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USING A MULTICRITERIA INTERACTIVE APPROACH IN SCHEDULING NON-CRITICAL ACTIVITIES

A typical project consists of many activities. Logical dependencies cause some of them to be critical and some non-critical. While critical activities have a strict start time, in some projects the problem of selecting the start time of a non-critical activity may arise. Usually, it is possible to use the "as soon as possible" or "as late as possible" rules. Sometimes, however, the result of such a decision depends on external factors, e.g., an exchange rate. In this paper, we consider the multi-criteria problem of determining the start time of a non-critical activity. We assume that the earliest start and the latest start times of the activity have been identified using the critical path method, but the project manager is free to select the time when the activity will actually be started. This decision, however, cannot be changed later, as it is associated with the allocation of key resources. The criteria that are usually considered in such a situation are cost and risk. We assume that the cost depends on an exchange rate. We also consider the risks of project delay and a decrease in quality. This paper formulates the selection of the start time for a non-critical activity as a discrete dynamic multicriteria problem. We solve it using an interactive procedure based on the analysis of trade-offs.

Keywords: *project scheduling, trade-offs, interactive approach, CRR binomial method*

1. Introduction

One of the most important processes in project management, executed during the planning phase, is scheduling. The critical path method (CPM), proposed in the late 1950s by Walker and Kelley [7], is one of the oldest tools for scheduling but is still widely used. Knowing activities' durations and the logical dependences between them, we can calculate the earliest start and latest finish time for each activity, which constitutes the schedule.

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The CPM defines two types of activities: critical and non-critical. Activities which have a strict start and finish time are critical. An activity is "critical" in the sense that any delay in its implementation results in a delay of the whole project. The start time for a non-critical activity can be selected from a specific period. In the literature, little attention is paid to the selection of the start time of a non-critical activity. Two classical approaches are usually applied:

- as soon as possible (ASAP),
- as late as possible (ALAP).

Figure 1 presents three versions of a schedule for a project consisting of four activities: A, B, C and D. While A, B and D are critical activities, C is a non-critical one. The graph on the left represents a schedule prepared according to the ASAP rule – activity C starts at the earliest possible start time. The graph presented on the right illustrates the schedule prepared in accordance with the ALAP rule – activity C is finished at the latest finish time. Finally, the graph presented in the middle illustrates a schedule in which activity C starts somewhere between the earliest and latest start times.

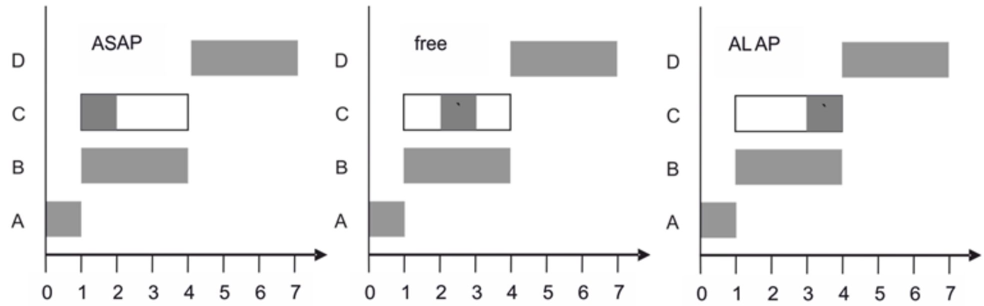

Fig. 1. Approaches to the choice of when to begin a non-critical activity (C)

ASAP is a more appropriate approach when it is important to complete a project within a stipulated time limit. Selecting this approach minimizes the risk of exceeding the deadline. ALAP, the more risky approach, may be chosen because of the availability of resources. There is also a third option: to start a non-critical activity at some time between these extremes. The aim of this paper is to propose a method for selecting this moment. In business practice, sometimes we are free to select the start time for non-critical activities anywhere between the earliest start time (ASAP) and the latest start time (ALAP). However, we need to be aware that we increase the risk of project delays by delaying the start of a non-critical activity.

