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Nasrullah KHAN[1]
Talat YASMIN[2]
Muhammad ASLAM[3]
Chi-Hyuck JUN[4]

ON THE PERFORMANCE OF MODIFIED EWMA CHARTS USING RESAMPLING SCHEMES

Two popular sampling schemes have been used to design control charts by means of a modified exponentially weighted moving average (EWMA) statistic. The structures of the proposed charts, using repetitive group sampling and multiple dependent state sampling, have been presented. The values of average run length have been determined by some specified control chart parameters. The performance of the proposed chart was illustrated via a simulation study. The efficiency of the proposed chart has been compared with the existing one. A practical example based on real data was also given to explain the application of the proposed chart in industry.

Keywords: *repetitive group sampling, multiple dependent state sampling, EWMA statistic, average run length, simulation*

1. Introduction

A process is a procedure consisting of some predefined and systematic activities to achieve a target. Compound procedures involve series of processes. One example of such a compound procedure is establishing a production line for some gadget, specialist

[1]Department of Statistics, College of Veterinary and Animal Sciences (Jhang Campus), University of Veterinary and Animal Sciences, Outfall Road Lahore 54000, Pakistan, e-mail address: nas_shan1@hotmail.com

[2]Faculty of Information Technology, University of Central Punjab, 1 Khayaban-e-Jinnah Road, Johar Town, Lahore, Pakistan, e-mail address: yas.talat@gmail.com

[3]Department of Statistics, Faculty of Sciences, King Abdulaziz University, Jeddah 21551, Saudi Arabia, e-mail address: aslam_ravian@hotmail.com

[4]Department of Industrial and Management Engineering, POSTECH, Chungamro 77, Pohang 37673, South Korea, e-mail address: chjun@postech.ac.kr

monitoring of the product's quality, designing marketing plans, and then managing sales. In spite of consistent monitoring of the whole procedure at each stage, it is not possible to completely remove natural variations from the processes involved. The use of statistical techniques to monitor and control process variation is called statistical process control (SPC). SPC makes use of information obtained from process data to control the variations in a process. Walter A. Shewhart was the first to develop graphical control-charts and used them to determine whether an industrial process is under control [33]. Since then, Shewhart control charts have been widely used as an efficient tool of SPC for monitoring and refining process performance by reducing the maximum possible variation [1]. Shewhart control charts achieve this goal by detecting the magnitude of variations in a process, categorizing the variations as acceptable or non-acceptable, classifying the causes of variations as common or special, and assisting in rectifying non-acceptable causes of such variations. The twenty first century has also witnessed the application of Shewhart charts in many other fields. Fugate et al. [2] highlighted the application of control-charts to vibration-based damage detection. Morton et al. [3] used control-charts for detecting and monitoring hospital-acquired infections. Anderson and Thompson [4] used control-charts for ecological and environmental monitoring. Woodall [5] emphasized the use of control-charts in health care and public health surveillance.

Roberts [6] introduced the concept of an exponentially weighted moving average control-chart (then called a geometric moving average control-chart). An EWMA chart is time weighted and is a powerful tool for detecting small shifts in a parameter of a process more rapidly than a Shewhart chart with an equal sample size. Each plotted point of an EWMA chart makes use of information obtained from several observations. This feature enables users of this chart to use the central limit theorem stating that the EWMA points are normally distributed and thus its control limits can be more clearly defined. Roberts [6] compared the average run lengths of his EWMA chart with the Shewhart control chart and some other simple moving average structures [7]. Together with a growing awareness of quality control in industry, the EWMA chart has become a popular tool for detecting and controlling minor inaccuracies. Muth [8] discussed the phenomenon that a time series is composed of two random components, one lasting for a single period and the other lasting for all subsequent periods. The EWMA can be thought of as the expected value of a time series. Freund [9] discussed the use of EWMA in industry. Escobedo et al. [10] used an EWMA chart for an empirical study on data obtained from the Behavioral Risk Factor Surveillance System (1984–1989) to study trends in the use of safety belts while driving. Steiner [11] added time-varying control limits based on the asymptotic limits of the EWMA chart and showed that his new chart is more reactive to minor shifts in a process than earlier asymptotic EWMA charts. In some recent developments, the idea of a modified EWMA chart was discussed by Khan et al. [12] and of a mixed EWMA-CUSUM chart by Aslam [13].

