of fuels. An accepted solution

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[1] In the data used for the simulations, the majority of participating countries are grouped into bigger regions, thus in the article the notions of countries, regions etc. are used as synonyms, in general meaning market participants.

should depend on their decisions, based on thorough economic optimization. This approach to emission reduction has been accepted by many countries of the world under the Kyoto Protocol.

When investigating the influence of the Kyoto Protocol limitations on the world economy, researchers build models of such a market and find optimal purchase/sale strategies for their countries. A market model, enabling the forecasting of the quantities and prices of traded emissions allowances and the cost of emission reduction for different countries is clearly needed. One important field is to build a transaction model and to solve many other problems associated with the credibility of and uncertainty regarding emission level reports, [6], [8], [10], [13].

The model of emission permits trading for CO_2 proposed earlier ([7]) does not include real transaction prices. Although theoretical prices are calculated via the model (by deriving the cost of emission reduction), they are not practically applicable, except at the equilibrium point. Actually, the main aim of optimization in this model is to find the equilibrium point, at which the prices faced by all the countries participating in the market are equal. The costs of emission reduction, prices and money transfer resulting from such a configuration fully describe such an ideal market. In the standard model neither negotiation of prices nor additional transaction costs are considered. The participants of this market are assumed to conduct the relevant transactions with the theoretical, optimal prices, and thus the optimal solution is given by the equilibrium point ([7]).

The new method, proposed in this paper, does not assume such an ideal market. Therefore, a more sophisticated, dynamic market model is introduced, to which some typical elements of a real market are added, such as the possibility of price negotiation and the influence of real prices on the resulting solutions (similar assumptions can be found in [3]). It is assumed that the market converges to the equilibrium after a sequence of transactions. The number of transactions between the start of the market and equilibrium is not known in advance. However, each transaction that is profitable for both parties[2] brings the market toward equilibrium. Thus convergence under the new model is assured if the considered parameters of the market are proper. Detailed assumptions describing the new market model are given in the next section.

The application of evolutionary algorithms (EAs) in economic simulation models ([3], [5]) has gained considerable attention, mainly due to the fact that economic systems may be quite easily modeled using this kind of tool. An evolutionary and agent-based approach to dynamic market modeling can be found in [3] and [11], where this method is used to simulate the very complicated market of the information sector ([3]) or gas trading ([11]).

[2] Only transactions profitable for both participants are accepted during simulations. This assumption seems to be reasonable, because the market considered is free and there is no compulsion to make unprofitable transactions.

EAs treat members of the population as agents or contract parties, who learn to behave almost optimally in their environment and adopt their strategies to get greater profits, playing their “market-game”. EA is a very good tool for simulating dynamic or non-stationary market models, where interactions between parties, price negotiations and contracts are repeated during the simulation and can be treated as small steps that lead the market state toward the global equilibrium point. The optimization properties of EA assure that the strategies of the trading parties and the equilibrium point of the market found in simulations are almost optimal.

2. A New market model

Although the Kyoto Protocol imposes severe constraints on the CO_2 emissions of the participating countries, it proposes mechanisms to exceed them. For example, the participants can buy additional permits from countries that emit less than their limit and due to this fact, some kind of market for emission permits arises. Thus, countries that would like to emit more than their assigned limit can do this, upon paying the selling countries for this opportunity.

For a purchasing country, trading is beneficial only when the price of permits is lower than the cost of reducing emissions of CO_2 by the appropriate level. The country that wants to offer permits on the market can also decrease its emission levels by more than it is obliged and sell excess permits (see Fig. 1). Decreasing its emissions by more than given by the Kyoto target is sometimes beneficial, but selling permits should bring more money than the costs of emission abatement.

