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LINEAR VERSUS NONLINEAR CAUSALITY FOR DAX COMPANIES

This study provides empirical evidence of the joint dynamics between stock returns and trading volume using stock data for DAX companies. Our research confirms the hypothesis that traditional linear causality tests often fail to detect some kinds of nonlinear relations, while nonlinear tests do not. In many cases, the test results obtained by use of empirical data and simulation confirm a bidirectional causal relationship, while linear tests did not detect such causality at all.

Keywords: *DAX companies, stock returns, trading volume, linear and nonlinear causality, simulation*

1. Introduction

Investors and analysts usually think that a stock price reflects investors' expectations regarding the future performance of a company based on the available information about this firm. Incoming information causes investors to change their expectations and is the main source of price changes. However, since investors vary in their interpretations of new information, prices may remain constant even though new information has been revealed to the market. This can take place if some investors think that the news is good, whereas others interpret it as bad news. Movements of prices therefore reflect the average reaction of investors to news. It is clear that stock prices can only change if there is a positive trading volume. As with prices, trading volume and changes in volume are related to the available set of relevant information on the market. Trading volume reacts in a different way to the reaction of stock prices. A revision in investors' expectations always leads to a rise in trading volume which therefore reflects the sum of investors' reactions to news. Observing the joint dyna-

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mics of stock prices and trading volume therefore improves the understanding of the microstructure of a stock market and can also have implications for research into options and futures markets.

A reliable answer to the question as to whether knowledge of one aspect of a financial market variable (e.g. trading volume) can improve short-run forecasts in other areas is very important for researchers, as well as market participants. Therefore, it is not surprising that there has recently been an increased interest in the relationship between trading volume, stock return and return volatility. The majority of previous empirical examinations have focused on the contemporaneous relationship between price changes and volume.

However, the dynamic relationship between these variables is more interesting than the contemporaneous one. One of the most important and fundamental topics in applied economics concerning such a dynamic dependence is the causal relationship between the variables in question. What researchers mean by causality and how it can be measured is an issue that is itself an important subject of research. The notion of causality in the Granger [11] sense is based on the idea that the past cannot be caused by the present or future. Therefore, if one event takes place before another event, causality can only lead from the first event to the second one. In order to prove causality Granger formulated a test statistic. By means of this test one can prove whether movements in one variable systematically precede movements in another variable. If including the past values of a variable in the information set can improve the prediction of another variable, then one can say that these variables are in a causal relation. In the case of a linear model, this means taking into account the dependency of one variable on its own past values and the past values of any potentially casual variable. If the coefficient estimates associated with the potentially causal variable are significant, then we have a causal (in the Granger sense) relationship from the causal variable to any variable under study. The first version of the Granger test was based on asymptotic distribution theory. But after a few years, Granger and Newbold [14] demonstrated by Monte Carlo simulations that if the process describing the dataset is not stationary, then regression analysis based on asymptotic distribution theory might not be a proper tool to analyze the data. The estimated dependencies can be spurious. Phillips [19] delivered an analytical proof of this observation. In order to cure this problem, a differencing of the data can be performed. However, as a result of differencing the long-run properties of the data can be lost. The existence of causality between two variables can be checked by means of a causality test. This test can be conducted on the basis of the vector error correction model (VECM) developed by Granger [12] and [13]. An assumption for this kind of testing is prechecking for unit roots and cointegration. These issues might not be interesting to the researcher in the context of causality. There are also a few papers, e.g. [20] and [21], which demonstrate that asymptotic distribution theory is not a proper basis for testing the causality of integrated variables by mean of the VAR model. This also holds true in the case where the variables cointegrate. Therefore, another concept of causality testing was developed based on a Wald test statistic.

However, the main goal of this study is a comparison of Granger linear and nonlinear causality test results. In the next section we give a short insight into the existing literature. In Section 3 we will characterize the dataset. In Section 4 we demonstrate the concept and testing of linear and nonlinear causality. In Section 5 we report the empirical results. Section 6 displays the results of simulation exercises concerning a comparison of the concepts of linear and nonlinear causality. The last section concludes the paper.

2. Literature overview

In recent years the subject of nonlinear causality tests has been raised many times (e.g., see [1] and [2]). One serious problem with the linear approach to testing for causality is that such tests can have low power to detect some kinds of nonlinear causal relations. This problem was raised in articles dedicated to nonlinear causality tests (see also Section 6 of this study).

A nonparametric statistical method for uncovering nonlinear causal effects presented by Baek and Brock [3] was the starting point for further investigations. Their approach used the correlation integral, an estimator of spatial probabilities across time based upon the closeness of points in hyperspace, to detect causal relations.

Hiemstra and Jones [9] made further modifications to this. Their concept improves the small-sample properties of the test and relaxes the assumption that the series to which the test is applied are i.i.d. The authors run some Monte Carlo simulations to prove that their test is robust to the presence of structural breaks in the series and contemporaneous correlations in the errors of the VAR model used to filter out linear cross- and auto-dependence.

Diks and Panchenko [8] propose another modification. They pay attention to the fact that the null hypothesis in the HJ (Hiemstra and Jones) test is generally not equivalent to Granger noncausality. With the help of some modifications they manage to avoid this problem with an adaptation of the null hypothesis. Furthermore, the authors find their test to have better performance than the HJ one, especially in terms of over-rejection and size distortion, which are quite often reported for the HJ test. Some practical recommendations, including bandwidth adaptation, are also provided.

The main goal of our dynamic investigations – as we pointed out above – was to answer the question as to whether knowledge of one financial market variable can improve the short-run forecasts of others, which is very important for both researchers and market participants.

