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GENERATING A SET OF COMPROMISE SOLUTIONS OF A MULTI-OBJECTIVE LINEAR PROGRAMMING PROBLEM THROUGH GAME THEORY

Most of real-life problems, including design, optimization, scheduling and control, etc., are inherently characterized by multiple conflicting objectives, and thus multi-objective linear programming (MOLP) problems are frequently encountered in the literature. One of the biggest difficulties in solving MOLP problems lies in the trade-off among objectives. Since the optimal solution of one objective may lead other objective(s) to bad results, all objectives must be optimized simultaneously. Additionally, the obtained solution will not satisfy all the objectives in the same satisfaction degree. Thus, it will be useful to generate a set of compromise solutions in order to present it to the decision maker (DM). With this motivation, after determining a modified payoff matrix for MOLP, all possible ratios are formed between all rows. These ratio matrices are considered a two person zero-sum game and solved by linear programming (LP) approach. Taking into consideration the results of the related game, the original MOLP problem is converted to a single objective LP problem. Since there exist numerous ratio matrices, a set of compromise solutions is obtained for MOLP problem. Numerical examples are used to demonstrate this approach.

Keywords: *multi-objective programming problem, game theory, compromise solution*

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1. Introduction

A large number of real-life problems can be modelled basically by LP, and a single-objective LP problem usually results in a single solution, called optimal solution. However, modelling real-life problems by LP is not realistic, because, it is increasingly difficult for people and institutions to interpret and judge the world around them in a one-dimensional manner and to treat them as if they were bound to a single criterion. In this approach, multi-objective decision making has come to the point of helping DMs to make decisions when they face problems expressed by multiple, unequally weighted [A1]and incompatible objectives.

Since there exist a lot of MOLP models arising in various industries, many approaches are developed to solve them. Das, Goswami and Alam focused on the solution procedure of the multiobjective transportation problem with cost coefficients of the objective functions and expressed the source and destination parameters as interval values [7]. Sabri and Beamon adopted multiobjective decision analysis to allow the use of a performance measurement system which includes cost, customer service levels (fill rates), and flexibility (volume or delivery) [23]. A mixed-integer LP to designing a multi-echelon and multi-objective supply chain network via optimizing commodity transportation and distribution was proposed by Paksoy, Özceylan and Weber [18]. Suprajitno presented the MOLP problems with interval numbers as coefficients and values of their variables which are also in the form of intervals and solved by a modified simplex method [26]. Shao and Ehrgott utilized algorithms developed to compute the entire set of nondominated points of multi-objective linear programmes to solve linear multiplicative programming problems [24]. Kasimbeyli et al. presented an analysis, characteristics and comparison of six commonly used scalarization methods in multiobjective optimization [11]. Cococcioni, Pappalardo and Sergeyev offered a new approach for solving lexicographic MOLP problems using a recently introduced computational methodology, allowing one to work numerically with infinities [6]. An LP-based algorithm was submitted for a class of optimization problems with a multi-linear objective function and affine constraints by Charkhgard, Savelsbergh and Talebian [5].

The Pareto set allows us to select the acceptable solution from a wide variety of options, which serves advantage to DM. With this motivation, in 1951, Koopmans first came up with the concept of Pareto efficient solution set, which effectively described the solution under the relationship of the partial order but not the total order [12]. Multi-objective optimization methods try to obtain solutions that are as close as possible to the Pareto optimal front and at the same time uniformly distributed solutions. Athan and Papalambros considered generalized weighted criteria methods that retain the advantages of the linear method without suffering from the limitation that capturing Pareto optimal points in a non-convex attainable region [2]. Deb and Saxena suggested an evolutionary multi-objective optimization procedure for solving large-objective (M) problems, which degenerate to possess a lower-dimensional Pareto-optimal front (lower than M) [8]. Huang, Galuski and Bloebaum offered the multi-objective Pareto concurrent subspace optimization method in which each discipline has substantial control over its own objective function during the design process, while still ensuring responsibility for constraint satisfaction in coupled subspaces [10]. Marler and Arora explored the fundamental significance of the weights in terms of preferences, the Pareto optimal set, and objective-function values [15]. Giagkiozis and Fleming investigated the implications of using decomposition-based methods over Pareto-based methods on algorithm convergence from a probabilistic point of view for multi-objective optimization [9].

