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COORDINATED PRODUCTION, ORDERING, SHIPMENT AND PRICING MODEL FOR SUPPLIER–RETAILER INVENTORY SYSTEM UNDER TRADE CREDIT

The objective of this article is to maximize the joint profit for supplier and retailer by constructing a combined supplier–retailer inventory model wherein supplier and retailer both have implemented trade credit policies, and some defective items are received by the retailer. The customer's demand is expressed as a function of time, price and credit period, which is appropriate for the products for which demand increases initially and after some time it starts to decrease. Production, directly proportional to the customer's demand rate, is considered as one of the decision variables for the purpose of reducing the holding cost of the supplier. The article estimates the optimum replenishment cycle, customer's credit period, retail selling price by a classical optimization technique. For validation of the derived model, various numerical examples are demonstrated. Finally, implementing sensitivity analysis on the decision variables by varying the inventory parameters, effective managerial insights are generated which is beneficial for the players of supply chain by practically gaining the maximum joint profit through advising to opt for the case $N \le M \le T$, i.e., when the customer's trade credit period offered by the retailer is lesser than the retailer's trade credit period offered by the supplier and which is lesser than the replenishment cycle length.

Keywords: *defective items, production inventory system, supply chain of two players, time-price and credit dependent demand*

1. Introduction

In order to maximize the profit, most initial work related to inventory problems focuses either on the viewpoints of a retailer or on that of the supplier. Recently, how-

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ever, to increase joint profit through strategic collaboration, coordinated inventory models widely have come into focus. Goyal [20] proposed a coordinated model to minimize the total cost of the system and to estimate the optimal order quantity of the retailer for a single supplier–retailer system. A combined lot size model by Banerjee [5] was developed for a retailer produced by a supplier, as based on the lot-for-lot ordering. Goyal [21] widespread Banerjee's model [5] by removing the lot-for-lot policy supposition of the supplier and proved that a considerable amount of inventory cost can be reduced, in the case where the supplier's production quantity can be expressed as a positive integer multiple of the retailer's purchase quantity. Lu [18] supposed that the demand rate is less than the supplier's production rate, and the delivery starts as soon as the quantity ordered by the retailer is produced, and lot-based goods are delivered afterwards. Kim and Lee [26] proposed an effective algorithm that successively estimates inbound ordering lot-sizes and a shipment schedule that minimizes the total cost including holding cost, ordering cost, and freight cost for multiple products to be shifted from a supplier to a warehouse by common freight containers.

Usually, in ideal inventory models, the concept of defective items is overlooked. But due to imperfect production or destruction in shipment, defective items can be created. The customer will return the goods, in case the retailer sells defective items without inspection. Porteus [34] and Rosenblatt and Lee [36] initially proposed a major relationship between quality imperfection and lot size. Later, a modified economic order quantity (EOQ) model with stochastic demand was proposed by Paknejad et al. [33] where the number of defective items (assumed as a random variable) returned to the supplier in the delivery time of the next batch. Economic production quantity (EPQ) model, including defective items with a supposition of having greater production rate for non-defective items than the demand rate, was developed by Salmeh and Jaber [7]. Ouyang and Chang [32] proposed an inventory model involving defective items production process with controllable lead time, dealing with investment in quality improvement. Many other articles, such as by Chung and Hou [19], Hou [23], Rahim and Al--Hajailan [35], Lin et al. [17], Wee et al. [46], etc., also relate to defective items.

Moreover, practically, to increase order quantity, the supplier offers trade credit to the retailer. From the retailer's point of view, granting trade credit not only increases sales and revenue, but also opportunity cost. Initially, an inventory model with the permissible delay in payments was introduced by Haley and Higgins [11]. Kingsman [27] highlighted the effects of various means of payment on stocking and ordering. An EOQ model was proposed by Goyal [21], along with the permissible delay in payments, with interest earned and paid. Aggarwal and Jaggi [3], referring to deteriorating items, modified Goyal's model.

All the inventory models listed above suppose one-level trade credit. Nowadays, more and more efforts are taken to raise collective advantage by constructing a coordinated model for both players rather than individual one in a supply chain. According to the assumption, a delay period is offered by a supplier to retailer, and within the trade credit period, the retailer could sell the goods, accumulate revenue and earn interest. But to enhance the customer's demand rate and to cut off on-hand stock cost, the customer should also be offered a trade credit period by the retailer. So, the supplier offers a credit period to the retailer and the retailer offers the credit period to his/her customers. To stimulate the demand, Huang [14] proposed an inventory model with the assumption that the retailer also permits a credit period to its customer which is shorter than the credit period offered by the supplier. Huang [15] modified Huang's [14] model under two levels of trade credit and limited storage space to estimate the retailer's inventory policy. Huang generalized his earlier model by developing the EPQ model to estimate the two-level trade credit policy. The EPQ model was presented by Mahata [28] for deteriorating items under the retailer's partial trade credit policy.

Kreng and Tan [24] altered Huang [14] by presenting the EOQ model under two levels of trade credit policy, depending on the order quantity by developing optimal wholesaler's replenishment decisions. A joint supplier–buyer inventory model was proposed by Ho et al. [13] with the assumption that demand is sensitive to the retail price and the supplier adopts a two-part trade credit policy. The EOQ model with advance sales discount and two-echelon trade credits were considered in Tsao [43]. Further, the EPQ model with defective items under trade credit policy was discussed in Kreng and Tan [25] to estimate the optimal replenishment decision. Later, Teng et al. [45] stretched the demand pattern from constant to increasing in time. The retailer's optimal EOQ is estimated by Chen et al. [8] when the supplier offers conditionally permissible delay in payments.

On the other hand, by expressing demand as a function of price and considering the difference between the purchase cost and selling price, Teng et al. [44] estimated the lot size and optimal price under allowable delay in payments. With the concept of permissible delay in payments, Shah and Shah [40] proposed a probabilistic EOQ inventory model for deteriorating items considering demand as a random variable and time and deterioration of units as continuous variables. Lin et al. [17] developed a united supplier–retailer inventory model with trade credit policy and allowing defective items by calculating optimal ordering and delivery policy. Many other significant articles, such as those by Mahata and Goswami [29], Arcelus et al. [4], Jamal et al. [16], Abad and Jaggi [1], Chang [7], Abuhommous [2], Wang [47], refer to trade credit.