The dynamic relationship between chosen stock variables has been the subject of research in recent years. Brailsford [4] and Lee and Rui [17] investigated the relationship between trading volume and price changes, mainly using index prices. The results of these studies vary, although a positive relationship is normally reported.

Gurgul and Majdosz [15] study the dynamic (causal) relationships between trading volume, stock returns and return volatility for the Polish stock market. Applying the linear Granger causality test, they observe a significant causal relationship between returns and trading volume in both directions and linear Granger causality from return volatility to trading volume. Furthermore, their findings show that knowledge of past stock price movements on the German, as well as the US stock market, improves short-run forecasts of current and future trading volume on the Polish stock market.

This paper relates to previous articles concentrated on the causal relationships between financial variables, e.g. the contribution by Mestel et al. [18] which concerns the empirical relationship between stock returns, return volatility and trading volume data of 31 companies from the Austrian stock market. The authors found evidence of a relationship (contemporaneous, as well as causal) between return volatility and trading volume. Their results indicate that return volatility precedes trading volume in about half of all cases, implying that information might flow sequentially rather than simultaneously into the market. In contrast, they point out that the relationship between stock returns and trading volume is mostly negligible. Knowledge of one of these variables cannot improve short-run forecasts of the other.

Gurgul et al. [16] perform similar investigations for DAX companies. Among other results (less concerned with the subject of this paper), their findings indicate that there is no evidence of a contemporaneous relationship between stock returns and trading volume. They also find the dynamic relations between these data to be mostly negligible. Taking these results into account, they conclude that short-run forecasts of current or future stock returns cannot be improved by a knowledge of recent volume data and vice versa. However, their research does not involve a nonlinear causality test, being based only on the traditional linear approach.

3. Dataset

In this section we give a short description of our dataset used in further computations. The considered dataset includes daily stock price and trading volume series for all the companies listed on the DAX. The main role of the DAX is to measure the performance of the 30 biggest German companies (see Table 1 in the Appendix). The investigation covers the period from January 2001 to November 2008. The full sample contains 2009 trading days and data were available for all these days in the case of

27 companies. All the companies considered had been quoted in the index for at least 1000 trading days over the period under study, thus we did not decide to reduce the number of companies for further analysis. A list of all the companies included in the sample, as well as their relevant period of quotation is included in the appendix at the end of the paper. We applied continuous returns (logarithmic returns). Table 2 contains descriptive statistics for percentage stock returns.

Table 2. Aggregated summary statistics for daily percentage stock return data from DAX companies (January 2001–November 2008)

	Mean	Std. deviation	Kurtosis	Skewness
Min	–0.1515	1.604247	3.182317	–0.67006
1st Quartile	–0.0488	1.967133	5.536662	–0.17087
Median	–0.0179	2.326153	6.722743	0.01388
3rd Quartile	0.0047	2.373798	9.544098	0.17428
Max	0.0919	2.890366	24.89462	0.94968

Directly from this table, we see that the mean daily stock returns over the period under study range from –0.1515% (Infineon) to +0.0919% (K&S) with a median of –0.0179%. The standard deviation is lowest for Henkel (1.604%) and highest for Commerzbank (2.89%).

Our data confirm the stylized fact of “fat-tailed and highly-peaked” distributions for return series. The median of stock return kurtosis is 6.72 and ranges from 3.18 (Merck) to 24.894 (Bayer). Return skewness is lowest for Deutsche Post (–0.67) and highest for Fresenius Medical Care (0.949). The median is 0.0138. From the results of Jarque-Bera and chi-square goodness-of-fit tests for normality, we may claim that these returns do not come from a normal distribution.

In this article we use the squared values of daily stock returns as a measure of volatility. The basic descriptive statistics for the time series of squared returns are contained in Table 3.

Table 3. Aggregated summary statistics for daily squared stock return data for DAX companies (January 2001– November 2008)

	Mean	Std. deviation	Kurtosis	Skewness
Min	0.10105	1.8007355	1634.10	32.7603
1st Quartile	0.12746	2.3985080	2542.94	41.7400
Median	0.16655	2.9005676	4344.87	55.6056
3rd Quartile	0.18111	3.4688685	6792.89	69.3173
Max	0.37487	6.0279713	9450.93	94.2585

The hypothesis of squared returns having a normal distribution is strongly rejected, as kurtosis reaches huge values. Let us underline the fact that these time series display the usual time dependence of stock returns regarding the second order moment (volatility persistence). This remark clearly suggests that returns cannot be assumed to be i.i.d.

To measure trading volume we use the natural logarithm of the daily number of shares traded. The mean trading volume varies from 12.34 (Merck) to 16.90 (Deutsche Telekom) with a median of 14.73. Some basic descriptive statistics for the time series of trading volume can be found in Table 4.

Table 4. Aggregated summary statistics for daily trading volume data from DAX companies (January 2001–November 2008)

	Mean	Std. deviation	Kurtosis	Skewness
Min	12.34037	0.41875	–0.56696	–1.12513
1st Quartile	13.83341	0.51551	0.29239	–0.06303
Median	14.73013	0.57667	0.81201	0.15567
3rd Quartile	15.40939	0.65303	1.06386	0.24100
Max	16.90416	1.02803	3.53313	0.62347

Because trading volume data is characterized by time-varying means (see [10]), we therefore decided to detrend our volume data. We do not restrict our interest only to detrended trading volume, the ideas of expected and unexpected trading volume are also included in our research. The details of the methods used to model these trading volume series are contained in the next section.