The basic idea to solve a MOLP problem is to determine a solution that represents an acceptable trade-off or compromise between objectives, or to determine a set of such solutions. The decision-maker is allowed to choose a favourable solution among the obtained solutions. Romero, Amador and Barco showed how multiobjective programming, compromise programming, and filtering techniques can be used to tackle some problems found in agricultural planning [21]. Lahdelma, Miettinen and Salminen enhanced a method that uses achievement functions for charting subsets of reference points that would support a certain alternative to be the most preferred one [13]. Biswal and Acharya paid regard to a MOLP problem where some of the right-hand side parameters of the constraints are multi-choice in nature. According to them, the selection from the sets should be in such a manner so that the combination of choices for each set should provide the best compromise solution [4]. Ronald, Figueira and Smet dealt with project portfolio selection evaluated by multiple experts and modelled as a multi-objective combinatorial optimization problem solved by two procedures based on inverse optimization. They offered a compromise among the group of experts [22].

In game theory literature, there is a limited number of studies about how to obtain the optimal solution set when the system is satisfied with all the components. Belenson and Kapur proposed to develop a useful technique which is based on a two-person zero--sum game with mixed strategies for solving linear programmes involving more than one objective function [3]. Rao presented graphical interpretations of the non-cooperative and cooperative game theory approaches for a two-criteria optimization problem [19]. Rao and Freiheit developed a modification to the introduced game theory in which the two optimization steps are combined and with an algorithm for its implementation [20]. Sim and Kim discovered the evolutionarily stable strategy as a solution to multi -objective optimization problems using a coevolutionary algorithm based on evolutionary game theory [25]. Annamdas and Rao proposed particle swarm optimization with which coupled game theory based algorithms to solve multi-objective engineering optimization problems involving continuous, discrete and/or mixed design variables [1]. Meng, Ye and Xie presented the game description of the multiobjective optimization design problem and took the design objectives as different players [17]. Lee focused on the development of a multi-objective game theory model for balancing economic and environmental concerns in reservoir watershed management and for assistance in the decision [14]. Matejas and Peric introduced a new iterative method based on the principles of game theory for solving MOLP problems with an arbitrary number of DMs [16].

In this paper, we present a solution procedure to a MOLP problem by using an approach based on the linear programming modelling of a game matrix. In [3], MOLP is solved by converting the problem to a two-person zero-sum game, by forming the traditional payoff matrix. Since it uses the normalization-based approach, it has the ability to generate only one game matrix and thus one solution for the original MOLP problem, whereas in our approach a modified payoff matrix is constructed by a feasible corner point and individual optima of the objectives. With the aim of obtaining more than one solution, a scaling-based approach is used instead of normalizing. Thus, all possible ratios are formed between all rows of the payoff matrix. These various ratio matrices are solved by traditional linear programming approach for a two-person zero-sum game. The objectives are weighted by the probabilistic results of the related game, and the original problem is converted to a single objective LP problem. Thus, a set of compromise solutions is obtained for MOLP problem and so our method is able to present various solutions to DM. Then, the DM can choose the most appropriate solution for the objective or objectives using the satisfaction degrees considering his prioritizes, preferences and requirements.

This paper is organized as follows: after introducing the MOLP problem and basic definitions in Section 2, our proposed approach is presented in Section 3. In the next section, numerical experiments are considered. The last section emphasizes our conclusions.

2. MOLP problem

The mathematical model of the MOLP problem can be written as follows:

$$\max_{x\in S}\left\{Z_1(x), Z_2(x), ..., Z_k(x)\right\}$$

where

$$S=\left\{x\in R^n \,\middle|\, Ax \le,=,\ge b;\ x\ge 0\right\} \text{ and } Z_i(x)=Z_i\left(x_1, x_2, ..., x_n\right)=\sum_{j=1}^{n} c_{ij}x_j$$

for $i=1,2,...,k$.

Since a minimization problem can be converted to a maximization problem by taking its negative form, we assumed all the objectives are in maximization direction.

Definition 2.1. $x^* \in S$ is said to be a Pareto-optimal solution if and only if there does not exist another $x \in S$ such that $Z_i(x) \geq Z_i(x^*)$ for all $i = 1, 2, \ldots, k$ and $Z_i(x) \neq Z_i(x^*)$ for at least one *i*.

Definition 2.2. An optimal compromise solution of MOLP problem is a feasible solution $x \in S$ at which DM's preferences value, taking into consideration various respective objectives, is maximum.

3. Our approach to MOLP problem

In this paper, we intended to generate a set of compromise solutions in order to present it to the DM. With this motivation, firstly, a modified payoff matrix is determined by individual optima of the objectives and a feasible corner point of the corresponding feasible region *S*.