As based on the assumption of fixed rate of demand over entire inventory cycle, various research work was undertaken, although, in a real-life scenario, there are several factors affecting the rate of demand such as the selling price of the products, time, customer's credit-period offered by retailer. Therefore, the demand rate is assumed to fluctuate as a function based on the selling price.

Initially, Silver et al. [41] derived an inventory model with time-varying rate of demand, later various researcher like; Silver [42], Chung et al. [9, 10], Hariga [12], Bose et al. [6], Mehta et al. [30, 31], Shah et al. [39], and Shah and Shah [40] expressed market demand rate as a function varying with respect to time in terms of linear, exponential or quadratic, etc.

This article proposes a combined supplier–retailer inventory model in which both the supplier and the retailer adopted trade credit policies, and the retailer receives a lot containing some defective items. The customer's demand is expressed as a function of time, price and credit period which is appropriate for the products for which demand increases initially and after some time it starts to decrease, and which is one of the additional concept in this article as compared to previous works in which demand was expressed either as constant or as a function of price or as a function of credit period. In order to reduce the holding cost of the supplier, the production is considered as one of the decision variables, which is directly proportional to the customer's demand rate. The aim of this paper is to maximize the joint profit for the supplier and the retailer. Some numerical examples are demonstrated for validation of the developed mathematical model. Finally, implementing sensitivity analysis on the decision variables by varying the inventory parameters, effective managerial insights are generated which is beneficial for the players of supply chain.

2. Notations and assumptions

The mathematical model is developed on the basis of the following notation and assumptions.

2.1. Model notations

$P(t)$ – the supplier's production rate per unit at any time t (a decision variable)

$D(t, N, p)$ – the retailer's demand rate per unit at any time t, the customer's trade credit period N, and selling price p

A_s – the supplier's setup cost per order in dollars

A_r – the retailer's ordering cost per unit ordered in dollars

G – transportation cost per delivery in dollars

h_s – the supplier's holding cost per item per unit time in dollars

h_{r_1} – the retailer's holding cost per non-defective item in dollars (excluding interest charges)

h_{r_2} – the retailer's holding cost per defective item in dollars (including treatment cost excluding interest charges)

C_{sc} – the retailer's unit screening cost in dollars

x – the retailer's screening rate per order in dollars

β – the supplier's defective rate in a production batch per unit, is given, $0 \le \beta < 1$

C_{st} – the supplier's unit treatment cost of defective items in dollars

C_s – the supplier's production cost per unit in dollars

C_r – unit price charged by the supplier to the retailer in dollars

p – unit retail price charged by the retailer to the customers in dollars (a decision variable)
M the retailer's trade credit period offered by the supplier per order in years
N – the customer's trade credit period offered by the retailer per order in years, where, $N \leq M$ (a decision variable)
I_{re} – the retailer's interest earned per dollars per unit time
I_{ro} – the retailer's capital opportunity cost per dollars per unit time
I_{so} – the supplier's capital opportunity cost per dollars per unit time
Q – the retailer's order quantity per order (non-defective items) (a decision variable)
q – size of the shipments from the supplier to the retailer in a production batch
n – number of shipments from the supplier to the retailer per production run; a positive integer (a decision variable)
T – the retailer's replenishment cycle length in years (a decision variable)
$T(N, p, T)$ – the supply chain's joint total profit per unit time in dollar with number of shipments n, credit period N, selling price p and cycle length T
$TPR(N, p, T)$ – total profit per unit time for the retailer in dollars
$TPS(N, p, T)$ – total profit per unit time for the supplier in dollars

2.2. Model assumptions

- The system consists of only one supplier: one retailer for only one product.
- Shortages are impermissible.
- The lot size for the seller is n.q.
- On attaining order, the examination of items is done by screen ratex and defective items are scrutinized in each lot and return back to the supplier in next batch delivery time. Hence, the retailer obtains an incoming portion comprising $(1-\beta)\,q = \int_0^T D$ non-defective items, i.e.

$$Q = n(1-\beta)q = n\int_0^T D(t, N, p)\,dt \tag{1}$$

with $P(t) = \frac{1}{T}\lambda\int_0^T D(t, N, p)dt$, where, $\lambda \geq 1$, the (non-defective) supplier's production rate is higher than the retailer's demand rate, i.e., $(1-\beta)P(t) > D(t, N, p)$ per cycle as shortages are not permitted. Let

$$C_s < C_r < p \tag{2}$$

Both players in supply chain assume trade-credit strategies, Let M and N represent the credit period offered by the supplier to the retailer, and that of the retailer to each customer, respectively, where, $M \geq N$.

- In [N, M], the retailer selling items utilizes the sales revenue to earn interest at a rate I_{re} and on the expiration of the permitted delay period M he pays to the supplier and has an opportunity cost at a rate of I_{ro} for the items still in stock and for the items sold but unpaid by the customers. A similar case is for the customer in [0, N].
- The supplier has to bear opportunity cost at a rate of I_{so} as he cannot receive the payment immediately after delivery of the items.

Let us assume that the demand rate for the item is

$$D(t, N, p) = a(1 + bt - ct^2)p^{-\eta}N^{\alpha}$$

a function of time; with p as the selling price and N as credit period of customer per unit, where, $a > 0$ is a scale demand, $0 \leq b < 1$ denotes the linear rate of change of demand with respect to time, $0 \leq c < 1$ denotes the quadratic rate of change of demand and η, α are mark up for selling price and trade credit, respectively, where, $\eta > 1$ and $\alpha > 1$.