4. Methodology

In this article we use both linear and nonlinear Granger causality tests to explore the dynamic relation between trading volume and stock returns, as well as between trading volume and volatility. As previously mentioned, our main goal was to compare the results obtained thanks to the application of linear tests with those gained with the help of the modified Hiemstra and Jones test (Panchenko and Diks's correction). At this point, we mention once again the previous results achieved by Gurgul et al. [16]. The authors investigated dynamic relationships between the mentioned variables for DAX companies using only linear causality tests. This article contains the results obtained by means of nonlinear methods, which throws new light on the investigated stock relations.

The definition of causality used in this paper is due to Granger. Let $\{X_t\}$ and $\{Y_t\}$ be two scalar-valued, stationary and ergodic time series. Furthermore, let $F\{X_t / I_{t-1}\}$ denote the conditional probability distribution of X_t given the bivariate information set

I_{t-1}. The latter consists of an L_X – lagged vector of X_t ($X_{t-L_X}^{L_X} := (X_{t-L_X}, X_{t-L_X+1}, ..., X_{t-1})$) and L_Y – lagged vector of Y_t ($Y_{t-L_Y}^{L_Y} := (Y_{t-L_Y}, Y_{t-L_Y+1}, ..., Y_{t-1})$). According to Granger [11], given lags L_X and L_Y, the time series $\{Y_t\}$ does not strictly cause the time series $\{X_t\}$, if:

$$F(X_t \mid I_{t-1}) = F(X_t \mid I'_{t-1}), \quad t = 1, 2, ..., \tag{1}$$

where I'_{t-1} denotes an information set including lagged values of X_t only. In other words, knowledge of the past values of the time series $\{Y_t\}$ improves the prediction of current and future values of $\{X_t\}$, if equality (1) does not hold. In this situation $\{Y_t\}$ is said to Granger cause $\{X_t\}$.

The linear Granger causality test used in this paper is based on the methodology of vector autoregression, which assumes the stationarity of the investigated time series. Thus, before we started our research on causality, we conducted some tests of stationarity. Based on previous results and research (see [10]), we checked first for linear and nonlinear time trends in the series of trading volumes, which in other words may be expressed as carrying out an analysis of the following regression model:

$$V_t = a + bt + ct^2 + \varepsilon_t, \tag{2}$$

where ε_t is assumed to be white noise. The residuals in the above equation represent the detrended trading volume (the symbol used is $V_{\text{detrended}}$) for the subsequent analysis.

The next step of our initial analysis is testing for stationarity. We use the augmented Dickey–Fuller unit root test, which is based on the following regression model:

$$\Delta z_t = a + bz_{t-1} + \sum_{i=1}^{k} c_i \Delta z_{t-i} + \varepsilon_t, \tag{3}$$

where $\{z_t\}$ denotes the time series being analyzed, k is the lag number, Δ is the differencing operator and ε_t is assumed to be white noise. The null hypothesis is simply described by the condition $b = 0$, and the one-sided alternative is $b < 0$. For the critical values we refer to Charemza and Deadman [6].

The ADF test results allow us to assume the stationarity of all the variables considered. For all companies and variables, the parameter b was found to be negative and statistically significant at sensible levels.

In the next step we define two important time series – expected trading volume (symbol used: EV) and unexpected trading volume (symbol used: UEV). Their definitions are given below:

$$\text{EV}_t = E[V_t \mid U_t], \tag{4}$$

$$\text{UEV}_t = V_t - E[V_t \mid U_t], \tag{5}$$

where V_t denotes trading volume on date t, and $E[V_t|U_t]$ denotes the predicted trading volume conditional on the information set U_t.

To create the time series of expected volume, we use the following regression model:

$$V_t = \mu + \sum_{j=1}^{k} \alpha_j V_{t-j} + \sum_{j=1}^{k} \beta_j R_{t-j} + \sum_{j=1}^{k} \gamma_j R_{t-j}^2 + \varepsilon_t \,, \tag{6}$$

where V_t, R_t, R_t^2 denote trading volume, return and squared return (which we use as a measure of volatility), respectively. It is worth mentioning that the idea of model (6) comes from the suggestions of Lee and Rui [17]. To choose the length of the lag (k), we use the Akaike Information Criterion (AIC), as well as the Schwarz–Bayesian Information Criterion (BIC). The computation scheme goes as follows. For each day we estimate (using the ordinary least squares method (OLS)) the parameters of equation (6), based on the data set of 100 trading days prior to day t. For every equation significance tests are conducted for all parameters. We omit those variables which are insignificant at the 5% significance level and then we finally estimate the restricted model once again. This is how the value of EV_t is obtained. It is obvious that the first realization of EV_t will occur on the 101$^{\text{st}}$ analyzed trading day, thus the sample size is smaller for models with variables EV_t or UEV_t.

To test for linear Granger causality, we use a vector autoregression model (VAR) of the form:

$$\begin{cases} y_t = \mu_y + \sum_{j=1}^{k} \alpha_{0,j} y_{t-j} + \sum_{j=1}^{k} \beta_{0,j} x_{t-j} + \varepsilon_t, \\ x_t = \mu_x + \sum_{j=1}^{k} \alpha_{1,j} x_{t-j} + \sum_{j=1}^{k} \beta_{1,j} y_{t-j} + \varepsilon_t, \end{cases} \tag{7}$$

where the time series $\{y_t\}$ and $\{x_t\}$ are assumed to be stationary and ergodic. For each company we create six VAR models describing the relations between trading volume and stock returns, as well as between trading volume and volatility (note that we consider three different ways of describing volume). We set up a maximum lag equal to 5 and for each VAR model we chose an optimal lag k (from the set {1, ..., 5}), using the AIC and BIC criteria. In most cases we find $k = 2$ to be the appropriate value, which ensures that the residuals do not exhibit serial correlation. Finally, to estimate the parameters in each VAR model we use the OLS method.