Sherman [14] introduced the repetitive sampling technique in acceptance sampling procedures. Unlike double sampling, where a decision is based on the combined information from two samples, if a decision is not made on the basis of the first sample alone, the repetitive sampling procedure is continued until a decision results from a single sample, such that no decision was taken on the basis of any earlier sample. In later years, quite a number of authors used repetitive sampling with control-charts, including Balamurali and Jun [15], Ahmad et al. [16], Aslam et al. [17], Aslam et al. [18], Azam et al. [19], and Aslam et al. [20].

Wortham and Baker [21] introduced multiple dependent/deferred sampling (MDS), based on which the decision of whether to accept, reject, conditionally accept or conditionally reject a produced lot is made. If a conditional decision is based on the nature of a previously studied lot, such a plan is called multiple dependent and if the decision is to be based on future related lots, such a plan is called multiple deferred. Some more work on MDS plans has been done by Varest [22], Soundararajan and Vijayaraghavan [23], Balamurali and Kalyanasundaram [24], and Balamurali and Jun [25]. In recent years, the concept of an MDS plan has been used by Aslam et al. based on a process capability index [26], for a new *np* control-chart [27], and for the exponential [28], COM_POISSON [29], as well as the Burr XII [30] distributions, and by Dobbah et al. [31] for the EWMA with mixed MDS sampling.

This paper uses a generalized modified EWMA statistic to design a control-chart with two types of sampling, namely repetitive group (RG) sampling and multiple dependent state (MDS) one. This statistic is based on the modified EWMA statistic first presented by Patel and Divecha [32], and first suggested by Khan et al. [12]. It was predicted that the proposed chart will show better performance than earlier proposed charts in terms of shorter average run lengths (*ARL*s) for the detection of minor shifts in a parameter of the process and thus it will prove to be a helpful tool for quality control in the future.

2. Design of the proposed modified EWMA chart

Step 1. Select n items randomly at a time or interval of time denoted by t, and measure their quality characteristics. Calculate the following EWMA statistic, denoted by M_t, based on the smoothing constant λ, and the control constant $k = -\lambda/2$ at time t:

$$M_t = (1-\lambda) M_{t-1} + \lambda \overline{Y}_t + k\left(\overline{Y}_t - \overline{Y}_{t-1}\right) \tag{1}$$

Here, $\overline{Y}_t$ is the mean quality characteristic of the sample of n observations obtained at time t.

Step 2. Declare the process to be out-of-control if $M_t \le LCL_1$ or $M_t \ge UCL_1$, where UCL_1 and LCL_1 are the upper and lower outer control limits, respectively. Declare the process to be under control if $LCL_2 \le M_t \le UCL_2$, where UCL_2 and LCL_2 are the upper and lower inner control limits, respectively. If no decision is made, go to Step 1 and repeat the process.

The mean and variance of this new EWMA statistic are given as follows:

$$E\left(M_t\right) = \mu,\ \ V(M_t) = \frac{\sigma^2(\lambda + 2\lambda k + 2k^2)}{n\,(2-\lambda)} \tag{2}$$

2.1. Design of the proposed chart using RG sampling

For a repetitive chart, with L_1 and L_2 as the control-chart coefficients (where $L_1 > L_2 > 0$), the two outer control limits are

$$LCL_1 = \mu - L_1\sigma\sqrt{\frac{\lambda + 2\lambda k + 2k^2}{n(2-\lambda)}}, \quad UCL_1 = \mu + L_1\sigma\sqrt{\frac{\lambda + 2\lambda k + 2k^2}{n(2-\lambda)}} \tag{3}$$

and the two inner control limits are

$$LCL_2 = \mu - L_2\sigma\sqrt{\frac{\lambda + 2\lambda k + 2k^2}{n\left(2-\lambda\right)}}, \quad UCL_2 = \mu + L_2\sigma\sqrt{\frac{\lambda + 2\lambda k + 2k^2}{n(2-\lambda)}} \tag{4}$$

For a single sample, the probability of accepting at time t that a process which is under control is out-of-control is given by

$$P_{\text{out, 0}} = P\left(M_t \ge UCL_1\right) + P\left(M_t \le LCL_1\right) \tag{5}$$