3. The mathematical model

3.1. Retailer's total profit per unit time

The retailer's total profit per unit time includes the following components calculated as follows:

Sales revenue. For each order quantity Q, the retailer is charged C_rQ from the supplier, and pQ is received from the customer. Therefore, the retailer's sales revenue per unit time is

$$SR_r = \frac{(p - C_r)Q}{nT} \tag{3}$$

Ordering cost. The retailer's ordering cost per unit ordered is A_r. Each batch is dispatched to the retailer in n equally sized shipments, and each replenishment cycle length is T, therefore, the ordering cost per unit time is

$$OC_r = \frac{A_r}{nT} \tag{4}$$

Transportation cost. The transportation cost per delivery is G, therefore, transportation cost per unit time is

$$CT = \frac{G}{T} \tag{5}$$

Holding cost. On arrival of the order, all the items are examined with the screening rate x by the retailer. Hence, the retailer's holding cost can be split into two components, i.e., non-defective holding cost and defective holding cost. With T as the replenishment cycle length h_{r1} as holding cost per non-defective items, the number of non-defective items is given by

$$(1-\beta)q = n\int_0^T D(t,N,p)\,dt \tag{6}$$

as non-defective inventory level, $I_r(t)$ reduces with respect to time and it depends on the rate of market demand. Therefore, the retailer's holding cost per unit time for non-defective items is given by

$$\frac{h_{r_1}}{T}\left[\int_0^T I_r(t)dt\right] = \frac{h_{r_1}}{T}\left[\int_0^T D(t,N,p)\left(\frac{T}{n}-t\right)dt\right]$$

and the holding cost for undetected defective items per unit time is given by $\frac{h_{r_1}}{T}\left[\beta\frac{q}{2}\times\frac{q}{x}\right]$, where h_{r_1} represents the retailer's holding cost per non-defective items, β represents the supplier's defective rate in a production batch per unit, q represents the size of the shipments from the supplier to the retailer in a production batch, x represents the retailer's screening rate per order and the duration of the screening period is q/x.

Hence, the retailer's holding cost of non-defective items and undetected defective items per unit time is

$$HC_{nd} = \frac{h_{r_1}}{T}\left[\int_0^T I_r(t)dt + \beta\frac{q}{2}\times\frac{q}{x}\right]$$

where,

$$I_r(t) = D(t,N,p)\left(\frac{T}{n}-t\right)$$

Therefore,

$$HC_{nd} = \frac{h_{r_1}}{T}\left[\int_0^T D(t,N,p)\left(\frac{T}{n}-t\right)dt + \beta\frac{q}{2}\times\frac{q}{x}\right] \tag{7}$$

Let βq be the number of defective items in each received lot, the screening rate is x, the duration of the screening period is q/x and the holding cost per defective items is h_{r_2}. The total number of defective items in a lot after the screening is $(\beta qT - \beta q^2/2x)$. Therefore, the retailer's holding cost of defective items per unit time is

$$HC_d = \frac{h_{r_2}}{T}\left(\beta qT - \frac{\beta q^2}{2x}\right) \tag{8}$$

Therefore, the total holding cost per unit time is

$$HC_r = HC_{nd} + HC_d$$

thus,

$$HC_r = \frac{h_{r_1}}{T}\left[\int_0^T D(t,N,p)\left(\frac{T}{n}-t\right)dt + \beta\frac{q}{2}\times\frac{q}{x}\right] + \frac{h_{r_2}}{T}\left(\beta qT - \frac{\beta q^2}{2x}\right) \tag{9}$$

Screening cost. The retailer's screening cost per unit time is

$$SC = \frac{C_{sc}\int_0^T D(t,N,p)dt}{T(1-\beta)} \tag{10}$$

Opportunity cost and interest earned. For the retailer, there are two opportunity costs, i.e., the cost for items kept in stock when the payment is paid, and cost for items sold without being paid. The retailer earns interest in using sales revenue during the allowable clearing period, before the due date M. After the completion of this period, the retailer does not earn interest, and the retailer starts paying for the charges on the items in stock.

Let T be the replenishment cycle length where the retailer sells out the stock and collects all returns at a time in every cycle, M be the retailer's credit period and N be the customer's credit period.

Case 1: $T + N \le M$. This case comprises the collection of all the payment of sales items at a time $T + N$ from the customers by the retailer. But the supplier's payment is done only after the end of credit period M so the retailer is not supposed to pay any of the above-stated opportunity costs, and the retailer's sales revenue is utilized to earn interest at a rate of I_{re}. If the sales revenue for the time period $[N + T + N]$ is similar to that of the time period $[0, T]$, the sales revenue generated for the period $[0, T]$ is given by $\frac{pI_{re}}{T}\int_0^T tD(t,N,p)dt$ and the sales revenue for the period $[T + N, M]$ is calculated as $(M - T - N)\int_0^T tD(t,N,p)dt$ and thus the retailer's earned per unit time is

$$IE_{r_1} = \frac{pI_{re}}{T}\int_0^T tD(t,N,p)dt + (M - T - N)\int_0^T tD(t,N,p)dt \tag{11}$$

and the opportunity cost per unit time is

$$IC_{r_1} = 0 \tag{12}$$

Case 2: $T \le M \le T + N$. This case explains that the supplier is paid by the retailer at the period M, the time after the retailer sold all the items and before the time that the retailer collects all returns. Specifically, the retailer cannot receive the payment instantly after the delivery of all the items to the customers, but pays off the supplier at the due date, M and has to bear an opportunity cost during the time interval $[M, T + N]$ at a rate of I_{ro}. Therefore, the retailer's opportunity cost per unit time is

$$IC_{r_2} = \frac{pI_{ro}}{T}\int_0^{T+N-M} I_r(t)dt$$

where,

$$I_r(t) = \left(\frac{T}{n} - t\right)D(t,N,p)dt \tag{13}$$

and during the interval $[N, M]$, the retailer uses the sales revenue to earn interest at a rate of I_{re}. Thus, the retailer's interest earned per unit time is

$$IE_{r_2} = \frac{pI_{re}}{T}\left(\int_0^{M-N} tD(t,N,p)dt\right) \tag{14}$$

Case 3: *T* ≥ *M*. In this case, before the inventory is fully exhausted, the retailer pays to the supplier at the end of the credit period M, so, in the interval $[M, T]$, the retailer still has some stock on hand. Therefore, the retailer's opportunity cost is given by

$$IC_{r_{31}} = \frac{C_r I_{ro}}{T} \int_0^{T-M} D(t,N,p)dt\left(\frac{T}{n}-t\right)dt \tag{15}$$