To test for linear Granger causality, we use the simple F test. If the null hypothesis $\beta_{0,j} = 0$ ($\beta_{1,j} = 0$) for $j = 1, ..., k$ is rejected at a sensible level of significance, then we may say that X Granger causes Y (Y Granger causes X). The F statistic is calculated thus:

$$F = \frac{SSE_0 - SEE}{SEE} \cdot \frac{N - 2k - 1}{k}, \tag{8}$$

where SSE_0 denotes the sum of squared residuals for a model with restrictions ($\beta_{0,1} = \beta_{0,2} = \beta_{0,k} = 0$ ($\beta_{1,1} = \beta_{1,2} = ... = \beta_{1,k} = 0$), SSE denotes the sum of squared residuals for a model without any restrictions and N denotes the number of observations. Under the null hypothesis, the statistic (8) has an F distribution with k degrees of freedom in the numerator and $N - 2k - 1$ degrees of freedom in the denominator.

Let us now make a brief reference to the nonlinear Granger test used in this article. As we mentioned in the introduction, in recent years there have been several modifications of the test proposed by Baek and Brock [3]. In this paper we use the approach proposed by Diks and Panchenko [8].

We will focus on the problem of investigating whether $\{X_t\}$ Granger causes $\{Y_t\}$ (one can perform an analogous analysis testing for causality in the opposite direction). For this purpose, let us define for t = 1, 2, ... the $L_X + L_Y + 1$-dimensional vector $W_t = (X_{t-L_X}^{L_X}, Y_{t-L_Y}^{L_Y}, Y_t)$. The null hypothesis that $\{X_t\}$ does not Granger cause $\{Y_t\}$ may be written in terms of density functions:

$$f_{X,Y,Z}(x, y, z) = f_{X,Z}(x, z) f_{Z|X,Y}(z \mid x, y) = f_{X,Z}(x, z) f_{Z|Y}(z \mid y), \tag{9}$$

where $f_X(z)$ stands for the probability density function of the random vector X at point z, $X = X_{t-L_X}^{L_X}$, $Y = Y_{t-L_Y}^{L_Y}$, $Z = Y_t$ for t = 1, 2, ..., with the meaning of the other symbols as already explained in this paper. The last equation may be presented in the more convenient forms:

$$\frac{f_{X,Y,Z}(x, y, z)}{f_{X,Y}(x, y)} = \frac{f_{Y,Z}(y, z)}{f_Y(y)} \tag{10}$$

and

$$\frac{f_{X,Y,Z}(x, y, z)}{f_Y(y)} = \frac{f_{X,Y}(x, y)}{f_Y(y)} \frac{f_{Y,Z}(y, z)}{f_Y(y)}. \tag{11}$$

Next let us define the correlation integral $C_W(\varepsilon)$ for the multivariate random vector W by the following expression:

$$C_W(\varepsilon) = P[\| W_1 - W_2 \| \le \varepsilon = \iint I(\| s_1 - s_2 \| \le \varepsilon) f_W(s_1) f_W(s_2) ds_2 ds_1, \tag{12}$$

where W_1, W_2 are independent with distributions in the equivalence class of the distribution of W, letter I denotes the indicator function (equal to one if the condition in brackets holds true, otherwise equal to zero), $\|x\| = \sup\{|x_i|: i = 1, ..., d_W\}$ denotes the supremum norm (d_W is the dimension of the sample space W) and $\varepsilon > 0$.

Hiemstra and Jones [9] claimed that the null hypothesis in Granger's causality test implies that for every $\varepsilon > 0$:

$$\frac{C_{X,Y,Z}(\varepsilon)}{C_{X,Y}(\varepsilon)} = \frac{C_{Y,Z}(\varepsilon)}{C_{Y}(\varepsilon)} \tag{13}$$

or equivalently:

$$\frac{C_{X,Y,Z}(\varepsilon)}{C_{Y}(\varepsilon)} = \frac{C_{X,Y}(\varepsilon)}{C_{Y}(\varepsilon)} \frac{C_{Y,Z}(\varepsilon)}{C_{Y}(\varepsilon)} . \tag{14}$$

They recommend calculating sample versions of correlation integrals and then testing whether the left-hand- and right-hand-side ratios differ significantly or not. They propose the use of the following formula as an estimator of the correlation integral:

$$C_{W,n}(\varepsilon) = \frac{2}{n(n-1)} \sum\sum_{i<j} I_{ij}^{W} , \tag{15}$$

where $I_{ij}^{W} = I(\| W_i - W_j \| < \varepsilon)$. As shown by Diks and Panchenko [7], testing relation (13) or (14) is not equivalent, in general, to testing the null hypothesis of Granger causality. The authors find exact conditions under which the HJ test is useful in investigations of causality. This fact is not the main point of our research, we refer to Diks and Panchenko [8] for more details on this issue.

As we have already noted, we used the modified version of the HJ test proposed by Panchenko and Diks. In terms of the expected value and density functions, they managed to bypass the problem mentioned above by testing based on the asymptotic theory of the modified test statistic [8]. Furthermore, they presented some advice concerning the proper way of choosing the bandwidth according to sample size. This adaptation is helpful in reducing the bias of the test, which is one of the serious problems which arise for long time series. We use all their remarks in our analysis and emphasize that the performance of the modified test is much better than that of the HJ test. We have also decided to use the same lags for each pair of time series analyzed, establishing the lag at the level of 2.