Thus the ARL when the process is under control is given by ARL_0, where

$$ARL_0 = \frac{1}{P_{\text{out, 0}}} \tag{6}$$

If the mean of this quality measure shifts to a new mean $\mu_1 = \mu + c\mu$, where c is the shift constant resulting from some unknown factors, the probability of inferring that the process is out-of-control is given by

$$P_{\text{out},1} = P\left(M_t > UCL_1 \mid \mu_1\right) + P\left(M_t < LCL_1 \mid \mu_1\right) \tag{7}$$

Based on RG sampling, the probability of repeating the sampling procedure at time t when a process is under control is

$$P_{\text{rep},0} = P\left(LCL_1 \le M_t \le LCL_2\right) + P\left(UCL_2 \le M_t \le UCL_1\right)$$

For the shifted process, this probability becomes

$$\begin{aligned} P_{\text{rep},1} &= P\left(LCL_1 \le M_t \le LCL_2 \mid \mu_1\right) + P\left(UCL_2 \le M_t \le UCL_1 \mid \mu_1\right) \\ &= P\left(M_t < LCL_2 \mid \mu_1\right) - P\left(M_t < LCL_1 \mid \mu_1\right) + P\left(M_t < UCL_1 \mid \mu_1\right) \\ &\quad - P\left(M_t < UCL_2 \mid \mu_1\right) \end{aligned} \tag{8}$$

Thus, at any given moment of time, the probability of concluding that an out of control process is out of control is given by

$$P_{\text{out}\,1,1} = \frac{P_{\text{out},1}}{1 - P_{\text{rep},1}} \tag{9}$$

The *ARL* for the shifted process, denoted by ARL_1, is given as follows

$$ARL_1 = \frac{1}{P_{\text{out}1,1}} \tag{10}$$

2.2. Design of the proposed chart with MDS sampling

When the current measure lies between the inner limits, then declare the process to be under control. When the current measure lies either between the outer and inner lower limits or between the inner and outer upper limits, declare the process to be under control if the m preceding measures were between the inner limits. Otherwise, declare the process to be out-of-control.

The probability of inferring a process which is under control to be under control, based on the MDS sampling scheme, is

$$P_{\text{in},0} = P\left(LCL_2 \le M_t \le UCL_2\right) + \left\{P\left(LCL_1 \le M_t \le LCL_2\right) + P\left(UCL_2 \le M_t \le UCL_1\right)\right\}\left[P\left(LCL_2 \le M_t \le UCL_2\right)\right]^m \tag{11}$$

Here, m is the number of preceding samples that may be considered.

For the shifted process, the probability of declaring the process to be in-control, $P_{\text{in},1}$, is given by

$$P_{\text{in},1} = P\left(LCL_2 \le M_t \le UCL_2 \mid \mu_1\right) + \left\{P\left(LCL_1 \le M_t \le LCL_2 \mid \mu_1\right) + P\left(UCL_2 \le M_t \le UCL_1 \mid \mu_1\right)\right\}\left[P\left(LCL_2 \le M_t \le UCL_2 \mid \mu_1\right)\right]^m \tag{12}$$

The expression for the *ARL* when the process is in-control is given by

$$ARL_0 = \frac{1}{1 - P_{\text{in},0}} \tag{13}$$

Denote the predefined ARL_0 to be r_0. Let $k = -\lambda/2$ be the point where the proposed chart exhibits the optimal results (based on the minimisation of ARL_1). The values of L_1, L_2, and *ARL* are determined using the following Monte Carlo simulation:

1. Generate 10 000 random samples from the normal distribution with mean 0 and variance 1. Compute the statistic M_t and the control limits.
2. Record the time of the first out-of-control call as the run length and repeat the process 10 000 times to obtain 10 000 run lengths.
3. Compute the average run length and choices of L_1, L_2 such that $ARL_0 \ge r_0$.
4. Generate 10 000 random samples from the normal distribution with a shift in the mean. Compute the statistic M_t and the control limits.
5. Record the time of the first out-of-control call as the run length and repeat the process 10 000 times to obtain 10 000 run lengths.
6. Compute the average run length for the shifted process.

The following trends are observed in our proposed charts.

- The *ARL* is equal to the value of r_0 when $c = 0$.
- For a fixed value of c, the *ARL* increases monotonically in λ.
- For a fixed value of λ, the *ARL* decreases monotonically in c.
- For fixed values of both c and λ, the *ARL* increases monotonically in r_0.
- The proposed chart reduces to the chart by Khan et al. [12] when $k_1 = k_2 = k$.