A simple, commonly used Walrasian model[3] of an emissions market is described by equations (1)–(3). Let us denote the total cost of decreasing emissions in a region (a country or a source) i down to x_i, by $C_i(x_i)$ (the abatement cost function). It is assumed that the cost functions $C_i(x_i)$ are positive, decreasing, continuous and differentiable for each region. The Kyoto limit imposed on region i is denoted K_i. The additional level of emissions permitted to participant i based on purchasing permits is expressed by s_i (s_i is negative if i is a net supplier of permits).

$$E = \min_{x_i} \sum_{i=1}^{n} C_i(x_i) \tag{1}$$

$$x_i \le K_i + s_i \tag{2}$$

[3] The Walrasian trade model assumes that prices are calculated by some market authority on the basis of supply and demand in a market and all transactions are conducted according to this price.

$$\sum_{i=1}^{n} s_i = 0 \tag{3}$$

where:

E – minimum total cost of decreasing emissions for all countries in the standard model;
$C_i(x_i)$ – the costs of decreasing emission in i from an initial value F_{0i} down to x_i;
s_i – the additional level of permits acquired by i;
K_i – Kyoto target for participant i;
n – number of participants;
x_i – emission of participant i.

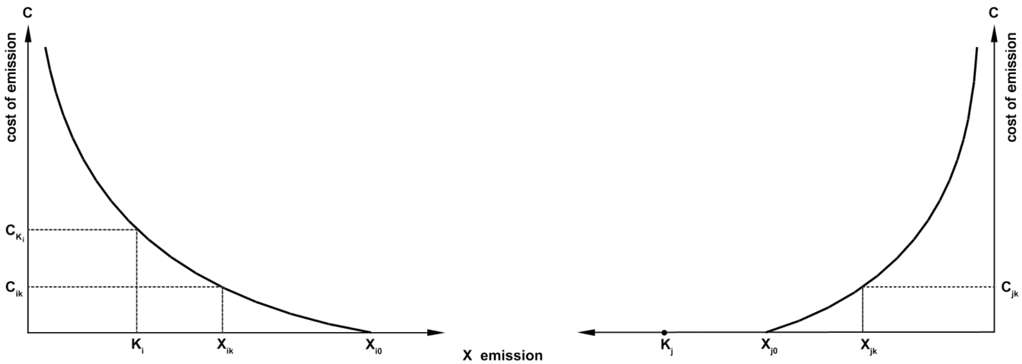

Fig. 1. Cost of emissions reduction for purchasing country i (left): without trade C_{K_i} and with trade C_k (gain C_i-C_k), K_i – Kyoto limit for country i, X_{ik} – emission after trade, X_{i0} – initial emission. Cost of emissions reduction for selling country j (right): emissions reduction without trade is equal to zero ($X_{j0} < K_i$) and after trade is equal to C_{jk} (gain C_{jk}) with emission level X_{jk}, K_j – Kyoto limit for country j.

The goal is to minimize the costs of reducing emissions to reach the overall Kyoto target, while fulfilling the needs of participants. However, the optimal value obtained from this model can be far from the real market optimum due to many inaccuracies in the parameter values used and simplification of real phenomena. Typically, many factors influence the prices of goods and the same mechanisms may play a role in the permits market. In the approach described the most important factors are, of course, the estimated costs of emissions reduction, but it is possible to consider different elements. The proposed new model with transactions and negotiated prices is closer to the free market, where countries independently make decisions on buying or selling permits taking into account prices and possible benefits, than the Walrasian model.

It is assumed in the new model that a transaction is finalized only when the negotiated price of a permit is lower than the cost of reduction for the buyer and higher than the cost for the seller. Otherwise, the transaction is not profitable to at least one of the participants. It is obvious that each party wants to maximize its profit.

In the further described evolutionary approach, maximization over s and π is performed in each transaction by genetic operators, while the total maximization over x is the overall task of the EA. As mentioned earlier, the model presented contains some important changes in relation to previous ones. The most important change is a different objective function, (4). This new objective function maximizes the difference between costs with no trading of permits and costs in the case of trading plus expenditures for the permits. This takes into account the purchase/sale price of permits, which considerably influences the profitability of transactions and the decision to buy/sell permits, i.e. whether it is better to reduce emissions rather than to buy permits.