In some cases, the result of an activity may depend on its completion time. An example is given by construction projects, where total costs depend on the prices of materials which vary seasonally. In such a situation, the problem of selecting the appropriate time to start a non-critical activity arises. This is an interesting research problem which raises the question of whether it is possible to determine the optimal starting time taking into account the history of changes in the factors that determine the result of the activities.

The financial literature proposes various solutions to a similar problem, called the timing problem, based on the valuation of financial options. A well-known solution is given by the Cox–Ross–Rubinstein (CRR) method [2], based on binomial trees. The next section presents the modelling of future changes in parameters by a binomial tree.

In this paper, the selection of the start time of a non-critical activity is defined as a dynamic multi-criteria decision making problem subject to risk. We consider three criteria: the expected cost of the activity, probability of delay and probability of a decline in quality. When multiple criteria are considered, it is usually impossible to identify a solution which is optimal in relation to all of the criteria. Instead, we can try to identify non-dominated solutions – ones for which it is not possible to improve the value according to one criterion without decreasing the value according to any of the others. Usually, the number of non-dominated solutions is so large that it is not easy to decide which one should be selected. Thus, solving a multicriteria problem requires information about a decision-maker's preferences. Two main approaches can be used in multiple criteria decision making [10]. The first assumes that the decision-maker articulates his/her preferences on an a priori basis. In such a case, the procedure is divided into two distinct phases: (1) acquisition of information on preferences, (2) computations. This approach is often criticized. First, the decision-maker has to consider all kinds of choices and trade-offs which might be relevant, and as this information is acquired before knowing whether the alternatives are influenced by these preferences, it may be redundant. Moreover, the decision-maker may find the choices he/she faces to be purely hypothetical, which results in a reduced level of concentration, thereby reducing the quality of the information obtained.

An interactive approach is an alternative to methods based on an a priori basis. Using such an approach, information on preferences is acquired step by step. At each iteration, the dialog and computation phases are repeated. The decision-maker is more closely involved in the process of solving the decision problem and, as a result, improves his/her knowledge about the structure of the problem.

Two main paradigms are used for gaining information on preferences: direct and indirect [6]. The former assumes that the decision-maker expresses his/her preferences in relation to the criteria themselves. Such an approach is used, e.g., by Benayoun et al. [1]. Indirect collection of information on preferences means that the decision maker has to determine which trade-offs between attributes are acceptable at each iteration, given the current candidate solution. The method proposed by Geoffrion et al. [3] is an example of such an approach. Techniques combining both approaches have also been proposed, for instance in [5]. As was shown in [8] and [9], trade-offs can also be used to solve a discrete stochastic multicriteria decision making problem.

This study is an extension of the work presented in papers [14] and [15], where a bicriteria problem was considered. Here, we propose a technique that can be used when more than two criteria are analysed. In a bi-criteria problem, the situation is clear: at a non-dominated solution improving the value of f_1 requires worsening the value of f_2 and vice versa. If both criteria require maximisation, it is quite sensible to identify a solution for which the increase in f_1 per unit decrease in f_2 is maximal.

However, when more than two criteria are analysed, the problem becomes more complicated. First, comparing trade-offs for various pairs of criteria requires evaluations to be standardized. Second, it can be possible to improve a solution according to more than one criterion at the same time. In this study, we propose a new interactive technique based on trade-offs that can be used when at least three criteria are considered. We use this technique for selecting the start time of a non-critical activity.

The paper is structured as follows. The problem is formulated in Section 2. In Section 3, we present the methodology. A numerical example illustrating the applicability of the procedure is presented in Section 4. The last section contains conclusions.