The individual optima of each objective are found by solving the MOLP problem as a single objective LP for each objective and ignoring all others as follows:

$$X_i^* = \max_{x \in S} Z_i(x), \; i = 1, 2, ..., k \tag{1}$$

A feasible corner point can be obtained in the following ways:

Using (2), a new objective function can be formed by adding the normalized objective functions up or

$$\max Z_{k+1}(x) = \sum_{i=1}^{k} \frac{Z_i(x)}{Z_i(X_i^*)} \tag{2}$$

Alternatively, a random nonzero feasible corner point can be determined by solving the original constraints with the zero objective function as well.

The optimal solution of (2) over the feasible region *s*, which is denoted by X_{k+1}^*, is determined by using

$$X_{k+1}^* = \max_{x \in S} Z_{k+1}(x) \tag{3}$$

Thus, the modified payoff matrix is constructed as in Table 1. We note that if the modified payoff matrix has at least one negative entry, it must be converted to a matrix whose entries are all positive by adding consecutive integer of the absolute value of the smallest negative entry of the matrix.

Table 1. The modified payoff matrix

	Z_1	Z_2	$\cdots$	Z_k
X_1^*	$Z_1(X_1^*)$	$Z_2(X_1^*)$	$\cdots$	$Z_k(X_1^*)$
X_2^*	$Z_1(X_2^*)$	$Z_2(X_2^*)$	$\cdots$	$Z_k(X_2^*)$
$\vdots$	$\vdots$	$\vdots$	$\iddots$	$\vdots$
X_k^*	$Z_1(X_k^*)$	$Z_2(X_k^*)$	$\cdots$	$Z_k(X_k^*)$
X_{k+1}^*	$Z_1(X_{k+1}^*)$	$Z_2(X_{k+1}^*)$	$\cdots$	$Z_k(X_{k+1}^*)$

For most of the MOLP problems that represent a real-life problem, there exist disparities between the values of objectives. Also, the units of measurement are not common for each objective. Thus, the entries of the payoff matrix will have to be scaled to compensate for these discrepancies. In this context, row ratios are formed between all rows of the modified payoff matrix. A ratio of two objective function values represents the relative efficiency between the corresponding points in the nominator and denominator, respectively. This ratio matrix is presented in Table 2.

Table 2. The ratio matrix

.	Z_1	Z_2	$\cdots$	Z_k
$R(X_1^*/X_2^*)$	$Z_1^{1/2}$	$Z_2^{1/2}$	$\ldots$	$Z_k^{1/2}$
$R(X_2^*/X_3^*)$	$Z_1^{2/3}$	$Z_2^{2/3}$	$\ldots$	$Z_k^{2/3}$
$\vdots$	$\vdots$	$\vdots$	$\iddots$	$\vdots$
$R(X_k^*/X_{k+1}^*)$	$Z_1^{k/(k+1)}$	$Z_2^{k/(k+1)}$	$\ldots$	$Z_k^{k/(k+1)}$

A ratio of two objective function values represents the relative efficiency between the corresponding points in the nominator and denominator, respectively. Namely, $R\left(X_r^*/X_s^*\right)$ and $Z_i^{r/s}=Z_i\left(X_r^*\right)/Z_i\left(X_s^*\right)$ in Table 2 represent which points' ratio is determined and corresponding relative efficiency values, respectively $\left(i, r \in \{1, 2, ..., k\},\ s \in \{2, ..., k+1\}\right)$. Thus, the size of the payoff matrix is reduced to $(k \times k)$ from $((k+1)\times k)$. It is obvious that the ratio matrix is dependent on the order of the rows, that is, different permutations yield different ratio matrices.

In literature, MOLP problem is solved by converting it to a two-person zero-sum game [3]. A two-person zero-sum game is characterized by the strategies of each player and the payoff matrix. Taking these relative efficiency values as the first player's strategies and the objective functions as the second player's strategies, the ratio matrix is considered as a game matrix.