In order to test for nonlinear Granger causality between two time series, say $\{EV_t\}$ and $\{R_t\}$, we apply the Panchenko and Diks test. We applied our calculations on the basis of the time series of residuals resulting from the applied VAR model (in this case for the $\{EV_t\}$ and $\{R_t\}$ model). The time series of residuals reflect nonlinear dependencies (linear causality is described by VAR). The time series of residuals are both standardized, thus they share a common scale parameter. According to the idea of bandwidth adaptation mentioned above, we set the bandwidth ε to be equal to 0.9 in all cases, despite the smaller sample size appearing in those VAR models which describe the dynamics of $\{EV_t\}$ or $\{UEV_t\}$. The latter did not have a significant influence on the value of the bandwidth and therefore the test results.

5. Empirical results

In this section we present the results of both linear and nonlinear Granger causality tests. These outcomes are helpful in describing the dynamic relationship between stock returns and trading volume, as well as between volatility and volume. The latter is helpful in developing a better understanding of the dynamics of stock markets.

The main point that essentially distinguishes our study from former contributions lies in the fact that we conduct not only a linear causality test, but also its nonlinear version, which in many cases leads to interesting conclusions.

One important problem in conducting linear Granger tests is their low power in detecting certain kinds of nonlinear causal relationship. As presented by Brock [5], linear causality tests, such as the Granger test, can often lack power in uncovering nonlinear relationships. He used the following model:

$$X_t = B \cdot Y_{t-p} \cdot X_{t-q} + \varepsilon_t \,, \tag{16}$$

where $\{Y_t\}$ and $\{\varepsilon_t\}$ are mutually independent and individually i.i.d. $N(0, 1)$ time series, B denotes a parameter and p and q are lag lengths. One can easily see that $\{X_t\}$ depends on past values of $\{Y_t\}$, but since all autocorrelations and cross correlations are zero the traditional linear test will incorrectly indicate that there is no lagged dynamic causal relation from $\{Y_t\}$ to $\{X_t\}$.

The results of linear and nonlinear Granger causality tests are showed in tables 12–17 in the appendix at the end of this paper.

Before we examine the general results, let us analyze a particular example of the usage of nonlinear Granger causality tests. The following table illustrates the p-values obtained from testing for linear and nonlinear Granger causality in the case of the *Infineon* company:

Table 5. Results of causality tests conducted for the *Infineon* company

Volume form / Null Hypothesis	Detrended Volume		Expected Volume		Unexpected Volume	
	Linear test	Nonlinear test	Linear test	Nonlinear test	Linear test	Nonlinear test
V_t do not cause R_t	0.6394	0.27	0.8041	0.09	0.5770	0.06
R_t do not cause V_t	0.8759	0.96	0.3611	0.01	0.2938	0.99
V_t do not cause R_t^2	0.3835	0.43	0.3170	0.26	0.6051	0.29
R_t^2 do not cause V_t	0.0404	0.99	0.0001	0.05	0.2847	0.99

As we can see, a traditional linear causality test fails to detect causality between expected trading volume and returns in any direction, whereas a nonlinear test reveals a bidirectional relationship. In the above example, only the nonlinear test finds evidence of Granger causality from unexpected volume to stock returns. On the other hand, the Granger causality from volatility to detrended volume is found to be linear. Similar results for other companies may be supplied by the authors upon request.

After analyzing tables 12–17, one can easily find that the conducted tests provide a strong basis for claiming that stock returns Granger cause expected trading volume. We found evidence for this thesis in all cases using the nonlinear test, while the traditional linear approach fails to detect this type of causality in two cases. It is interesting that linear causality in the opposite direction was found in six cases only, nonlinear causality was found in ten cases, and no instance of both linear and nonlinear causality was revealed. This example shows that causality from expected volume to returns was found in 16 cases. One should note that the sum of the number of rejected null hypotheses in linear causality tests and the corresponding number of rejections in tests for strictly nonlinear relations gives the total number of causal relations observed.

Even stronger evidence is found for causality from volatility to expected trading volume. This kind of relationship was detected for all companies with the use of a traditional linear test, while a nonlinear test confirmed it in all but one case. A bidirectional relationship was found in 20 cases, including 6 strictly linear, 5 strictly nonlinear and 9 cases with both bidirectional types of causality. One more issue worth mentioning is the average p-value observed while investigating the relations between volatility and expected volume. Since this is very close to zero in most cases both for linear and nonlinear tests (especially when testing causality from volatility to expected volume), there is solid evidence for the existence of a dynamic relationship between these variables. We therefore conclude that short-run forecasts of expected trading volume may be improved by knowledge of the recent values of stock returns, as well as the recent values of volatility. As mentioned above, we found strong evidence of this fact. Proofs of feedback relations are more convincing when both linear and nonlinear methods are considered rather than linear tests alone.

The analysis of the relations between unexpected trading volume and stock returns or volatility indicates that the test results supply evidence of a very weak causal relationship. In spite of this, a few facts are worth underlining. The first issue is that causality from unexpected trading volume to stock returns was only detected by the linear test in two cases, while the nonlinear test rejected the null hypothesis in 11 cases. Similarly, there were 5 instances of causality from unexpected trading volume to volatility which were found only by nonlinear testing. This may be taken as support for the well known thesis of linear causality tests having low power in detecting some kinds of nonlinear causal relations. We mention here once again the Brock example presented above.