Formulae (5)–(9) are constraints ensuring that the market model has realistic properties:

- a participant cannot emit more than its Kyoto obligation plus acquired permits – (5),
- additional permits can be bought only from participants in the market, no extra permits are available – (6),
- the number of units traded in one transaction is limited to $s_{\max}$ to avoid large perturbations of permit prices – (7),
- the sale and the purchase prices are the same – (8),
- the numbers of units traded in a transaction are negative for sellers and positive for buyers and their absolute values are the same – (9).

The new model is described by the following formulae:

$$G = \sum_{j=1}^{T} \max_{x_{ji}} \sum_{i=1}^{n} \max_{s_{ji}\pi_{ji}} (C_{j-1,i}(x_{j-1,i}) - (C_{ji}(x_{ji}) - s_{ji}\pi_{ji})), \tag{4}$$

$$x_{Ti} \le K_i + \sum_{j=1}^{T} s_{ji}, \tag{5}$$

$$\sum_{i=1}^{n} \sum_{j=1}^{T} s_{ji} = 0, \tag{6}$$

$$0 \le s_{ji} \le s_{\max}, \tag{7}$$

$$\pi_{ji} = \begin{cases} 0 & \text{for parties not trading in transaction } j \\ \pi_{ji} & \text{for parties trading in transaction } j \end{cases} \tag{8}$$

$$s_{ji} = \begin{cases} 0 & \text{for parties not trading in transaction } j \\ -s_{ji} & \text{for the party selling in transaction } j \\ s_{ji} & \text{for the party buying in transaction } j \end{cases} \tag{9}$$

where:

- G – minimum expenditures of decreasing emissions resulting from trading,
- T – number of transactions of permits conducted,
- $C_{ji}(x_{ji})$ – the costs of decreasing emissions in region i from initial value F_{0i} to value x_{ji} after j transactions,
- K_i – Kyoto target for country i,
- n – number of participants,
- x_{ji} – emissions of participant i after j transactions,
- s_{ji} – the number of units of emissions acquired by participant i in transaction j,
- $s_{\max}$ – the maximum number of units allowed to be traded in one transaction,
- π_{ji} – price of permits bought/sold by participant i in transaction j.

Using the objective function (4), it is possible to find the solution which maximizes the difference between the cost without trade and the cost with trade, in other words the overall profit from emission trading. According to objective function (1), the cost of emission reduction, not including the trading of permits, is minimized. However, the purchasing costs may be considerable in comparison to the cost of CO_2 reduction if there were no trade. Also, a different method of setting permit prices is accepted to prevent the situation in which the price of a permit is zero. The authority, or rather the participants of a market, must set a minimal price, below which the price of a permit cannot decrease. According to the model described, the process of negotiations is simulated by generating a random value between the shadow prices of the contracting parties and it is possible (but with a very small probability) at early stages of the market's evolution that zero is selected as the price of a permit in a contract. It is possible that the shadow price of the selling country is zero when this country reports initial emissions below the Kyoto level ($K_i > X_0$) – see the right hand side of Fig. 1. Therefore, its price for one unit of emissions should not be the derivative of the abatement cost, but the derivative with the minimal value (15). In practical cases, price negotiations prevent a situation in which the price of permits drops to zero, because no country would like to sell them for free. Similar situations may also occur in the standard model, when the needs of the buying countries are less than the surplus of the selling countries and equilibrium of the market will establish emission levels lower than the Kioto target ($X < K_i$) for selling countries. Thus, it seems reasonable that the models described should have some kind of protection against such cases and imposing a minimal price for permits is one of the possible solutions.

The second important change is the introduction of transactions. Transactions are performed iteratively until none can be conducted (since there is no benefit to at least one participant). The number of transactions – T in formula (4) – is not known in advance, before the simulations. It depends on local agreements between regions and may be different in simulations with the same market parameters (EA are random algorithms). However, it can be proved that the value T is finite and the model is convergent to the optimal solution with equal shadow prices of buyers and sellers. This can be derived from the fact that the cost functions are decreasing and monotonic and only profitable transactions are accepted. In consequence, transactions decrease the difference between the shadow prices of contracting parties and lead to the optimal solution with equal prices[4].