2. Formulation of the problem

Let us assume that the cost of an activity is expressed in a foreign currency (e.g., EUR). The cost in the domestic currency (e.g., PLN) depends on the exchange rate, which is constantly fluctuating. Since we assume that this activity is non-critical, the problem of selecting its start time arises. If the probability that the exchange rate will fall is greater than the probability of its increase, it is quite clear that (based purely on the criterion of expected cost) the activity should be started as late as possible. On the other hand, the later the activity is started, the higher the risk that it will not be completed on time.

In this paper we consider a multi-criteria problem of scheduling a non-critical activity. Our assumptions are as follows:

- The cost of an activity is expressed in foreign currency, and does not depend on the actual completion time.
- The minimal completion time ($t_{\min}$) and the latest finish time (LF) for the activity considered have been estimated.
- For organizational reasons, the activity can only be started at the beginning of one of the following periods: $k = 1, 2, ..., LF - t_{\min}$.
- The actual cost of the activity in domestic currency depends on the exchange rate at the end of the period in which the activity is started.
- For each period, expert estimates of the probability that the activity is finished on time, assuming that it is started at the beginning of period n, are available.
- For each period, expert estimates of the probability that a decline in quality occurs, are available.
- The problem consists of deciding when to start the activity taking into account three criteria based on: f_1 – the cost of the activity, f_2 – the probability that the activity

is delayed and f_3 – the probability that a decline in quality occurs. Our goal is to minimize the values of all three functions.

2.1. Modelling the future using of a binomial tree

We assume that the future value of our parameter (X) can be modelled using stochastic differential equations. For this purpose, we choose geometric Brownian motion (GBM) based on the equation:

$$dX(t) = \mu X(t)dt + \sigma X(t)dW(t) \tag{1}$$

where: $W(t)$ is the Wiener process, $X(t)$ is the value of the parameter X, at time t, μ is the drift parameter, σ is the volatility parameter which determines the variability of the process.

Implementation of this process is shown in Fig. 2 up to the point $t = 0$. The same figure shows simulations of three paths of the process after the point $t = 0$. This continuous process can be approximated by a discrete structure, namely a binomial tree. In Fig. 2, we present such a tree using arrows which cover future changes in the process, starting from the point $t = 0$.

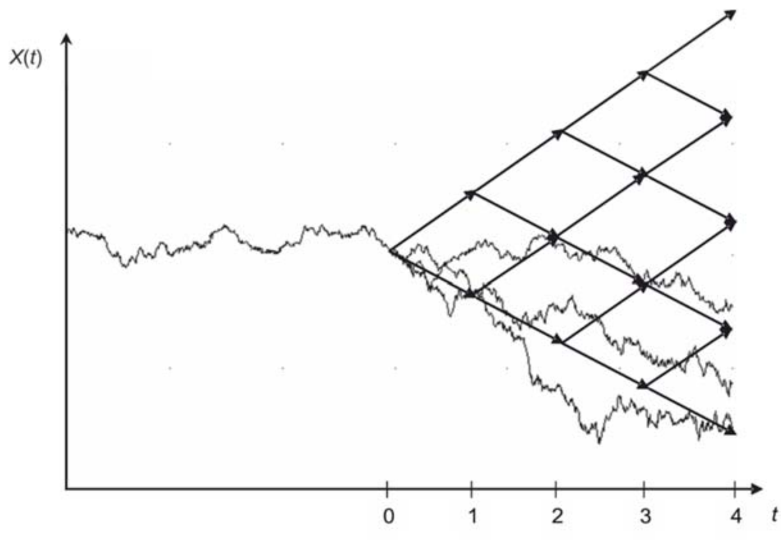

Fig. 2. Binomial tree covering the stochastic process

One problem that arises is to select an appropriate model of a stochastic process and then to calibrate a binomial tree. This issue was discussed in [13] and previously in [12]. The nodes of such a graph can be calculated from the formula:

$$x_{i,k} = x_{0,0} \mathrm{e}^{(k-2i)\hat{\sigma}\sqrt{\Delta t_p}} \tag{2}$$

where: $x_{i,k}$ is the value of the parameter x after k periods and i declines, Δt_p is the amount of time in years represented by one period in the tree, $\hat{\sigma}$ is the estimated volatility parameter for GBM. We can estimate such parameters on the basis of historical data. The estimated volatility $\hat{\sigma}$ of the process is calculated on the basis of historical data regarding their variability:

$$\hat{\sigma} = \frac{\sigma_d}{\sqrt{\Delta t_d}} \tag{3}$$

where: Δt_d is the amount of time in years between observations, σ_d is the standard deviation in historical data.

Knowing this, we can calculate the typical growth factor (u) (together with the fall factor $1/u$):

$$u = \mathrm{e}^{\hat{\sigma}\sqrt{\Delta t_p}} \tag{4}$$

The probability of an increase can be calculated using the following formula:

$$q = \frac{1}{2} + \frac{\mu\sqrt{\Delta t_m}}{2\hat{\sigma}} \tag{5}$$

We can also calculate the probability of reaching node (i, k), i = 0, 1, ..., k, after k periods [4]:

$$P\left\{ x \text{ at } (i, k) \right\} = \frac{k!}{i!(k-i)!} q^{k-i} (1-q)^i \tag{6}$$

This leads us directly to the expected value of the parameter X at stage k:

$$\mathbf{E}\left[X(k) \right] = \sum_{i=0}^{k} \frac{k!}{i!(k-i)!} q^{k-i} (1-q)^i x_{i,k} \tag{7}$$

Using formula (7), we can calculate the expected cost of the activity when it starts at a particular moment k, which gives us the objective function for the first criterion:

$$f_1(a_k) = K\left[X(k) \right] \tag{8}$$

where K denotes the fixed cost in EUR, and the parameter X is the EUR/PLN exchange rate.

2.2. Modelling the risk of a delay

The second criterion is risk of delay, measured as the probability of a delay. A non--critical activity must be finished before the latest completion time. A longer delay causes a delay in the entire project. This probability can be derived from the expected value and standard deviation of the duration, estimated using the PERT method (program evaluation and review technique) [11] but it is better to inform this calculation using expert knowledge and intuition. We ask an expert to define the probability of delay for each alternative start time.

In the example presented below, it is assumed that a non-critical activity can be started between January and October (Table 1). When the activity is started in January (alternative a_1), there is a 1% chance of a delay past the end of the year, but when it starts in October (alternative a_{10}), there is a 20% chance that not only the analysed activity, but also the entire project, will be delayed.

2.3. Modelling the risk of bad quality

The third criterion is the risk of poor quality, measured as the probability of such quality. In some situations, the value of this probability is influenced by the start time of an activity. For example, in a construction project, the risk of poor quality depends on the weather, which changes during the year. Similarly to the risk of delay, we assume that the risk of poor quality is estimated by an expert.

In the example presented below, it is assumed that the probability of poor quality is the lowest if the activity is completed in the summer months.

3. Multicriteria procedure for scheduling non-critical activities

To solve this problem, we use the interactive approach widely discussed in [9]. Let $\mathbf{A} = \{a_1, a_2, ..., a_m\}$ be the set of efficient (non-dominated) alternatives representing the period in which the activity is started and $F = \{f_1, f_2, ..., f_n\}$ be the set of objective functions for each criterion. By $f_j(a_i)$ we denote the evaluation of alternative a_i with respect to criterion f_j.