We also beg to note once again that no bidirectional linear causal relation between unexpected trading volume and stock returns was found for any company, neither between unexpected trading volume and volatility. The application of nonlinear methods does not change this at all, which strongly supports the hypothesis of no bidirectional causal relationship between the pairs of variables mentioned above.

The analysis of the results of causality tests conducted for detrended volume and stock returns or volatility also provides some interesting observations. It must be noted that our findings from linear tests confirmed that stock returns caused detrended trading volume for 21 companies. The nonlinear test found just one case of this type of causality (incidentally, this one was not indicated by the linear test). These results convinced us to accept the hypothesis that the past values of stock returns are useful for improving the short-run forecast of detrended trading volume and also that the nature of this process is rather linear. If we look at the results of tests of causality in the opposite direction, we will see that in this case no linear relationship was found for any company. However, in 9 cases the nonlinear test indicated a causal relationship from detrended trading volume to stock returns, which seems to be a good starting point for further investigation.

Although volatility was found to be a linear cause of detrended trading volume in 19 cases and a nonlinear effect only in 2 cases, the analysis of these relations in the opposite direction led us to different conclusions in comparison to the previously mentioned pair of variables (R_t and $V_{\text{detrended}}$). In this case a linear Granger causality test rejected the null hypothesis 15 times, while the modified HJ test reported such results only 3 times in the absence of a significant linear relationship (see Table 13 in the appendix).

These results are clearly in line with the outcomes presented for stock returns and detrended trading volume and lead to interesting conclusions. As the causal relationship from $V_{\text{detrended}}$ to R_t was found to have a rather nonlinear nature, then the evidence of linear causality from detrended trading volume to volatility (squared returns) may in some way justify the sense of exploring the mentioned nonlinear relation in terms of polynomials of order two.

6. Simulation exercises

In order to compare the performance of both linear and nonlinear causality tests, we conducted some simulation exercises. We assumed the null hypothesis: $\{Y_t\}$ does not Granger cause $\{X_t\}$; and that $\{Y_t\}$ and $\{\varepsilon_t\}$ are mutually independent and individually i.i.d. $N(0, 1)$ time series (except in points e and f); number of repetitions: 50; maximal lag length (VAR models): 3, lag length (same for $\{X_t\}$ and $\{Y_t\}$ in nonlinear tests): 2 (in points a, c, e and f) and 4 (in points b and d); bandwidth adaptation: according to Panchenko and Diks):

Simulation results for chosen nonlinear models (percentage values denote the significance levels) are displayed below in a)–f):

a) $X_t = \frac{\sqrt{2}}{2} \cdot X_{t-1} \cdot Y_{t-1} + \varepsilon_t, \quad X_0 \sim N(0, \sqrt{2})$

Table 6. Simulation results for model a)

		N = 2000		N = 5000		N = 10000	
		Linear	Nonlinear	Linear	Nonlinear	Linear	Nonlinear
Example *p*-values		0.95	0.0011	0.93	0.0001	0.56	0.0061
		0.22	0.0041	0.19	0.0001	0.59	0.0033
		0.61	0.0001	0.46	0.0091	0.21	0.0002
Number of rejected null hypothesis	10%	13	50	7	50	6	50
	5%	11	48	5	49	6	47
	1%	4	45	1	45	3	47

b) $X_t = \frac{\sqrt{2}}{2} \cdot X_{t-1} \cdot Y_{t-3} + \varepsilon_t, \quad X_3 \sim N(0, \sqrt{2})$

Table 7. Simulation results for model b)

		N = 2000		N = 5000		N = 10000	
		Linear	Nonlinear	Linear	Nonlinear	Linear	Nonlinear
Example *p*-values		0.33	0.0051	0.49	0.0009	0.22	0.0071
		0.35	0.0004	0.18	0.0023	0.76	0.0002
		0.85	0.0022	0.77	0.0052	0.85	0.0045
Number of rejected null hypothesis	10%	15	50	11	50	11	50
	5%	11	49	4	50	6	48
	1%	1	47	1	49	2	48

c) $X_t = \frac{1}{2} \cdot X_{t-1} + \frac{1}{2}(Y_{t-1})^2 + \varepsilon_t, \quad X_0 \sim N(1, \sqrt{2})$

Table 8. Simulation results for model c)

		N = 2000		N = 5000		N = 10000	
		Linear	Nonlinear	Linear	Nonlinear	Linear	Nonlinear
Example *p*-values		0.22	0.0001	0.78	0.0077	0.34	0.0031
		0.17	0.0005	0.18	0.0001	0.35	0.0002
		0.49	0.0032	0.28	0.0059	0.69	0.0047
Number of rejected null hypothesis	10%	8	49	12	50	14	50
	5%	5	48	6	50	10	46
	1%	0	48	3	50	6	46

d) $X_t = \frac{1}{2} \cdot X_{t-1} + \frac{1}{2}(Y_{t-3})^2 + \varepsilon_t, \quad X_3 \sim N(1, \sqrt{2})$

Table 9. Simulation results for model d)

		N = 2000		N = 5000		N = 10000	
		Linear	Nonlinear	Linear	Nonlinear	Linear	Nonlinear
Example *p*-values		0.86	0.0034	0.43	0.0098	0.25	0.0002
		0.22	0.0001	0.26	0.0012	0.97	0.0001
		0.93	0.0024	0.24	0.0056	0.74	0.0005
Number of rejected null hypothesis	10%	12	50	8	50	7	50
	5%	10	49	4	50	7	48
	1%	5	49	0	48	2	47

e) $X_t = 0.88502X_{t-1} + 0.61325(Y_{t-1})^4 + \varepsilon_t,\ X_0 \sim N(1, \sqrt{5})$, (this time $Y_t \sim N\left(0, \frac{1}{2}\right)$, the other assumptions are unchanged):