The prices of and number of units in a transferred permit are negotiated. Unfortunately, real negotiations are difficult to simulate in a computer program, thus randomly generated numbers from a normal distribution tailored to appropriate price intervals play the role of the uncertainty of negotiations in computer simulations.

It should be noted that the presented market model is dynamic, contrary to the standard static one and has been prepared specially for application to an agent or evolutionary system. This model emphasizes the process of reaching the equilibrium via negotiations and transactions. It enables simulation of the behavior of the market and is capable of finding the equilibrium, contrary to the standard approach which can only find the equilibrium and prices at that point. The evolutionary method is probably slower than many typical optimization methods in solving the standard formulation, but it is difficult to adopt optimization methods to the new model (dynamic programming could probably be an alternative to EA, but this method is also not very fast). Thus, the evolutionary method is a natural way to perform computer simulations under the new model. This method is presented in the next section.

3. Evolutionary algorithm method in computer simulations

Evolutionary algorithms are based on the phenomenon of natural evolution. Similarly to nature, they operate on a population of individuals (also agents, members or solutions), which are reproduced, modified, evaluated and the best are selected over a sequence of generations. Individuals are encoded solutions of the problem to be solved. They are modified by evolutionary operators at each step of the algorithm, emulating random changes (mutation) and recombination of parent genes in the genome of natural organisms. The best ones are selected as members of the next generation using some selection method, an equivalent of natural selection, promoting the best individuals. A standard evolutionary algorithm works in the manner shown in

[4] In the simulations conducted the final prices are not exactly equal because transactions are conducted on integer numbers of units (packets of permits) and due to the accepted models of negotiations.

Algorithm 1, but this simple scheme requires many problem specific improvements to work efficiently.

1. Random initialization of the population of solutions.
2. Reproduction and modification of solutions using genetic operators.
3. Evaluation of the solutions obtained.
4. Selection of individuals for the next generation.
5. If a stop condition not satisfied, go to 2.

Algorithm 1. The evolutionary algorithm.

The adaptation of the genetic algorithm to the problem considered requires an appropriate encoding of solutions, definitions of specialized genetic operators for the problem to be solved, the data structure and the fitness function to be optimized by the algorithm.

Thus, a specialized evolutionary algorithm is applied to solve the problem. One individual of the EA population considered here contains information about all the countries participating in the market, so it is a complete solution of our problem. It is possible to create as many individuals as necessary (in the simulations about 400) and obtain the same number of mainly different solutions. Of course, the best one is the most important, but in some circumstances several of the remaining solutions can also be used.

Another method may also be applied when each country constitutes a separate agent [1], [5] and the population of solutions in the evolutionary algorithm corresponds to the countries participating in trade. However, in the case considered (5 countries and groups of countries – regions) this population is too small for the evolutionary algorithm to work efficiently, thus it was not used.

Each individual in the EA population codes the actions to be used by each country. The information needed to describe these actions (or, in other words: to encode a solution to the problem and obtain a population member) is as follows and is encoded as a vector of 8 floating point numbers

- the theoretical price of a country's own permits (shadow price),
- the real current price of a permit for sale/purchase,
- the real current value of a permit for sale/purchase,
- current number of units for sale/purchase,
- the net number of units sold/purchased,
- current emissions level,
- previous emissions level (before the present transaction),
- present and previous value of the objective function.

To modify solutions, the following genetic operators were used:

- competition – the country considered offers a number of permits for sale, other countries offer to buy, the best option is chosen and the solution is modified,

• bilateral sale – two randomly chosen countries conduct negotiations and if they agree, the solution is modified.