and the opportunity cost for the items previously sold but not yet paid during the time interval $\left[T, T+N\right]$

$$IC_{r_{32}} = \frac{pI_{ro}}{T} \int_0^{T+N-M} D(t,N,p)\left(\frac{T}{n}-t\right)dt - \int_0^{T-M} D(t,N,p)dt\left(\frac{T}{n}-t\right)dt \tag{16}$$

$$\begin{aligned} IC_{r_3} &= IC_{r_{31}} + IC_{r_{32}} \\ &= \frac{C_r I_{ro}}{T}\left(\int_0^{T-M} D(t,N,p)\left(\frac{T}{n}-t\right)dt\right) \\ &\quad + \frac{pI_{ro}}{T}\left(\int_0^{T+N-M} D(t,N,p)\left(\frac{T}{n}-t\right)dt - \int_0^{T-M} D(t,N,p)dt\right) \end{aligned} \tag{17}$$

The retailer's interest earned per unit time, during the credit period by utilizing sales revenue is given by

$$IE_{r_3} = \frac{pI_{re}}{T} \int_N^M D(t,N,p)(t-N)dt \tag{18}$$

Therefore, the total profit per unit time for the retailer is stated as below:

$$TPR(N,p,T) = \begin{cases} TPR_1(N,p,T) & \text{if} \quad T+N \le M \\ TPR_2(N,p,T) & \text{if} \quad M-N \le T \le M \\ TPR_3(N,p,T) & \text{if} \quad T \ge M \end{cases} \tag{19}$$

where

$TPR_i(N, p, T) =$ sales revenue – ordering cost – transportation cost

– holding cost – screening cost + interest earned

$$TPR_i(N,p,T) = SR_r - OC_r - CT - HC_r - SC + IE_{r_i} - IC_{r_i},\ i = 1, 2, 3 \tag{20}$$

3.2. The supplier's total profit per unit time

The supplier's total profit per unit time can be calculated as follows:

Sales revenue. The supplier's production for one unit C_s is and he sells to the retailer for C_r. Therefore, the supplier's sales revenue per unit time is

$$SR_s = \frac{(C_r - C_s)}{T}\int_0^T P(t)dt \tag{21}$$

Setup cost. The supplier's setup cost for batch is A_s and each production cycle length is nT, therefore, the supplier's set up cost per unit time is

$$SC = \frac{A_s}{nT} \tag{22}$$

Treatment cost of defective items. Let $n\beta q$ be the number of defective items in each production run and the treatment cost per defective items is C_{st}. Therefore, the treatment cost of defective items per unit time is

$$TCD = \frac{C_{st}\beta q}{T} \tag{23}$$

Holding cost. The supplier's inventory level rises, as the supplier's production rate for the perfect items is greater than the retailer's demand rate. The supplier halts production when the required production unit nq is reached. The supplier gives the delivery of an average of $(1-\beta)q/\int_0^T D(t,p,N)dt$ units of time on a continuous basis until the inventory level of the supplier falls to zero with q as the supplier's shipment quantity in each lot. Hence, the inventory level of the supplier per cycle can be calculated by removing the stored inventory level of the retailer from the stored inventory level of the supplier is stated as:

$$HC_s = nq\left[\frac{q}{P(t)} + \frac{(n-1)(1-\beta)q}{\int_0^T D(t,N,p)dt} - \frac{nq}{2} \times \frac{nq}{P(t)}\right]$$

$$= \frac{nq^2}{P(t)} + \frac{n(n-1)(1-\beta)q^2}{2\int_0^T D(t,N,p)dt} - \frac{n^2q^2}{2P(t)} \tag{24}$$

Opportunity cost. Let M be the credit period offered by the supplier to the retailer, the supplier will not obtain the payment until M. Hence, the opportunity cost per unit time for the supplier is

$$OC_s = \frac{C_r I_{so}(1-\beta)qM}{T} \tag{25}$$

Therefore, for the fixed payment date M, the total profit per unit time for the supplier can be expressed as

$$TPS(N,p,T) = SR_s - SC_s - TCD - HC_s - OC_s \tag{26}$$

3.3. The supply chain total profit per unit time

In order to increase the supplier–retailer's profit gain and cash flow to acquire an optimal scenario, the coordinated supplier–retailer inventory model is applied and the total profit per unit time is given by the sum of the total profit of the retailer as well as a total profit of the supplier for all the three respective cases

$$TP(N,p,T) = TPR_i(N,p,T) + TPS(N,p,T) \text{ for } i = 1, 2, 3 \tag{27}$$

Now, to maximize the total profit, we apply the following necessary and sufficient condition:

$$\frac{\partial TP(N,p,T)}{\partial N} = 0, \quad \frac{\partial TP(N,p,T)}{\partial p} = 0, \quad \frac{\partial TP(N,p,T)}{\partial T} = 0 \tag{28}$$

That results in a system of non-linear equations either, containing a finite number of solutions, having no solution, or having infinitely many solutions.

Therefore, whether the given system is consistent or not can be checked by supplying the hypothetical numerical values to the inventory parameters and utilizing the Maple Software 18 on solving the three equations, and in case of consistent system, adopting the algorithm the below for the solution:

Step 1. Allocate hypothetical values to the inventory parameters.

Step 2. Solve Eq. (28) simultaneously by mathematical software Maple 18.

Step 3. Check second order (sufficient conditions)

$$\frac{\partial^2 TP(N,p,T)}{\partial N^2}<0,\quad \frac{\partial^2 TP(N,p,T)}{\partial p^2}<0,\quad \frac{\partial^2 TP(N,p,T)}{\partial T^2}<0 \tag{29}$$

Step 4. Compute profit per unit time from equation (27), order quantity, number of shipments, optimum selling price, optimum credit period for the customer, optimum replenishment cycle length.

As such it is difficult to prove analytically the optimum solution for the decision variables, so we prefer a numerical and graphical solution which helps us to show the concavity nature of the profit function. The objective is to make a total profit per unit time maximum with respect to selling price, the customer's credit period and cycle time. Now, we examine the working of the model with numerical values for the inventory parameters.