In the procedure described below, we will also use standardized evaluations of the realizations with respect to each criterion $g_j(a_i)$ which are determined from the following formula:

$$g_j(a_i) = \frac{\max\limits_{i \in \overline{1,m}} \{f_j(a_i)\} - f_j(a_i)}{\max\limits_{i \in \overline{1,m}} \{f_j(a_i)\} - \min\limits_{i \in \overline{1,m}} \{f_j(a_i)\}} \tag{9}$$

Let $A^{(l)}$ be the set of alternatives considered in iteration *l*. In each iteration, a candidate alternative $a^{(l)}$ and a potency matrix $M^{(l)}$ is presented to the decision maker (DM). The potency matrix consists of two rows: the first contains the best values according to the criteria attained within the set $A^{(l)}$, and the second one, the worst ones:

$$M^{(l)} = \begin{bmatrix} \underline{f}_1^{(l)} & \cdots & \underline{f}_n^{(l)} \\ \overline{f}_1^{(l)} & \cdots & \overline{f}_n^{(l)} \end{bmatrix} \tag{10}$$

Since in this study we assume that all the criteria involve minimisation, the following formulas are used for determining the best and worst, respectively, values according to each criterion:

$$\underline{f}_n^{(l)} = \min_{a_i \in A^{(l)}} f_j(a_i),\ j \in \overline{1,n} \tag{11}$$

$$\overline{f}_n^{(l)} = \max_{a_i \in A^{(l)}} f_j(a_i),\ j \in \overline{1,n} \tag{12}$$

Our procedure consists of the following steps:

Preliminary phase

1. Using formula (9), for each alternative a_i calculate the standardized values of the evaluations with respect to each criterion.

2. Determine the first candidate, alternative $a^{(1)}$, using the min-max criterion:

For each alternative, determine the minimum of the standardized evaluations with respect to the criteria:

$$g^{\min}(a_i) = \min_{j \in \overline{1,n}} \{g_j(a_i)\} \tag{13}$$

Set the alternative a_i that maximizes the value $g^{\min}(a_i)$ to be the first candidate $a^{(1)}$.

3. Set $l = 1$ and $A^{(1)} = A$ and start the first iteration.

Iteration *l*

1. Determine the potency matrix $M^{(l)}$.

2. Present the values of the objective functions obtained for alternative $a^{(l)}$ and the potency matrix $M^{(l)}$ to the DM. If the DM is satisfied with the proposal, end the procedure.

3. Ask the DM to assign each criterion to one of the following three sets:

F_1 – the set of criteria according to which improvement is required in comparison with alternative $a^{(l)}$.

F_2 – the set of criteria according to which there should not be any deterioration in comparison with $a^{(l)}$.

F_3 – the set of criteria according to which deterioration is acceptable in comparison with alternative $a^{(l)}$.

4. Determine the set $A^{(l+1)}$ consisting of all the alternatives from the set $A^{(l)}$ which satisfy the following conditions:

$$\underset{f_j \in F_1}{\forall} f_j(a_i) < f_j(a^{(l)}) \tag{14}$$

$$\underset{f_j \in F_2}{\forall} f_j(a_i) \le f_j(a^{(l)}) \tag{15}$$

5. If $A^{(l+1)} = \varnothing$, inform the decision maker that no alternative exists satisfying the requirements specified in step 4. Return to step 2.

6. If $A^{(l+1)}$ only consists of one alternative, take this alternative as the next proposal $a^{(l+1)}$. Proceed to step 10.

7. For each alternative $a_i \in A^{(l+1)}$ and each pair of criteria (f_j, f_k) such that $f_j \in F_1$, $f_k \in F_3$ and $f_k(a_i) > f_k(a^{(l)})$, calculate the value of the trade-off $t_{jk}(a_i)$ using the following formula:

$$t_{jk}(a_i) = \frac{g_j(a_i) - g_j(a^{(l)})}{g_k(a^{(l)}) - g_k(a_i)} \tag{16}$$

8. For each pair of criteria (f_j, f_k) such that $f_j \in F_1$, $f_k \in F_3$, check whether there exists at least one alternative $a_i \in A^{(l+1)}$, for which the value of $t_{jk}(a_i)$ was calculated in step 7. If so, then for each alternative $a_m \in A^{(l+1)}$ such that $f_k(a_m) \le f_k(a^{(l)})$, assume

$t_{jk}(a_i)$ to be twice as great as the maximal value of the trade-off calculated for the pair (f_j, f_k) in step 7. If there does not exist an alternative $a_i \in A^{(l+1)}$ such that the value of $t_{jk}(a_i)$ was calculated in step 7, assume that $t_{jk}(a_m) = 1$ for all $a_i \in A^{(l+1)}$.