The prices of and numbers of units covered by traded permits are randomly chosen to emulate the process of negotiation. The number of units traded is an integer chosen from the interval {1, ..., 5}[5] and the price of the permit is a value between the purchase offer and the sale offer with the expected value being the average of these two values.

The fitness function (in the domain of EAs an objective function is called a fitness function, because it is often somewhat modified – scaled or moved according to the requirements of the EA) for the EA is simply the objective function given by formula (4).

The population initializing procedures and genetic operators are designed so as to obey constraints (5)–(9) and forbidden solutions cannot appear in the population of solutions.

The use of specialized genetic operators requires the application of some method of sampling them at each iteration of the algorithm. In the approach used, see [12], [14], it is assumed that an operator that generates good results should have a large probability of being applied and thus more frequently affect the population. But it is very likely that an operator that is good for one individual, gives worse effects for another, for instance because of its location in the domain of possible solutions. Thus, every individual may have its own preferences. Every individual solution is associated with a vector of floating point numbers, besides the encoded solution. Each number corresponds to a genetic operator. It is a measure of quality of the genetic operator. The higher the number is, the higher the probability of executing the operator.

This set of probabilities, or in different words ranking of qualities, is based on the experience of each individual and according to it, each individual chooses an operator at each stage of the algorithm. Due to the experience gathered, individuals can maximize the chances of their offspring surviving.

This method of computing quality factors is based on reinforcement learning [4] (one of the algorithms used in machine learning). A member of the EA population (an individual, a solution) is treated as an agent, whose role is to select and call one of the evolutionary operators[6]. When the i-th operator is applied, it can be regarded as an agent's action a_i leading to a new state s_i, which, in this case, is a new solution. An agent receives a reward (also reinforcement value or payoff) or penalty r_t depending

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[5] It is not profitable to trade too large a number of units in one transaction, because this can have a large impact on permit prices, thus a limitation on the maximum number of units sold is introduced.

[6] The described mechanism of operator selection is universal and can be applied in a wide range of evolutionary algorithms, not only those connected with market simulations. The notions "reward", "agent", "strategy" and "policy" are typically used in the domain of reinforcement learning and have a different meaning to similar notions used in economics or in the domain of market games.

on the quality (the value of the fitness function) of the new state (solution). The aim of each agent is to perform the actions (the set of actions performed constitutes a strategy or decision policy, Π), which give the highest long term discounted cumulative reward (or total discounted reinforcement over its lifetime) V^*:

$$V^* = \max_{\Pi} (V^{\Pi}) \tag{10}$$

$$V^{\Pi} = E_{\Pi}\left\{\sum_{k=0}^{\infty} \gamma^k r_{t+k+1}\right\}. \tag{11}$$

The following formula can be derived from (11) and (12) and is used for evaluation purposes:

$$V_{ij}(s_{t+1}) = V_{ij}(s_t) + \alpha \cdot (r_{i,j,t+1} + \gamma \cdot V^*(s_{t+1}) - V_{ij}(s_t)) \tag{12}$$

where:

- Π – represents the strategy of the agent,
- V^{Π} – represents the discounted cumulative reward obtained using strategy Π,
- E – represents the expected value,
- k – represents consecutive time steps,
- t – represents the current time,
- $V_{ij}(s_t)$ – is a quality factor or the discounted cumulative reward of the i-th member of the EA population valued for the j-th genetic operator at moment (iteration) t,
- V^* – estimated value of the best quality factor (in the experiments the value obtained using the best operator),
- α – is a learning factor,
- γ – is a discount factor,
- $r_{i,j,t+1}$ – represents the reward obtained when the i-th population member executes the j-th genetic operator, which is equal to the improvement in the quality of the solution (the value of the fitness function of this solution) after execution of the evolutionary operator: $r_{i,j,t+1} = G_{i,j,t+1} - G_{i,j,t}$ (G is the value of the objective (fitness) function, as in formula (4)).

In the experiments presented here the values of α and γ were set to 0.1 and 0.2, respectively.