3. Advantages of the proposed charts

3.1. Comparison with Khan et al. [12]

The obtained values of the *ARL*s for existing charts and our proposed chart, together with the differences between the corresponding *ARL*s, for given values of r_0, the constant c, and λ, are given in the following tables. Tables 1–4 correspond to the RG sampling scheme and Tables 5–7 correspond to the MDS sampling scheme.

Table 1. *ARL*s for the proposed chart using RG sampling; $\lambda = 0.10$

c	$r_0 = 300$		$r_0 = 370$	
	L_1	L_2	L_1	L_2
	2.539751	2.071434	2.61895	2.11554
	ARL_1	ARL_{1sd}	ARL_1	ARL_{1sd}
0	306.31	301.64	371.75	365.01
0.05	192.71	188.26	227.88	216.49
0.10	90.96	83.05	105.02	94.68
0.15	49.52	41.56	53.83	45.98
0.20	30.80	23.28	32.96	25.24
0.25	21.37	14.34	22.66	15.48
0.30	16.08	9.69	17.19	10.69
0.40	10.46	5.34	10.88	5.62
0.50	7.76	3.50	8.02	3.55
0.60	6.08	2.40	6.33	2.53
0.70	5.01	1.82	5.24	1.86
0.80	4.34	1.46	4.50	1.51
0.90	3.81	1.20	3.95	1.23
1.00	3.41	1.00	3.53	1.03
2.00	2.03	0.17	2.04	0.20

Table 2. *ARL*s for the proposed chart using RG sampling; $\lambda = 0.20$

c	$r_0 = 300$		$r_0 = 370$	
	L_1	L_2	L_1	L_2
	2.708651	2.132171	2.77381	2.03164
	ARL_1	ARL_{1sd}	ARL_1	ARL_{1sd}
0	309.50	308.57	372.34	363.49
0.05	223.73	222.41	258.94	251.62
0.10	118.68	115.55	136.69	130.15
0.15	64.99	60.33	73.18	67.60
0.20	38.91	34.60	42.58	37.77
0.25	25.59	21.31	27.67	23.12
0.30	18.06	13.85	19.26	14.95
0.40	10.58	6.91	11.02	7.44

Table 2. *ARL*s for the proposed chart using RG sampling; $\lambda = 0.20$

0.50	7.33	4.22	7.56	4.35
0.60	5.48	2.78	5.65	2.91
0.70	4.35	1.97	4.56	2.05
0.80	3.70	1.51	3.80	1.57
0.90	3.21	1.19	3.32	1.25
1.00	2.85	0.94	2.92	0.97
2.00	2.00	0.06	2.00	0.06

Table 3. *ARL*s for the proposed chart using RG sampling; $\lambda = 0.50$

c	$r_0 = 300$		$r_0 = 370$	
	L_1	L_2	L_1	L_2
	2.85464	2.41788	2.92071	2.436848
	ARL_1	ARL_{1sd}	ARL_1	ARL_{1sd}
0	305.89	302.76	370.13	361.29
0.05	255.09	249.40	305.68	304.17
0.10	166.02	165.78	198.04	195.82
0.15	103.36	103.09	119.72	118.45
0.20	64.45	61.60	74.36	72.56
0.25	41.76	40.46	47.24	46.05
0.30	28.56	27.06	31.83	30.20
0.40	14.44	12.81	15.99	14.44
0.50	8.65	6.97	9.41	7.77
0.60	5.84	4.34	6.18	4.64
0.70	4.30	2.75	4.42	2.89
0.80	3.41	1.91	3.54	2.06
0.90	2.91	1.36	2.94	1.41
1.00	2.55	0.99	2.58	1.01
2.00	2.00	0.02	2.00	0.02

Table 4. *ARL*s for the proposed chart using RG sampling; (EWMA); $\lambda = 1.0$, $k = 0$

c	$r_0 = 300$		$r_0 = 370$	
	L_1	L_2	L_1	L_2
	2.931244	1.926294	3.007656	2.268241
	ARL_1	ARL_{1sd}	ARL_1	ARL_{1sd}
0	303.61	301.18	373.86	371.82
0.05	289.05	288.50	361.54	352.54
0.10	247.03	247.40	297.22	295.69
0.15	192.43	191.82	237.78	242.45
0.20	147.35	146.06	183.51	177.95
0.25	109.39	109.19	137.10	135.51
0.30	83.26	83.00	102.67	101.75
0.40	49.75	48.17	58.53	55.78