9. For each $a_i \in A^{(l+1)}$, determine the average of the trade-offs calculated in steps 7 and 8. Set the alternative a_i maximizing this average to be the next proposal $a_i \in A^{(l+1)}$.

10. Set $l = l + 1$ and proceed to the next iteration.

The first candidate is determined using the min-max criterion. In each iteration, the evaluations of the objective functions for the proposed alternative and the potency matrix are presented to the DM. The DM can either accept the proposed alternative as the solution of the problem, or else determine the direction of improvement by indicating the following:

- According to which criteria are improvements required in comparison to the candidate?
- According to which criteria should there be at least no deterioration in comparison to the candidate?
- According to which criteria can there be deterioration in comparison to the candidate?

Since we are operating within the set of efficient alternatives, the decision maker must indicate at least one criterion according to which deterioration is permissible.

This procedure continues until the decision maker is satisfied with the proposed alternative (step 2). If, as a result of this analysis, the set of options considered is reduced to one, the decision-maker may accept it, or consider the alternatives proposed at an earlier step once again and decide to select one of them.

4. A numerical example

We consider a non-critical activity that should be completed by no later than December 31st. The cost of the activity is 50 million € and does not depend on the completion time. The initial PLN/€ rate is 4.1472. As there is not enough space to show how real data can be used to estimate u using formula (4), we assume that the probability of an increase q is equal 0.4. The nominal completion time is 3 months. Obviously, the sooner the activity is started, the lower the risk that the activity will be delayed. In the initial phase, a binomial tree is used to generate the probability distributions of the PLN/EUR rate according to the amount of time that passes. Next, these distributions are used to identify the distributions of the activity's cost. Table 1 presents the expected costs for various starting times and the values of the two other objective functions for each alternative.

The expected cost, calculated according to the procedure described in 2.1, decreases over time. The probability of delay grows with time, but the exact values must be declared by an expert. The probability of low quality is lowest during the summer months, as we are considering a construction project. It is quite clear that all of the alternatives are non-dominated, since the later the activity starts, the lower is the expected cost and the higher the risk of delay. Identification of the final solution proceeds according to the following scenario presented below.

Table 1. The set of alternatives

Alternative	Starting month	Expected cost (10^6 PLN)	Probability of delay	Probability of poor quality
a_1	January	206.635	0.01	0.80
a_2	February	205.913	0.02	0.50
a_3	March	205.194	0.04	0.40
a_4	April	204.476	0.10	0.20
a_5	May	203.762	0.11	0.05
a_6	June	203.050	0.15	0.05
a_7	July	202.340	0.16	0.05
a_8	August	201.633	0.18	0.15
a_9	September	200.928	0.19	0.20
a_{10}	October	200.226	0.20	0.30

Source: authors' calculations.

Preliminary phase. Using formula (9), we calculate the standardized values of the evaluations of the efficient alternatives with respect to each of the criteria $g_j(a_i)$, as presented in Table 2.

Table 2. The standardized values $g_j(a_i)$

Alternative	Starting month	$g_1(a_i)$	$g_2(a_i)$	$g_3(a_i)$	$\min\limits_{j\in 1,n}\{g_j(a_i)\}$
a_1	January	0.00	1.00	0.00	0.00
a_2	February	0.11	0.95	0.40	0.11
a_3	March	0.22	0.84	0.53	0.22
a_4	April	0.34	0.53	0.80	0.34
a_5	May	0.45	0.47	1.00	0.45
a_6	June	0.56	0.26	1.00	0.26
a_7	July	0.67	0.21	1.00	0.21
a_8	August	0.78	0.11	0.87	0.11
a_9	September	0.89	0.05	0.80	0.05
a_{10}	October	1.00	0.00	0.67	0.00

Source: authors' calculations.