The selection method applied consists of two methods with different properties: histogram selection (increases the diversity of the population) and deterministic roulette (strongly promotes the best individuals), [14], which are selected at random during the execution of the algorithm. The probability of executing these selection methods is obtained from formula (13).

$$p_{his}(t+1) = \begin{cases} p_{his}(t)\cdot(1-a) \text{ for } & R(t) > 3\cdot\sigma(F(t)) \\ p_{his}(t)\cdot(1-a)+0.5\cdot a \text{ for } & R(t) \geq 0.5\cdot\sigma(F(t)) \wedge R(t) \leq 3\cdot\sigma(F(t)) \\ p_{his}(t)\cdot(1-a)+a \text{ for } & R(t) < 0.5\cdot\sigma(F(t)) \end{cases} \tag{13}$$

$$R(t) = \max(F_{av}(t) - F_{\min}(t), F_{\max}(t) - F_{av}(t))$$

where:

$p_{his}(t+1)$, $p_{his}(t)$ – probability of histogram selection in consecutive iterations;

$1 - p_{his}(t)$ – probability of selecting the deterministic roulette method, $p_{\det}(t)$;

$F_{av}(t)$, $F_{\min}(t)$, $F_{\max}(t)$ – average, minimal and maximal values of the fitness function in the population;

$\sigma(F(t))$ – standard deviation of the values of the fitness function in the population of solutions;

a – a *small* value for changing the probability $p_{his}(t)$, in simulations $a = 0.05$.

If the standard deviation of the fitness function in the population ($\sigma(F(t))$) is too small with respect to the extent of this function ($\max(F_{av}(t) - F_{\min}(t), F_{\max}(t) - F_{av}(t))$), then it is desirable to increase the probability of the appearance of histogram selection. In the opposite case, the probability of deterministic roulette selection is increased. If the parameters of the population are located in some range considered as profitable, it is beneficial to keep approximately the same probabilities for the appearance of both methods of selection. It is important that always $p_{his}(t) + p_{\det}(t) = 1$, i.e. some method of selection must be executed.

4. Computer simulation results

The computer simulations were conducted on a standard set of participants, as in [2], [9] and [13]. The following participants are taken into account: USA, EU, Japan, Canada–Australia–New Zealand (CANZ) and the former Soviet Union (FSU). The data presented and used are rather approximate. For instance, data for the USA are considered, although this country has not signed the Kyoto protocol yet. Since it would be difficult in practice to start the CO_2 permit market omitting the country with the highest CO_2 emissions level in the world, the USA were usually considered in simulations. The results presented in this section also take this country into account to preserve compatibility with earlier results, but the CO_2 permit market has finally started without the USA. This means that the real prices of permits are lower than those obtained from various models assuming the presence of the USA, because of the significantly lower demand from buying countries.

The costs of emissions abatement depend on the value of emission reduction in the following way (a quadratic cost function), see [2], [9]:

$$C_i(x) = \begin{cases} a_i * (X_{i0} - x_i)^2 & \text{for} \quad x_i < F_{i0} \\ 0 & \text{for} \quad x_i \geq X_{i0} \end{cases} \tag{14}$$

where:
$C_i(x)$ – cost function for emissions abatement for country i,
a – cost function parameter,
X_{i0} – initial emission,
x_i – current emission.

The marginal prices are derivatives of the cost function, with a small modification – the introduction of the value min_*p* which is the minimal price for permits, preventing the situation in which permits are sold at price 0, which may occur when the costs of emission reduction are 0 for a country with $X_{i0} < K_i$

$$c_i(x) = \begin{cases} \max((2 * a_i * (X_{i0} - x_i)), \min\_p) & \text{for} \quad x_i < X_{i0} \\ \min\_p & \text{for} \quad x_i \geq X_{i0} \end{cases} \tag{15}$$

where:
$c_i(x)$ – modified marginal price of emissions permit,
min_*p* – minimum price of permits,
the remaining symbols have the same meaning as in formula (14).

Table 1 describes the coefficients of the participants' cost functions, data come from [2], [9] and [13].