4. Numerical examples and sensitivity analysis

In this section, we provide some numerical examples to illustrate the above mathematical model. We use mathematical software Maple 18 to solve these examples with a computation time of 18–20 seconds each. The purpose of choosing this software is a better visualization of the mathematical problems, which enhances the research. Maple 18 software is also used to solve the highly nonlinear system of equations generated from the derived inventory system by Eq. (28), which demonstrates the computational complexity of the proposed algorithm and assesses the consistency of the system.

Example 1. Assume the following values in the developed model:
$a = 20\,000$ units, $b = 85\%$, $c = 41\%$, $\lambda = 1.0001$, $\eta = 1.001$,
$\alpha = 0.001$, $M = 0.6$ years, $C_r = \$20$/unit,
$C_s = \$10$/unit, $h_s = \$0.5$/unit/year, $h_{r1} = \$0.1$/unit/year,
$A_s = \$500$/order, $A_r = \$1000$/order,
$h_{r2} = \$0.5$/unit/year, $I_{so} = 5\%/\$$/year, $I_{re} = 9\%/\$$/year,
$I_c = 13\%/\$$/year, $I_{ro} = 5\%/\$$/year,

G = \$25/item/year, C_{st} = \$5/unit, C_{sc} = \$5/unit,
x = 12%/order, β = 10%/\$/year.

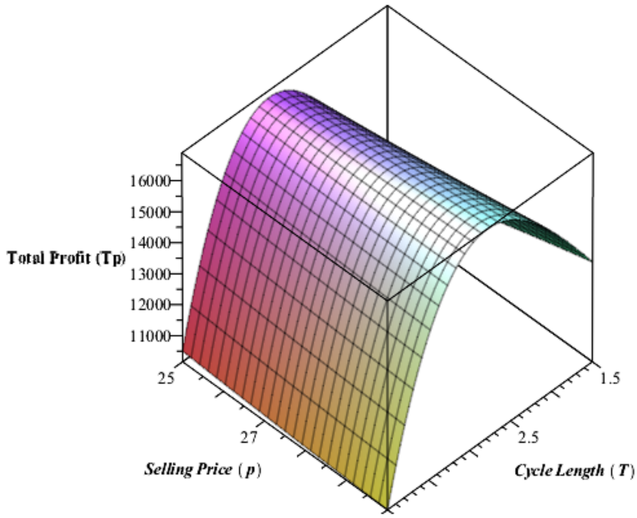

Fig. 1. Concavity of the total profit in dependence of the cycle time (T) selling price (p) and customer's credit period (N) for $N \leq M \leq T$

As shown in Fig. 1, the customer's optimal credit period N is 0.586 years, optimal cycle time T is 2.9492 years, and optimal retail price p is \$24.73 and the corresponding joint total profit $TP = \$16\,665.34$. Clearly, here $N \leq M \leq T$.

Example 2. Assume the following values in the developed model:
a = 20 000 units, b = 0.001%, c = 37%, λ = 1.001,
η = 1.1, α = 0.001, M = 0.6 years, C_r = \$20/unit,
C_s = \$10/unit, h_s = \$0.15/unit/year, h_{r_1} = \$0.5/unit/year,
A_s = \$500/order, A_r = \$1000/order,
h_{r_2} = \$0.6/unit/year, I_{so} = 5%/\$/year, I_{re} = 9%/\$/year,
I_c = 13%/\$/year, I_{ro} = 5%/\$/year,
G = \$15/item/year, C_{st} = \$5/unit, C_{sc} = \$5/unit,
x = 15%/order, β = 10%/\$/year.

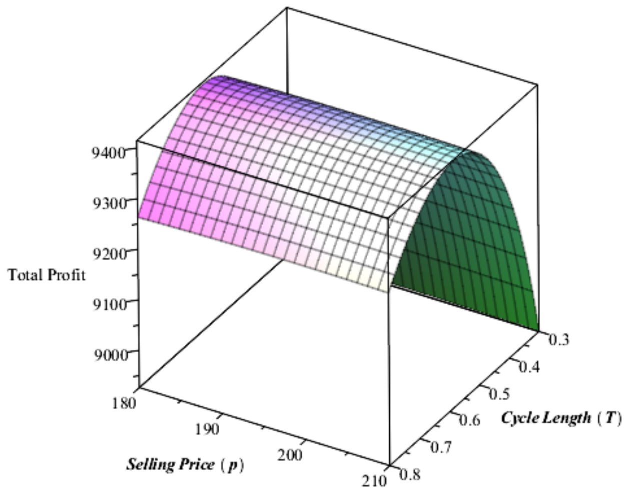

Fig. 2. Concavity of the total profit in dependence of the cycle time (T) selling price (p) and customer's credit period (N) for $N \le T \le M$

As shown in Fig. 2, the customer's optimal credit period N is 0.459 years, optimal cycle time T is 0.581 years, and the optimal retail price p is \$197.28 and the corresponding joint profit TP is \$9400. Clearly, here the $N \le T \le M$ case is satisfied. Now, for Example 1, we examine the effects of various inventory parameters on decision variables, cycle time, selling price and total profit by varying them as from –20% to 20% (Figs. 3–6).