Table 4. *ARL*s for the proposed chart using RG sampling; (EWMA); $\lambda = 1.0$, $k = 0$

0.50	29.53	27.93	34.27	33.25
0.60	18.98	17.32	21.77	20.02
0.70	12.74	11.09	14.18	12.72
0.80	8.90	7.24	9.98	8.47
0.90	6.69	5.25	7.34	5.74
1.00	5.14	3.55	5.46	3.85
2.00	2.06	0.26	2.07	0.28

Table 5. *ARL*s for the proposed chart using MDS sampling; $\lambda = 0.10$, $k = -\lambda/2$

c	$m = 2$				$m = 3$			
	$r_0 = 300$		$r_0 = 370$		$r_0 = 300$		$r_0 = 370$	
	L_1	L_2	L_1	L_2	L_1	L_2	L_1	L_2
	2.555761	1.857878	2.623627	2.1202	2.55773	2.152859	2.631644	1.781694
	ARL_1	ARL_{1sd}	ARL_1	ARL_{1sd}	ARL_1	ARL_{1sd}	ARL_1	ARL_{1sd}
0	313.08	313.68	370.67	359.54	322.47	316.54	374.64	371.20
0.05	200.63	189.56	231.27	219.47	193.11	185.15	225.71	216.92
0.10	93.22	85.31	102.50	94.07	94.10	82.79	105.78	95.52
0.15	50.34	42.28	54.96	47.49	50.93	40.96	55.65	45.37
0.20	30.56	23.28	32.53	24.78	31.10	23.83	33.34	25.60
0.25	21.63	14.62	22.85	15.64	21.65	14.47	23.10	15.74
0.30	16.37	9.97	17.22	10.62	16.32	9.91	17.13	10.42
0.40	10.60	5.48	11.00	5.72	10.55	5.38	11.01	5.63
0.50	7.69	3.42	7.94	3.52	7.77	3.52	8.08	3.62
0.60	6.19	2.50	6.38	2.56	6.15	2.46	6.37	2.52
0.70	5.13	1.90	5.29	1.94	5.10	1.83	5.26	1.87
0.80	4.38	1.46	4.51	1.49	4.36	1.43	4.50	1.47
0.90	3.82	1.19	3.93	1.22	3.83	1.21	3.94	1.23
1.00	3.43	1.01	3.53	1.03	3.44	1.02	3.54	1.04
2.00	2.03	0.17	2.04	0.19	2.03	0.17	2.04	0.19

Table 6. *ARL*s for the proposed chart using MDS sampling; $\lambda = 0.20$, $k = -\lambda/2$

c	$m = 2$				$m = 3$			
	$r_0 = 300$		$r_0 = 370$		$r_0 = 300$		$r_0 = 370$	
	L_1	L_2	L_1	L_2	L_1	L_2	L_1	L_2
	2.712104	2.130452	2.785545	2.062592	2.704668	2.023787	2.793424	2.151082
	ARL_1	ARL_{1sd}	ARL_1	ARL_{1sd}	ARL_1	ARL_{1sd}	ARL_1	ARL_{1sd}
0	316.28	315.22	380.22	371.75	310.25	309.99	382.77	369.72
0.05	222.94	222.00	269.65	268.31	218.76	218.11	270.78	268.80
0.10	121.48	116.22	140.52	134.67	119.88	114.94	142.14	138.29
0.15	65.43	60.32	74.03	67.91	64.57	59.44	73.81	67.84
0.20	38.23	33.86	42.46	37.74	37.92	33.71	43.35	38.17
0.25	25.84	21.75	27.98	23.61	25.63	21.56	28.20	23.40

Table 6. *ARL*s for the proposed chart using MDS sampling; $\lambda = 0.20$, $k = -\lambda/2$