Table 1. The data applied to the calculations

Country (region)	Initial emissions (F_0) MtC/y	Cost function parameter (a) MUSD/(MtC/y)^2	Kyoto Limit (K_i) MtC/y
USA	1820.3	0.2755	1251
EU	1038.0	0.9065	860
Japan	350.0	2.4665	258
CANZ	312.7	1.1080	215
FSU	898.6	0.7845	1314

The results obtained using traditional optimization methods (under the assumption of a perfect market) are presented in Table 2 (see [2]).

Table 2. Results under the assumption of a perfect permit market (the column "Final price" shows the shadow price at the equilibrium point)

Country (region)	Final emissions MtC/y	Final price USD/tC	Number of permit units acquired Mt/y	Expenditure on permits MUSD/y	Costs of reducing emissions MUSD/y	Total costs Permits+ reduction MUSD/y
USA	1561.6	142.5	310.8	44289.0	18433.0	62722.0
EU	959.4	142.5	99.1	14121.75	5602.0	19723.75
Japan	321.1	142.5	63.5	9048.75	2059.0	11107.75
CANZ	248.4	142.5	32.9	4688.25	4583.0	9271.50
FSU	807.8	142.5	–506.3	–72147.75	6473.0	–65674.75
Total	3988.3	–	0	0	37150.0	37150.0

The respective results obtained under the new model are presented in Tables 3 and 4.

The application of an EA to simulation of the permit market gives some additional benefits, because the result is not only one set of parameters, but a set of possible scenarios. The EA operates on a population of different solutions and computations are conducted in a non-deterministic way. In particular, the negotiation of permit prices is modeled using random numbers from a modified normal distribution, adapted to the desired interval (prices are generated from the interval which is profitable to both countries, if there is no such interval, no transaction is made). Thus the different scenarios depend mainly on the prices negotiated (i.e. randomly generated in the simulation). This is also the reason for presentating the results obtained in two tables. Table 3 presents results from a scenario with high prices, imposed by the sellers (the mean value of the appropriate normal distribution is situated closer to the buyer's shadow price), while Table 4 shows the results for runs with low prices, imposed by the buyers (the mean value of the appropriate normal distribution is situated closer to the seller's shadow price). As one can notice, the differences between them are not very great, except for the columns "Permit expenditures" and "Permits+reduction cost". The results presented are averages for the 10 simulations conducted.

When analyzing the data in Tables 2 (old model), 3 (new model with high prices), and 4 (new model with low prices), one can observe that after the introduction of permit prices to the trading model, although there is a similar level of permits trade among countries, permit-related expenditures are quite different. This is not profitable for the USA, in particular. This conclusion seems to be reasonable, because free market prices are a bit higher than in the optimal Walrasian model. The USA buy permits for the largest amount of emissions (310.8 units in the standard model and 311 units in the new one, three times more than the second region – EU) and because of this, the USA's costs are so high. It is also worth noticing that the expenditure of Japan is very sensitive to the level of permit prices, but those of CANZ are rather robust. This fact is probably caused by the shape of their cost curves.

Table 3. Results of simulations using the new model (the column "Final price" denotes the shadow price at the point where trade finished due to a lack of any further benefits) under the assumption that the sellers impose prices

Country (region)	Final Emission MtC/y	Final price USD/tC	Number of imported permits Mt/y	Permits expenditures MUSD/y	Emission reduction cost MUSD/y	Permits+ reduction cost MUSD/y
USA	1562.0	142.3	311	50222.9	18381.1	68604.0
EU	959.0	143.2	99	15014.8	5657.5	20672.3
Japan	321.0	143.1	63	14420.0	2074.3	16494.3
CANZ	248.0	143.4	33	1741.7	4638.2	6379.9
FSU	808.0	142.2	–506	–81399.4	6439.5	–74959.8
Total	3898.0	–	0	0	37190.6	37190.6