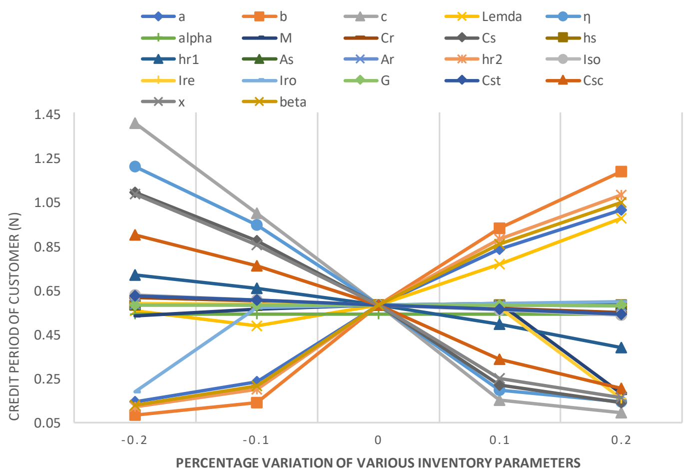

Fig. 3. Credit period in dependence of various inventory parameters

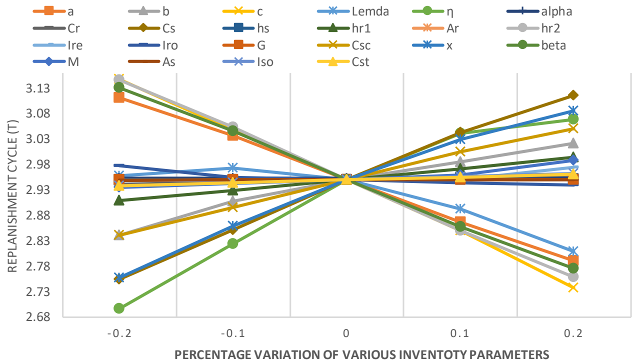

Fig. 4. Replenishment cycle in dependence of various inventory parameters

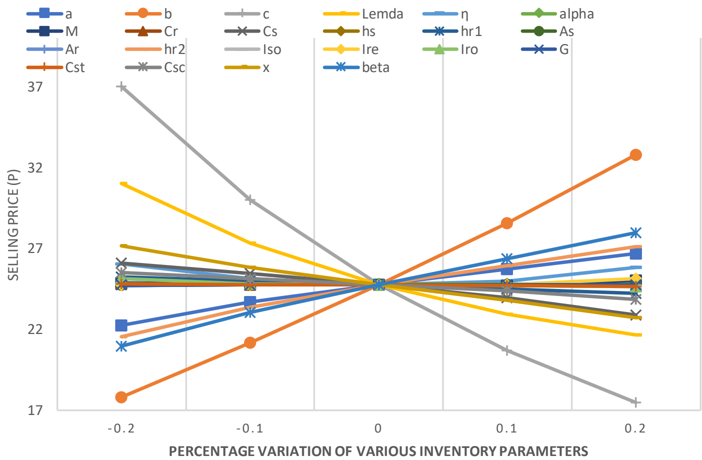

Fig. 5. Selling price in dependence of various inventory parameters

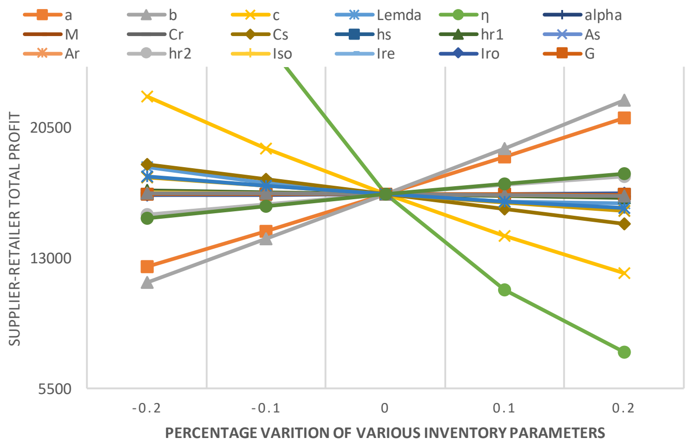

Fig. 6. Supplier–retailer total profit versus variation of various inventory parameters

Table 1. Impact on decision variables on increasing the inventory parameters (±20%)

Inventory parameters	Customer's credit period N	Replenishment cycle T	Customer's selling price p	Total profit TP
Scale demand, a	↑	↓	↑	↑
Linear demand rate, b	↑	↑	↑	↑
Quadratic demand rate, c	↓	↓	↓	↓
Supplier's defective rate, β	↑	↓	↑	↑
Retailer's credit period, M	↑	↑	↓	↓
Mark-up for selling price, η	↓	↑	↓	↓
Lambda, λ	↑	↓	↓	↓
Supplier's unit treatment cost of defective items, C_{st}	↓	↑	↓	↓
Retailer's unit screening cost, C_{sc}	↓	↑	↓	↓
Supplier's production cost, C_s	↓	↑	↓	↓
Unit charged by the supplier to retailer, C_r	↓	↑	↓	↓
Retailer's capital opportunity cost per dollar per unit time, I_{ro}	↑	↓	↓	↑
Retailer's interest earned per dollar per unit time, I_{re}	↓	↑	↑	↓
Supplier's capital opportunity cost per dollar per unit time, I_{so}	↓	↑	↓	↓
Retailer's screening rate per order, x	↓	↑	↓	↓
Retailer's holding cost per non-defective item per unit time, h_{r_1}	↓	↑	↓	↓
Retailer's holding cost per defective item per unit time, h_{r_2}	↑	↓	↑	↑
Supplier's setup cost, A_s	↓	remains constant	↑	↓
Retailer's ordering cost per unit ordered, A_r	↓	↑	↑	↓
Transportation cost per delivery, G	↓	remains constant	↑	↓

Sensitivity analysis of the decision variables by increasing the inventory parameters

Scale demand a. It is advisable to vary the scale demand which increases the customer's trade credit period and customer's selling price by shortening the replenishment cycle length and hiking the total joint profit of the inventory system.

Linear demand rate b. On varying linear demand rate, the retailer-customer joint profit rises by hiking the customer's trade credit, selling price but by the elongation of the replenishment cycle length. So, in order to hike the total profit, the linear demand rate is preferred to hike.

Quadratic demand rate c. On varying the quadratic demand, even by shortening the cycle length, the customer's trade credit period decreases with a decline in the total profit of the system. Therefore, an increase in the quadratic term in the demand function is not a favourable one.

Supplier's defective rate β. On varying the supplier's defective rate, the cycle length shortens and by opting the increment in customer's credit period and increasing the selling price, some result in the hike in total profit is observed. As a result, it is preferable to reduce the defective rate.

Retailer's credit period M. By increasing the retailer's credit period, obviously, the customer's credit period increases, but with the decrease in customer's selling price the total profit declines with lengthening the replenishment cycle.

Mark-up for selling price η. The customer's selling price decreases along with the decrease in the customer's credit period and lengthening in the replenishment cycle resulting in lowering the total profit. Thus, the value of markup for selling price should not be increased.