These game matrices can be solved by the traditional linear programming approach as follows:

$$
\begin{aligned}
&\min \quad v \\
&q_1 Z_1^{1/2} + q_2 Z_2^{1/2} + \cdots + q_m Z_k^{1/2} \le v \\
&q_1 Z_1^{2/3} + q_2 Z_2^{2/3} + \cdots + q_m Z_k^{2/3} \le v \\
&\vdots \\
&q_1 Z_1^{k/(k+1)} + q_2 Z_2^{k/(k+1)} + \cdots + q_m Z_k^{k/(k+1)} \le v \\
&q_1 + q_2 + \cdots + q_m = 1 \\
&q_1, q_2, \ldots, q_m \in [0,1]
\end{aligned}
\tag{4}
$$

where $q_1, q_2, ..., q_m$ denote the mixed strategy solutions of column player, i.e., objectives of the MOLP, and v is the value of the game.

For finding a Pareto-optimal solution by using mixed strategy solutions, the following well-known theorem is presented:

Theorem. If a point $x^* \in S$ is a Pareto-optimal solution, then there exists a vector $\lambda^* \in R^n$ such that $\sum_{i=1}^{k} \lambda_k^* = 1$, $\lambda_k^* \ge 0$ and x^* is the solution of the following equivalent LP problem [3]:

$$\max_{x \in S} \hat{Z}(x) = \sum_{i=1}^{k} \sum_{j=1}^{n} \lambda_k^* c_{ij} x_j$$

where $S = \left\{ x \in R^n \,\middle|\, Ax \le, =, \ge b;\ x \ge 0 \right\}$.

In this paper, we obtained the vector $\lambda^* \in R^n$ as the result of (4). Thus, MOLP problem is converted to a single objective LP problem.

By different permutations of the ratio matrix rows, it will be possible to have more than one ratio matrix. Thus, we will enable to generate various solutions to present the DM. Thus, the DM may choose any compromise solution from the set.

4. Numerical experiments

Example 1. The following MOLP problem is considered [27]:

$$\max Z_1(\mathbf{x}) = -x_1 + 2x_2$$
$$\max\ Z_2(\mathbf{x}) = 2x_1 + x_2$$
$$-x_1 + 3x_2 \le 21$$
$$x_1 + 3x_2 \le 27$$
$$4x_1 + 3x_2 \le 45$$
$$3x_1 + \ x_2 \le 30$$
$$x_1, x_2 \ge 0$$

After obtaining individual optima of the objectives, a random nonzero feasible corner point is determined by solving the original constraints with the zero objective function. The modified payoff matrix is given in Table 3.

Table 3. The modified payoff matrix of Example 1

	Z_1	Z_2
$X_1^* = (0, 7)$	14	7
$X_2^* = (9, 3)$	–3	21
$X_3^* = (10, 0)$	–10	20

The modified payoff matrix has negative entries. Since the smallest entry is -10, we should add 11 to all entries. The positive modified payoff matrix is given in Table 3.

Table 3. The positive modified payoff matrix of Example 1

	Z_1	Z_2
$X_1^* = (0, 7)$	25	18
$X_2^* = (9, 3)$	8	32
$X_3^* = (10, 0)$	1	31

By using the values in Table 3, the first ratio matrix can be obtained as in Table 4.

Table 4. The first ratio matrix of Example 1

	Z_1	Z_2
$R\left(X_1^*/X_2^*\right)$	$Z_1^{1/2} = 3.125$	$Z_2^{1/2} = 0.562$
$R\left(X_2^*/X_3^*\right)$	$Z_1^{2/3} = 8.000$	$Z_2^{2/3} = 1.032$

In Table 4, $Z_1^{1/2} = 3.125$ means that the objective function value at X_1^* is 3.125 times efficient relative to that at X_2^* for the first objective. The remaining ratio values may be interpreted similarly. By using the ratio matrix in Table 4, the following LP problem can be constructed for the solution of the related game:

$$\begin{aligned} &\min \quad v \\ &3.125q_1 + 0.562q_2 \le v \\ &8q_1 + 1.032q_2 \le v \\ &q_1 + q_2 = 1 \\ &q_1, q_2 \in [0, 1] \end{aligned} \tag{5}$$

where q_1 and q_2 denote the mixed strategy solutions of column player, i.e., objectives. The optimal solution of (5) is $(q_1^*, q_2^*) = (0, 1)$. Finally, MOLP problem can be converted to (6) which has only one objective:

$$\max_{x \in S_1} \hat{Z}(\mathbf{x}) = q_1^*(-x_1 + 2x_2) + q_2^*(2x_1 + x_2) \tag{6}$$

where S_1 is the feasible region of Example 1. Since $(q_1^*, q_2^*) = (0, 1)$, $\hat{Z}(\mathbf{x})$ is equal to $Z_2(\mathbf{x})$ and the optimal solution of (6) is $X_2^* = (9, 3)$.