2. Alternative a_5 is assumed to be the first proposal, $a^{(1)}$.

3. We set $l = 1$ and $A^{(1)} = A$.

Iteration 1

1. We calculate the potency matrix $M^{(1)}$.

2. The potency matrix (Table 3) and the candidate solution $a^{(1)} = a_5$ are presented to the decision-maker.

Table 3. Potency matrix in iteration 1

	Expected cost f_1	Probability of delay f_2	Probability of low quality f_3
Present proposal, a_5	203.762	0.11	0.05
Optimistic value	200.226	0.01	0.05
Pessimistic value	206.635	0.20	0.8

Source: authors' calculations.

The decision-maker is not satisfied with the proposal.

3. The decision-maker is willing to accept an increase in the value of f_3, wants to improve f_2 and to retain the value of f_1. So we have: $F_1 = \{f_2\}$, $F_2 = \{f_1\}$, $F_3 = \{f_3\}$.

4. We determine $A^{(l+1)}$.

5. $A^{(l+1)} = \varnothing$, so we go back to step 2.

6. The potency matrix (Table 3) and the candidate solution $a^{(1)} = a_5$ are again presented to the decision-maker. The decision-maker again is not satisfied with the proposal.

7. The decision-maker decides to improve the expected cost f_1, and retain the value of f_3. So we have $F_1 = \{f_1\}$, $F_2 = \{f_3\}$, $F_3 = \{f_2\}$.

8. $A^{(2)} = \{a_6, a_7\}$.

9. The trade-offs for the pair of criteria (f_1, f_3) are calculated for $a_i \in A^{(2)}$ (Table 4). Alternative a_7 is identified as the new candidate solution.

Table 4. Trade-offs in iteration 1

Alternative	a_6	a_7
Trade-off	0.53	0.84

Source: authors' calculations.

10. $l := 2$ and the procedure goes to the next iteration.

Iteration 2

1. The potency matrix $M^{(2)}$ is calculated.

2. The potency matrix (Table 5) and the candidate solution $a^{(2)} = a_7$ are presented to the decision-maker:

Table 5. Potency matrix in iteration 2

	Expected cost f_1	Probability of delay f_2	Probability of low quality f_3
Actual proposal a_7	202.340	0.16	0.05
Optimistic value	202.340	0.15	0.05
Pessimistic value	203.050	0.16	0.05

Source: authors' calculations.

The decision-maker is satisfied with this proposal.

As the DM is satisfied with the proposed alternative, we end the procedure. According to the DM's preferences, the best option is to start the non-critical activity in July. The expected cost is equal to 202.340 million PLN, the probability of low quality is minimised and the probability of delay is at an acceptable level (0.16).

5. Conclusion

The problem of specifying the start time of a non-critical activity has been defined as a multiple criteria dynamic decision making problem under risk. The main and original contribution of our work is a new interactive procedure that can be used for solving such problems. It uses trade-offs to identify proposals for the decision maker. Considering more than two criteria creates additional problems in the analysis of trade-offs. These problems are solved using the method presented in the paper.

We have considered a three-criteria problem, assuming that the decision maker is interested in minimizing the cost of the activity, the risk of delay and the risk of low quality. This procedure uses a binomial tree to model the stochastic process describing the change in the activity's cost. On the other hand, we assume that experts estimate the risk of delay and the risk of low quality.

The latter assumption may be considered as a weakness of the proposed approach. In future research, we plan to consider more sophisticated methods for risk evaluation, taking into account previous experience and the evaluations of multiple experts.

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