Cycle length λ. The customer's selling price decreases along with the shortening of the cycle length. There is a remarkable reduction in the total profit of the system. Therefore, the value of should not be increased.

Supplier's unit treatment cost of defective items C_{st}. By lengthening the replenishment cycle length, the trade credit period and selling price decreases, resulting in a decrease in the total joint profit.

Retailer's unit screening cost C_{sc}. The cycle length increases but the reduction in selling price and credit period results in a decrease of the total profit of the system, which is undesirable.

Supplier's production cost C_s. The cycle length increases but the reduction in selling price and credit period results in a decrease of the total profit of the system, which is undesirable.

Unit charged by the supplier to the retailer C_r. There is a decrease in the selling price and credit period with an increase in the replenishment cycle; the total profit of the inventory system decreases, which is not preferable.

Retailer's capital opportunity cost per dollar per unit time I_{ro}. The customer's selling price decreases with shortening the cycle length and increasing the credit period while raising the total profit of the system.

Retailer's interest earned per dollar per unit time I_{re}. Credit period decreases with an increase in selling price and replenishment cycle length by an undesirable decrement in total profit.

Supplier's capital opportunity cost per dollar per unit time I_{so}. By increasing the cycle length, decrease in customer's selling price and credit period result in lowering the total profit of the system.

Retailer's screening rate per order x. The cycle length elongates with the reduction in selling price and credit period, resulting in the drop in the total profit, which is undesirable.

Retailer's holding cost per non-defective item per unit time h_{r_1}. The reduction in selling price and credit period with the increase of the cycle length, resulting in a decrease in the total profit, which is not preferable.

Retailer's holding cost per defective item per unit time h_{r_2}. It is preferable to vary holding cost per defective item per unit time as total profit margin increases by lowering the cycle length and increasing the selling price as well as credit period.

Supplier's setup cost A_s*, retailer's ordering cost per unit ordered* A_r*, transportation cost per delivery G.* There is a decrease in the customer's credit period, and with the hike in selling price the total profit declines. Consequently, it is undesirable that the supplier's setup cost, the retailer's ordering cost, transportation cost per delivery vary.

5. Conclusions

Offering credit period to the players of supply chain is the best promotional tool on the business market. In this article, we study an inventory model in which the supplier offers credit period to the retailer and so does the retailer to the customer for the purchased products. In the model, we assume that the retailer returns defective products to the supplier during cycle time as well as production is demand-dependent, which is a crucial situation on the competitive market. The model deals with credit period as well as price sensitive quadratic demand. Quadratic demand can be seen when products are in demand for some time and after that the demand for that product decreases regularly for various reasons. The non-linear profit function is obtained. The necessary conditions are expressed to arrive at optimal solution based on evidence. We maximize the coordinated total profit for selling price, trade credit and cycle time. Numerical examples and sensitivity analysis are provided to make beneficial decisions for the managers, suggesting to gain maximum joint profit by advising to opt for the case, i.e., the situation when the customer's trade credit period offered by the retailer is shorter than the retailer's trade credit period offered by the supplier and which is shorter than the replenishment cycle length.

For future research, this concept can be extended to stochastic demand. Allowing shortages may also be an option. One can also add deterioration of inventory products in prospective studies.

References

[1] ABAD P.L., JAGGI C.K., *A joint approach for setting unit price and the length of the credit period for a seller when end demand is price sensitive*, Int. J. Prod. Econ., 2003, 83 (2), 115–122.

[2] ABUHOMMOUS A.A., *The impact of offering trade credit on firms' profitability*, J. Corp. Acc. Fin., 2017, 28 (6), 29–40.

[3] AGGARWAL S.P., JAGGI C.K., *Ordering policies of deteriorating items under permissible delay in payments*, J. Oper. Res. Soc., 1995, 46 (5), 658–662.

[4] ARCELUS F.J., SHAH N.H., SRINIVASAN G., *Retailer's pricing, credit and inventory policies for deteriorating items in response to temporary price/credit incentives*, Int. J. Prod. Econ., 2003, 81–82, 153–162.

[5] BANERJEE A., *A joint economic-lot size model for purchaser and vendor*, Dec. Sci., 1986, 17 (3), 292–311.

[6] BOSE S., GOSWAMI A., CHAUDHURI K.S., *An EOQ model for deteriorating items with linear time dependent demand rate and shortages under inflation and time discounting*, J. Oper. Res. Soc., 1995, 46 (6), 771–782.

[7] CHANG C.T., *An EOQ model with deteriorating items under inflation when supplier credits linked to order quantity*, Int. J. Prod. Econ., 2004, 88, 307–316.

[8] CHEN S.C., TENG J.T., SKOURI K., *Economic production quantity models for deteriorating items with up-stream full trade credit and down-stream partial trade credit*, Int. J. Prod. Econ., 2014, 155, 302–309.

[9] CHUNG K.J., TING P.S., *A heuristic for replenishment of deteriorating items with a linear trend in demand*, J. Oper. Res. Soc., 1993, 44 (12), 1235–1241.

[10] CHUNG K.J., TING P.S., *On replenishment schedule for deteriorating items with time proportional demand*, Prod. Plan. Control, 1994, 5 (4), 392–396.

[11] HALEY C.W., HIGGINS R.S., *Inventory policy and trade credit financing*, Manage. Sci., 1973, 20 (4), 464–471.

[12] HARIGA M., *An EOQ model for deteriorating items with shortages and time-varying demand*, J. Oper. Res. Soc., 1995, 46 (4), 398–404.

[13] HO C.H., OUYANG L.Y., SU C.H., *Optimal pricing, shipment and payment policy for an integrated supplier–buyer inventory model with two-part trade credit*, European J. Oper. Res., 2008, 187 (2), 496–510.

[14] HUANG Y.F., *Optimal retailer's ordering policies in the EOQ model under trade credit financing*, J. Oper. Res. Soc., 2003, 54 (9), 1011–1015.

[15] HUANG Y.F., *An inventory model under two-levels of trade credit and limited storage space derived without derivatives*, Appl. Math. Model., 2006, 30, 418–436.