By constructing the all possible ratio matrices, the corresponding LP problems can be written for the related games. The obtained Pareto-optimal solutions and their objective values are presented in Table 5.

Table 5. Compromise solution set of Example 1

(x_1, x_2)	Z_1	Satisfaction degree [%]	Z_2	Satisfaction degree [%]
(9, 3)	–3	29.2	21	100
(6, 7)	8	75	19	90.5
(0, 7)	14	100	7	33.3
(5.03, 7.32)	7.32	81.7	6.91	82.8

As it is seen in Table 5, our approach generates more than one Pareto-optimal solutions different from Zimmermann's [27] solution to offer the DM. Thus, DM may choose one of them by considering the economic conditions, market conditions, his own status, etc.

Example 2. Let us consider the following MOLP problem [16]:

$$\max Z_1(\mathbf{x}) = 50x_1 + 100x_2 + 17.5x_3$$
$$\max Z_2(\mathbf{x}) = 50x_1 + 50x_2 + 100x_3$$
$$\max Z_3(\mathbf{x}) = 20x_1 + 50x_2 + 100x_3$$
$$\max Z_4(\mathbf{x}) = 25x_1 + 75x_2 + 12x_3$$
$$12x_1 + 17x_2 \le 1400$$
$$3x_1 + 9x_2 + 8x_3 \le 1000$$
$$10x_1 + 13x_2 + 15x_3 \le 1750$$
$$6x_1 + 16x_3 \le 1325$$
$$12x_2 + 7x_3 \le 900$$
$$9.5x_1 + 9.5x_2 + 4x_3 \le 107$$
$$x_1, x_2,\ x_3 \ge 0$$

where $x=(x_1, x_2, x_3)$.

The modified payoff matrix of Example 2 is presented in Table 6.

Table 6. The modified payoff matrix of Example 2

	Z_1	Z_2	Z_3	Z_4
$X_1^* = (44.937, 50.633, 41.772\ )$	8041.139	8955.696	7607.595	5422.152
$X_2^* = (22.276, 31.566, 74.459)$	5573.413	10137.987	9469.697	3817.839
$X_3^* = (0,\ 26.693,\ 82.813)$	4118.490	9615.885	9615.885	2995.703
$X_4^* = (10.417, 75.000, 0)$	8020.833	4270.833	3958.333	5885.417
$X_5^* = (45.221, 49.612, 43.523)$	7983.866	9093.890	7737.266	5373.676

We note that $X_5^* = (45.221, 49.612, 43.523)$ is obtained by solving the corresponding linear programming problem to (2). By constructing all the possible ratio matrices, the corresponding LP problems can be written for the related games. The obtained Pareto-optimal solutions and their objective values are presented in Table 7. Also, Table 8 gives the satisfaction degrees of the objectives.

Table 7. Compromise solution set of Example 2

(x_1, x_2, x_3)	Z_1	Z_2	Z_3	Z_4
(45.22, 49.61, 43.52)	7983.87	9093.89	7737.26	5373.68
(44.94, 50.63, 41.77)	8041.14	8955.70	7607.59	5422.15
(22.28, 31.57, 74.46)	5573.41	10137.99	9469.70	3817.84

Table 8. Satisfaction degrees of the objectives in Example 2

(x_1, x_2, x_3)	Z_1	Z_2	Z_3	Z_4
(45.22, 49.61, 43.52)	99.3	89.7	80.5	91.3
(44.94, 50.63, 41.77)	100	88.3	79.1	92.1
(22.28, 31.57, 74.46)	69.3	100	98.5	64.9

Tables 7 and 8 can be used so that the DM decides according to his own preferences.

5. Conclusions

Because of the problems related to one or more than one objective originate from several disciplines, using a single optimization technology is not sufficient to deal with real-life problems. Here, the trade-off among objectives can be interpreted as the biggest difficulties in solving MOLP problems. Because the obtained solution will not satisfy all the objectives in the same degree, it will be useful to generate various, but not many, solutions in order to present them to the DM. Indeed, if only one solution is obtained for MOLP problem, then the DM has no choice but to take this solution into consideration. With this perceptiveness, we presented a game theory-based approach to generate a set of compromise solutions of the MOLP problem. In addition, the number of solutions generated by our method will not be numerous, since the square ratio matrix of order k is permuted where k is the number of the objectives. The leading feature of our approach is the ability to offer more than one solution for the MOLP problem. Numerical examples are used to illustrate the applicability of the approach.

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