0.30	18.05	13.71	19.47	15.00	17.91	13.59	19.68	15.05
0.40	10.64	7.14	11.27	7.55	10.58	7.09	11.21	7.34
0.50	7.31	4.20	7.62	4.40	7.28	4.19	7.66	4.45
0.60	5.48	2.78	5.70	2.90	5.47	2.77	5.71	2.88
0.70	4.41	1.99	4.57	2.07	4.40	1.99	4.55	2.09
0.80	3.70	1.48	3.81	1.51	3.69	1.47	3.83	1.56
0.90	3.21	1.18	3.30	1.22	3.20	1.17	3.32	1.24
1.00	2.88	0.96	2.95	0.99	2.87	0.96	2.93	0.98
2.00	2.00	0.06	2.01	0.07	2.00	0.06	2.00	0.07

Table 7. *ARL*s for the proposed chart using MDS sampling; $\lambda = 1.0$, $k = 0$

c	$m = 2$				$m = 3$			
	$r_0 = 300$		$r_0 = 370$		$r_0 = 300$		$r_0 = 370$	
	L_1	L_2	L_1	L_2	L_1	L_2	L_1	L_2
	2.933451	2.145911	3.000289	2.176561	2.937483	2.190685	2.993971	2.060003
	ARL_1	ARL_{1sd}	ARL_1	ARL_{1sd}	ARL_1	ARL_{1sd}	ARL_1	ARL_{1sd}
0	300.15	299.66	366.74	362.64	304.08	303.94	359.77	355.64
0.05	287.12	278.41	350.40	339.45	290.83	281.66	345.38	336.06
0.10	241.07	242.73	294.51	293.95	243.21	243.78	289.96	289.88
0.15	190.89	189.33	232.44	235.46	193.72	193.15	227.10	228.52
0.20	145.95	140.58	175.57	168.27	147.13	142.08	172.55	165.78
0.25	112.50	109.21	134.49	131.43	113.41	109.81	132.14	130.22
0.30	84.81	81.63	101.15	99.02	85.81	82.80	99.50	97.67
0.40	49.95	48.71	58.28	57.59	50.27	49.14	57.34	56.64
0.50	29.36	27.83	34.33	33.26	29.65	28.01	33.74	32.48
0.60	19.25	17.57	21.70	19.82	19.40	17.71	21.41	19.53
0.70	12.81	11.36	14.36	12.85	12.90	11.42	14.23	12.73
0.80	8.96	7.40	9.90	8.24	9.01	7.45	9.76	8.11
0.90	6.55	4.99	7.13	5.62	6.58	5.02	7.08	5.57
1.00	5.12	3.53	5.50	3.91	5.14	3.54	5.46	3.87
2.00	2.07	0.27	2.08	0.29	2.07	0.27	2.08	0.29

The trends observed in the *ARL*s for our proposed charts, as mentioned in the previous section, are also evident in existing charts.

Observing the differences between the values of the *ARL*s for existing and proposed charts (*ARL* existing – *ARL* proposed), the following observations are clear.

1. The differences are all positive, which clearly indicates the higher degree of efficiency of the proposed charts compared to existing ones.

2. The differences follow the same pattern for all the cases of the RG sampling scheme, as well as the MDS sampling scheme.

3. For a fixed value of λ and varying the value of c in the interval [0.00, 1.00], the difference between the *ARL*s initially increases and attains a maximum value at certain value of c and then starts decreasing.

4. The maximum difference between *ARL*s, 20.06, is obtained using MDS sampling with $m = 3$, $r_0 = 300$, at $\lambda = 1.0$, and $c = 0.2$ (Table 6).

5. For a fixed value of c and increasing the value of λ from 0.1 to 1.0 by discrete amounts, with other parameters fixed, the difference between *ARL*s keeps on increasing.

4. Simulation study

Using computer simulation, an under-control process is generated with zero mean and unit variance based on repetitive group sampling and 20 observations.