Table 10. Simulation results for model e)

		N = 2000		N = 5000		N = 10000	
		Linear	Nonlinear	Linear	Nonlinear	Linear	Nonlinear
Example *p*-values		0.81	0.0009	0.95	0.0001	0.35	0.0091
		0.73	0.0071	0.88	0.0042	0.92	0.0001
		0.99	0.0611	0.43	0.0011	0.77	0.0022
Number of rejected null hypothesis	10%	8	49	7	50	8	50
	5%	5	46	5	49	3	49
	1%	3	45	2	47	0	46

f) $X_t = \frac{1}{2}X_{t-1} + \frac{1}{2}(Y_{t-1})^2 + \varepsilon_t,\ X_0 \sim N(1, \sqrt{2})$, (this time $\varepsilon_t \sim \frac{1}{2}\chi_1^2 + N\left(-\frac{1}{2}, \frac{\sqrt{2}}{2}\right)$ (independent addends), the other assumptions are unchanged):

Table 11. Simulation results for model f)

		N = 2000		N = 5000		N = 10000	
		Linear	Nonlinear	Linear	Nonlinear	Linear	Nonlinear
Example *p*-values		0.89	0.0001	0.95	0.0001	0.55	0.0013
		0.36	0.0001	0.88	0.0042	0.38	0.0001
		0.74	0.0056	0.43	0.0011	0.91	0.0045
Number of rejected null hypothesis	10%	11	50	13	50	7	49
	5%	6	50	9	47	5	48
	1%	4	49	2	46	2	48

Comparing the results of both linear and nonlinear tests, it is clear that a nonlinear test is more appropriate than a linear test by far when the actual dependencies are nonlinear, which is usually the case in economics and finance.

7. Conclusions

This paper concerns the joint dynamics of daily trading volume, stock returns and volatility for German companies listed on the DAX. In order to test for Granger causality, we used both linear and nonlinear methods, which surely distinguishes our research from previous work on this issue. Besides the standard *F* test we use a nonlinear test (developed by Hiemstra and Jones and modified by Panchenko and Diks). The main goal of this paper was to examine whether knowledge of the past values of one of the mentioned time series is helpful in improving short-run forecasts of another one. The application of nonlinear methods is not the only issue which distinguishes our results from results reported in the literature. We conducted our research in several versions, i.e. for detrended, expected and unexpected trading volume. This thorough investigation led us to some interesting conclusions.

Our findings indicate strong a causal relationship from stock returns to expected trading volume, as well as from volatility to expected volume. Another interesting fact is that whenever causality from expected trading volume to returns or to volatility was detected it was always a bidirectional relation (feedback). Our findings make these relationships even stronger as they are found to manifest an explicitly nonlinear nature too. As causality from volatility to volume was generally found to be stronger than in the reverse direction, we interpret this phenomenon as an indication that the arrival of new information on the market may be described in terms of a sequential rather than a simultaneous process.

In contrast, our findings demonstrate that there is a weak causal relationship between unexpected trading volume and stock returns or volatility. By means of linear Granger causality tests we find these variables to actually have no causal relationship at all. The nonlinear test does not provide as clear conclusions as the linear one. By means of Panchenko and Diks's modification of the HJ test we found a causal relation from unexpected volume to returns or volatility in about one third of the total number of DAX companies investigated.

Our findings confirm the hypothesis that traditional linear causality tests often fail to detect some kinds of nonlinear relationships. In many cases the results obtained by use of nonlinear methods allowed the identification of a bidirectional causal relationship, whereas a linear test did not find causality in any direction. The significance level used in all tests was set at 10% and we should note that *p*-values gained from nonlinear tests (especially for detrended and expected trading volume cases) were often just a bit higher (in any case much lower than in the corresponding linear tests) than the significance level, which makes our findings even more distinct. Some simulation exercises performed for this article evaluate the performance of linear and nonlinear tests in uncovering particular nonlinear causalities. We believe that our findings are a good starting point for further research. We hope our effort can help in a better understanding of the structure of the stock market.

APPENDIX

Table 1. Companies included in the sample, symbol legend and period of quotation

Company	Symbol	Period
Adidas	ADS	01/2001 – 11/2008
Allianz	ALV	01/2001 – 11/2008
Basf	BAS	01/2001 – 11/2008
Bayer	BAY	01/2001 – 11/2008
Bayerische Motoren Werke	BMW	01/2001 – 11/2008
Commerzbank	CBK	01/2001 – 11/2008
Continental	CON	01/2001 – 11/2008
Daimler	DAI	01/2001 – 11/2008
Deutsche Bank	DBK	01/2001 – 11/2008
Deutsche Börse	DB1	02/2001 – 11/2008
Deutsche Lufthansa	LHA	01/2001 – 11/2008
Deutsche Post	DPW	01/2001 – 11/2008
Deutsche Postbank	DPB	06/2004 – 11/2008
Deutsche Telekom	DTE	01/2001 – 11/2008
E.ON	EOAN	01/2001 – 11/2008
Fresenius Medical Care	FME	01/2001 – 11/2008
Henkel	HEN3	01/2001 – 11/2008
Hypo Real Estate Holding	HRX	10/2003 – 11/2008
Infineon	IFX	01/2001 – 11/2008
Linde	LIN	01/2001 – 11/2008
MAN	MAN	01/2001 – 11/2008
Metro	MEO	01/2001 – 11/2008
Merck	MRK	01/2001 – 11/2008
Münchener Rückversicherung	MUV2	01/2001 – 11/2008
RWE	RWE	01/2001 – 11/2008
SAP	SAP	01/2001 – 11/2008
K&S	SDF	01/2001 – 11/2008
Siemens	SIE	01/2001 – 11/2008
ThyssenKrupp	TKA	01/2001 – 11/2008
Volkswagen	VOW	01/2001 – 11/2008

In tables 12–17 the numbers in brackets denote the number of cases in which a linear test accepted the null hypothesis while a nonlinear one did not.