[16] JAMAL A.M.M., SARKER B.R., WANG S., *Optimal payment time for a retailer under permitted delay of payment by the wholesaler*, Int. J. Prod. Econ., 2000, 66, 59–66.

[17] LIN Y.J., OUYANG L.Y., DANG Y.F., *A joint optimal ordering and delivery policy for an integrated supplier–retailer inventory model with trade credit and defective items*, Appl. Math. Comp., 2012, 218 (14), 7498.

[18] LU L., *A one-vendor multi buyer integrated inventory model*, European J. Oper. Res., 1995, 81, 312–323.

[19] CHUNG K.L., HOU K.L., *An optimal production run time with imperfect production process and allowable shortages*, Comp. Oper. Res., 2003, 30, 483–490.

[20] GOYAL S.K., *An integrated inventory model for a single supplier-single customer problem*, Int. J. Prod. Res., 1976, 15 (1), 107–111.

[21] GOYAL S.K., *Economic order quantity under conditions of permissible delay in payments*, J. Oper. Res. Soc., 1985, 36, 335–339.

[22] GOYAL S.K., *A joint economics-lot-size model for purchaser and vendor: A comment*, Dec. Sci., 1988, 19 (1), 236–241.

[23] HOU K.L., *Optimal production run length for deterioration production system with a two-state continuous-time Markovian process under allowable shortages*, J. Oper. Res. Soc., 2005, 56, 346–350.
[24] KRENG V.B., TAN S.J., *The optimal replenishment decisions under two levels of trade credit policy depending on the order quantity*, Exp. Syst. Appl., 2010, 37 (7), 5514–5522.
[25] KRENG V.B., TAN S.J., *Optimal replenishment decision in an EPQ model with defective items under supply chain trade credit policy*, Exp. Syst. Appl., 2011, 38 (8), 9888–9899.
[26] KIM B.S., LEE W.S., *A multi-product dynamic inbound ordering and shipment scheduling problem at a third-party warehouse*, Int. J. Ind. Eng., 2013, 20 (1–2), 36–46.
[27] KINGSMAN B.G., *The effect of payment rules on ordering and stocking in purchasing*, J. Oper. Res. Soc., 1983, 34 (11), 1085–1098.
[28] MAHATA G.C., *An EPQ-based inventory model for exponentially deteriorating items under retailer partial trade credit policy in supply chain*, Exp. Syst. Appl., 2012, 39 (3), 3537–3550.
[29] MAHATA G.C., GOSWAMI A., *An EOQ model for deteriorating items under trade credit financing in the fuzzy sense*, Prod. Planning and Control, 2007, 18 (8), 681–692.
[30] MEHTA N.J., SHAH N.H., *An inventory model for deteriorating items with exponentially increasing demand shortages under inflation and time discounting*, Investigación Oper., 2003, 23 (1), 103–111.
[31] MEHTA N.J., SHAH N.H., *An inventory model for deteriorating items with exponentially decreasing demand shortages under inflation and time discounting*, Industrial Engineering Journal, 2004, 33 (4), 19–23.
[32] OUYANG L.Y., CHANG H.C., *Impact of investing in quality improvement on (Q, r, L) model involving imperfect production process*, Prod. Plan. Control, 2000, 11 (6), 598–607.
[33] PAKNEJAD M.J., NASRI F., AFFISCO J.F., *Defective units in a continuous review (s, Q) system*, Int. J. Prod. Res., 1995, 33 (10), 2767–2777.
[34] PORTEUS E.L., *Optimal lot sizing, process quality improvement and setup cost reduction*, Oper. Res., 1986, 34 (1), 137–144.
[35] RAHIM M.A., AL-HAJAILAN W.I., *An optimal production run for an imperfect production process with allowable shortages and time-varying fraction defective rate*, Int. J. Adv. Manuf. Techn., 2006, 27 (11–12), 1170–1177.
[36] ROSENBLATT M.J., LEE H.I., *Economic production cycles with imperfect production processes*, IIE Trans., 1986, 18, 48–55.
[37] SALAMEH M.K., JABER M.Y., *Economic production quantity model for items with imperfect quality*, Int. J. Prod. Econ., 2000, 64 (3), 59–64.
[38] SHAH N.H., SHAH Y.K., *A discrete-in-time probabilistic inventory model for deteriorating items under conditions of permissible delay in payments*, Int. J. Sys. Sci., 1998, 29, 121–126.
[39] SHAH N.H., GOR A.S., JHAVERI C., *Integrated optimal solution for variable deteriorating inventory system of vendor-buyer when demand is quadratic*, Canadian J. Pure Appl. Sci., 2009, 3 (1), 713–717.
[40] SHAH N.H., SHAH B.J., *EPQ model for time-declining demand with imperfect production process under inflationary conditions and reliability*, Int. J. Oper. Res., 2014, 11 (3), 91–99.
[41] SILVER E.A., MEAL H.C., *A simple modification of the EOQ for the case of a varying demand rate*, Production and Inventory Management, 1969, 10 (4), 52–65.
[42] SILVER E.A., *A simple inventory replenishment decision rule for a linear trend in demand*, J. Oper. Res. Soc., 1979, 30 (1), 71–75.
[43] TSAO Y.C., *Retailer's optimal ordering and discounting policies under advance sales discount and trade credits*, Comp. Ind. Eng., 2009, 56, 208–215.
[44] TENG J.T., CHANG C.T., GOYAL S.K., *Optimal pricing and ordering policy under permissible delay in payments*, Int. J. Prod. Econ., 2005, 97 (2), 121–129.
[45] TENG J.T., MIN J., PAN Q., *Economic order quantity model with trade credit financing for non-decreasing demand*, Omega, 2012, 40 (3), 328–335.

[46] WEE H.M., YU J., CHEN M.C., *Optimal Inventory model for items with imperfect quality and shortages backordering*, Omega, 2007, 35 (1), 7–11.
[47] WANG Z., LIU S., *Supply chain coordination under trade credit and quantity discount with sales effort effects*, Math. Probl. Eng., 2018, https://doi.org/10.1155/2018/2190236.

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