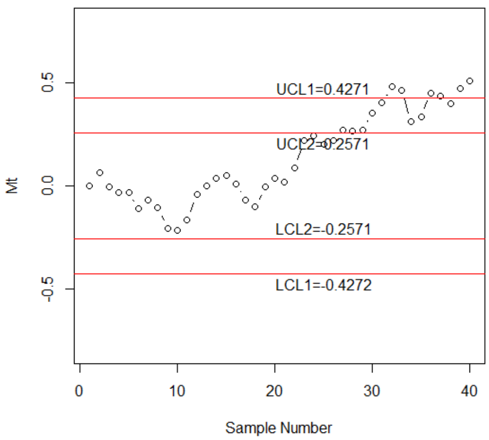

Fig. 1. Simulated control chart
for the proposed procedure using RG sampling

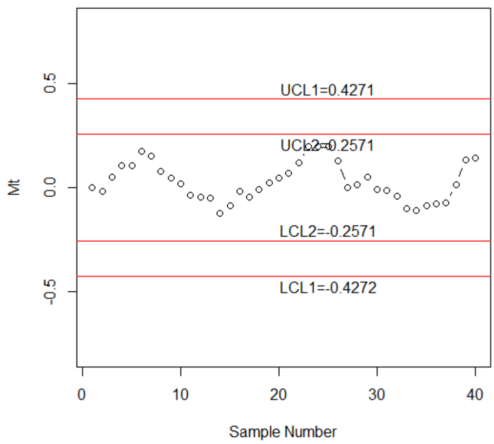

Fig. 2. Simulated control chart
for the existing process using RG sampling

Next, 20 observations are generated, again using repetitive group sampling, but for a shifted process with mean $\mu_1 = \mu + c\mu$ using $c = 0.2$, and $\lambda = 0.20$. These parameters are based on Table 3 for the value of $ARL = 13.13$, as a shift is detected on average more quickly (Table 8). A graph of the control chart for these 40 observations is constructed in Fig. 1 showing when the process is inferred to be out-of-control, together with the observations obtained when the mean is shifted. The statistic M_t is also plotted on an existing control chart using RG sampling in Fig. 2, which infers that there is no shift in the process.

Repeating the simulation procedure using MDS sampling with $m = 2$, the first 20 observations are selected from an under-control process and the next 20 from a shifted process with the same parameters as those used for RG sampling (Table 3) and the graph of the control chart for these 40 simulated observations with MDS sampling is constructed in Fig. 3. The graph shows that the process is inferred to be out-of-control after the process mean has shifted. The statistic M_t is also plotted on an existing control chart using MDS sampling in Fig. 4, which shows that no shift in the process is inferred.