Table 4. Results of simulations using the new model (the column "Final price" denotes the shadow price at the point where trade finished due to a lack of any further benefits) under the assumption that buyers have bigger market power

Country (region)	Final Emission MtC/y	Final price USD/tC	Number of imported permits Mt/y	Permits expenditures MUSD/y	Emission reduction cost MUSD/y	Permits+ reduction cost MUSD/y
USA	1562.0	142.3	311	32055.6	18381.1	50436.7
EU	959.0	143.2	99	10593.3	5657.5	16250.8
Japan	321.0	143.1	63	6760.6	2074.3	8834.9
CANZ	248.0	143.4	33	1695.1	4638.2	6333.3
FSU	808.0	142.2	–506	–51104.6	6439.5	–44665.1
Total	3898.0	–	˜0	0	37190.6	37190.6

It should be noticed that the final equilibrium price for the market is obtained as a result of small steps – transactions between market participants, not as in the traditional approach – a result of a global calculation. The results obtained are different because the price does not depend only on the shadow price, but also on the difference between the shadow prices of market participants and also on "ability to negotiate", modeled here using a random variable. Thus, there are several local equilibrium points for any particular trade between regions and the market simulation stops when no profitable transaction can be made, after a number of transactions denoted by T.

5. Conclusions

Generally, the final results presented in this paper are similar to these obtained using the standard model. Expenditures on permits constitute the only significant difference, but these costs are important for regions taking part in the CO_2 market, because they constitute the biggest part of financial means engaged. These differences originate from the fact that permit costs are calculated using the more realistic assumptions of the market model applied here.

The main advantage of the new model is its ability to take additional factors into consideration, for instance the inclusion of prices for emission permits, negotiations and different models of auctions (in the future) in the ideal market, without the necessity of completely changing the method of solution.

Nowadays, EA are often applied in economic simulations, mainly due to the fact that economic systems, with many interactions between their elements, may be quite easily modeled and simulated. The EA-based approach presented in this paper seems to be a good tool for analyzing economic phenomena ([1], [3], [11], [15]), especially for dynamic market models with elements of uncertainty (negotiated prices). Static models can be simulated using easier and probably faster methods, for instance linear programming. Adapting non-evolutionary methods (for instance dynamic programming) to dynamic models is quite difficult. Thus, evolutionary and agent methods are becoming more popular.

The introduction of several additional effects, like better models for price negotiations, auctions and uncertainty regarding the level of emissions reported by participants will be a challenge for further research in this domain.

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Symulacja rynku handlu pozwoleniami na emisję CO_2, z uwzględnieniem cen zakupu, przy użyciu algorytmu ewolucyjnego

Przedstawiono nowe podejście do symulacji rynku pozwoleń na emisję CO_2. Pierwsza z prezentowanych w tej pracy nowych koncepcji polega na jawnym wprowadzeniu cen zakupu/sprzedaży do modelu rynku. Czynnik ten, pomijany w dotychczas stosowanych modelach, może mieć znaczny wpływ na rynek, a zwłaszcza na podejmowanie decyzji kupna/sprzedaży i – w konsekwencji – także na ilości sprzedanych pozwoleń. Dlatego też powstał model oparty na bardziej realistycznych założeniach, który został porównany z modelem tradycyjnym. Zastosowano w nim kilka istotnych modyfikacji, takich jak zmodyfikowana funkcja celu i transakcje z negocjacjami cen pozwoleń na emisję.

Kolejną innowacją jest zastosowanie algorytmu ewolucyjnego do symulacji rynku. Algorytmy ewolucyjne są obecnie dość często używane nie tylko jako efektywne algorytmy optymalizacyjne, ale stosuje się je również do symulacji różnego typu systemów ekonomicznych, gier i rynków. Takie zastosowania algorytmów ewolucyjnych znane są pod angielską nazwą ***A****gent-Based* ***C****omputational* ***E****conomics* (ACE).

Słowa kluczowe: *rynek pozwoleń na emisję CO_2, Protokół z Kioto, algorytm ewolucyjny*