Table 12. Number of rejected null hypotheses based on Granger's test for causality between stock returns and detrended trading volume (significance level is 10%)

Alternative hypothesis	$R_t \overset{G.C}{\rightarrow} V_{\text{detrended}}$	$V_{\text{detrended}} \overset{G.C}{\rightarrow} R_t$	$V_{\text{detrended}} \overset{G.C}{\leftrightarrow} R_t$
Linear test	21	0	0
Nonlinear test	1 (1)	9 (9)	0 (0)

Table 13. Number of rejected null hypotheses based on Granger's test for causality between volatility and detrended trading volume (significance level is 10%)

Alternative hypothesis	$R_t^2 \overset{G.C}{\rightarrow} V_{\text{detrended}}$	$V_{\text{detrended}} \overset{G.C}{\rightarrow} R^2$	$V_{\text{detrended}} \overset{G.C}{\leftrightarrow} R^2$
Linear test	19	15	9
Nonlinear test	2 (1)	5 (3)	0 (0)

Table 14. Number of rejected null hypotheses based on Granger's test for causality between stock returns and expected trading volume (significance level is 10%)

Alternative hypothesis	$R_t \overset{G.C}{\rightarrow} EV_t$	$EV_t \overset{G.C}{\rightarrow} R_t$	$EV_t \overset{G.C}{\leftrightarrow} R_t$
Linear test	28	6	6
Nonlinear test	30 (2)	10 (10)	10 (10)

Table 15. Number of rejected null hypotheses based on Granger's test for causality between volatility and expected trading volume (significance level is 10%)

Alternative hypothesis	$R^2 \overset{G.C}{\rightarrow} EV_t$	$EV_t \overset{G.C}{\rightarrow} R^2$	$EV_t \overset{G.C}{\leftrightarrow} R^2$
Linear test	30	15	15
Nonlinear test	29 (0)	14 (5)	14 (5)

Table 16. Number of rejected null hypotheses based on Granger's test for causality between stock returns and unexpected trading volume (significance level is 10%)

Alternative hypothesis	$R_t \overset{G.C}{\rightarrow} UEV_t$	$UEV_t \overset{G.C}{\rightarrow} R_t$	$UEV_t \overset{G.C}{\leftrightarrow} R_t$
Linear test	3	2	0
Nonlinear test	0 (0)	11 (9)	0 (0)

Table 17. Number of rejected null hypotheses based on Granger's test for causality between volatility and unexpected trading volume (significance level is 10%)

Alternative hypothesis	$R^2 \overset{G.C}{\rightarrow} UEV_t$	$UEV_t \overset{G.C}{\rightarrow} R^2$	$UEV_t \overset{G.C}{\leftrightarrow} R^2$
Linear test	0	6	0
Nonlinear test	0 (0)	7 (5)	0 (0)

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Liniowa i nieliniowa przyczynowość dla spółek z indeksu DAX

W badaniach empirycznych, prezentowanych w literaturze a dotyczących zależności pomiędzy wielkością obrotów, stopami zwrotu i ich zmiennością, jest o wiele mniej wyników dotyczących przyczynowości nieliniowej niż liniowej. Naszą pracę wyróżnia spośród innych prac przede wszystkim to, że w artykule są przedstawione nie tylko wyniki z zakresu przyczynowości liniowej, ale i nieliniowej. Stosując testy przyczynowości liniowej i nieliniowej dla giełdy frankfurckiej zbadano, czy znajomość wielkości obrotów może być pomocna w prognozowaniu stóp zwrotu i ich zmienności. Badanie przeprowadzono w trzech wersjach: dla wielkości obrotów z usuniętym trendem, dla oczekiwanej wielkości obrotów i nieoczekiwanej wielkości obrotów. Badania, przeprowadzone zarówno za pomocą testu przyczynowości liniowej, jak i nieliniowej, potwierdzają istnienie przyczynowości od oczekiwanej wielkości obrotów do stóp zwrotu i ich zmienności. Drugim empirycznie stwierdzonym interesującym faktem jest równoczesne występowanie statystycznie istotnej zależności w odwrotnym kierunku. Natomiast w przypadku uwzględnienia w badaniach nieoczekiwanej wielkości obrotów przyczynowości są słabe, a w większości nieistotne statystycznie. Jednakże w przypadku tej wersji wielkości obrotów test nieliniowy wykrywa więcej istotnych wypadków niż test liniowy. W pracy, w celu porównania poprawności wskazań liniowych i nieliniowych testów przyczynowości, przeprowadzono też badania symulacyjne na bazie sześciu wybranych modeli nieliniowych.

Słowa kluczowe: *spółki DAX, stopy zwrotu, wielkość obrotów, przyczynowość liniowa i nieliniowa, symulacje*