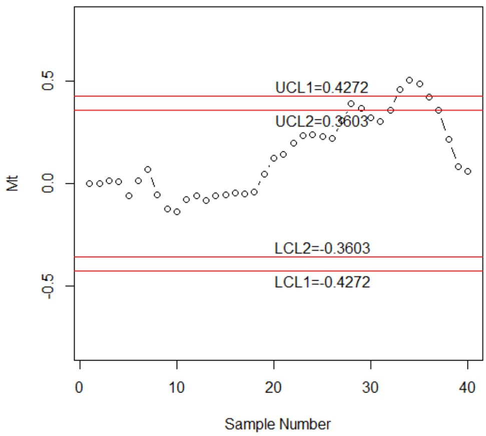

Fig. 3. Simulated control chart
for the proposed procedure using MDS sampling

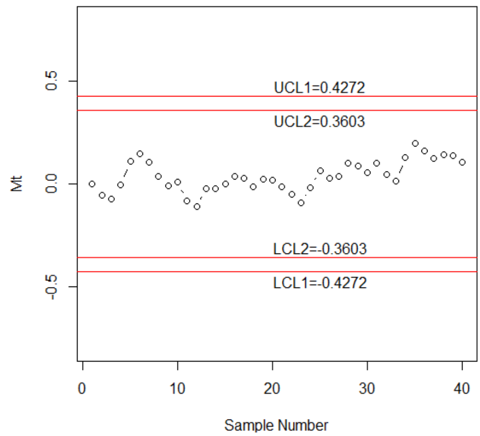

Fig. 4. Simulated control chart
for the existing procedure using MDS sampling

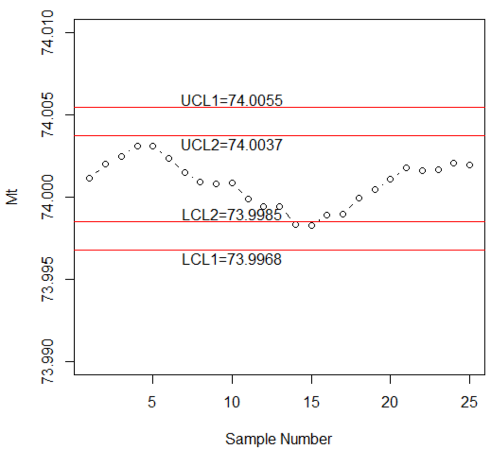

Fig. 5. Control chart for real data
using RG sampling

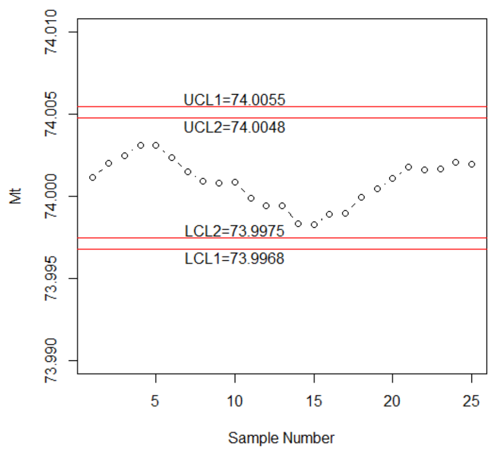

Fig. 6. Control chart for real data
using MDS sampling

To demonstrate the application of these charts in practice, the data from Montgomery [1] are used. These data refer to a sample of 25 measurements of the inside diameters of piston rings (in mm) made by a forging process for automotive engines. The measurements are made using a dial gauge. With a subgroup size of 5, the values of the modified EWMA statistic M_t based on Eq. (1) are presented in Table 8. The existing control chart uses LCL = 73.9968 and UCL = 74.0055. The four control limits used in conjunction with our RG sampling are LCL_1 = 73.9968, LCL_2 = 73.9985, UCL_2 = 74.0037, UCL_1 = 74.0055 and those used for MDS sampling are LCL_1 = 73.9968, LCL_2 = 73.9975, UCL_2 = 74.0047, UCL_1 = 74.0055. The proposed control chart based on RG sampling is given in Fig. 5 and the chart based on MDS sampling in Fig. 6.

Table 8. Realisations of the modified EWMA statistic

Sr#	Mean	M_t	Sr#	Mean	M_t	Sr#	Mean	M_t
1	74.010	74.00112	10	73.998	74.00085	19	73.998	74.00046
2	74.001	74.002	11	73.994	73.99988	20	74.009	74.00107
3	74.008	74.0025	12	74.001	73.9994	21	74	74.00175
4	74.003	74.0031	13	73.998	73.99942	22	74.002	74.0016
5	74.003	74.00308	14	73.99	73.99834	23	74.002	74.00168
6	73.996	74.00236	15	74.006	73.99827	24	74.005	74.00205
7	74.000	74.00149	16	73.997	73.99892	25	73.998	74.00194
8	73.997	74.00089	17	74.001	73.99893			
9	74.004	74.00081	18	74.007	73.99995			

The design of our proposed charts defines a process to be under-control if $LCL_2 \le M_t \le UCL_2$. Using RG sampling, this interval has a span of 0.0052, whereas using MDS sampling it is 0.0073. The span of the interval based on the existing chart is 0.0087. These results clearly indicate that both of our charts are more efficient than the existing chart in detecting a minor variation in a process. Our chart based on RG sampling shows two values of the statistic which marginally indicate that the process is out-of-control for subgroups 14 and 15. As these values fall within the control limits of the existing chart, we can slightly relax LCL_2 to include these points, just for comparison. Even then, the span of our control limits will be narrower than for the existing chart and our chart will still enjoy the merit of better efficiency in detecting minor variations.

5. Concluding remarks

A modified EWMA statistic, previously suggested by Khan et al. [12], has been revisited using two sampling schemes, repetitive group sampling and multiple dependent sampling. *ARL* tables have been constructed for multiple settings of the values of the parameters for the purpose of comparison with Khan et al. [12]. For all the studied settings, it was found that the *ARL*s obtained for the proposed charts are shorter than Khan's and the difference obtained can be as great as 20.06 for a specific set of parameters using MDS sampling. This fact establishes the efficiency of the proposed control-charts compared with Khan's chart using either sampling scheme. The simulation study also suggested that the proposed chart is more sensitive in detecting minor shifts in the values of parameters. Real data were used to show the applicability of the proposed chart in industry. The results obtained are encouraging enough to state that our proposed chart is a powerful addition to the set of SPC tools and is a good candidate for further research to make it even more efficient.

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