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EFFICIENCY MEASUREMENT IN DYNAMIC TWO-STAGE NETWORK STRUCTURES WITH FLEXIBLE INTERMEDIATE MATERIALS

Data envelopment analysis (DEA) is a nonparametric method for evaluating the relative efficiency of decision making units (DMUs) described by multiple inputs and multiple outputs. Since DEA was introduced in the 1970s, it has been widely applied to measure the efficiency of a wide variety of production and operation systems, including two-stage production systems with a series or parallel structure. The outputs from the first stage to the next stage are called intermediate factors (or measures). In some real applications, an intermediate material or some part of it can become the final output or input to the second stage of production. Previously existing models cannot be employed directly to measure the efficiency of such systems. The authors introduce a dynamic DEA model that identifies the structure of flexible intermediate factors to maximise the measure of overall system efficiency.

Keywords: *data envelopment analysis, flexible factors, dynamic two-stage network, efficiency*

1. Introduction

Following a seminal work by Farrell [11] on the measurement of production efficiency and work by Charnes, Cooper, and Rhodes [3], data envelopment analysis (DEA)

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became a popular empirical method for measuring the relative efficiency of a set of homogeneous decision making units (DMUs). In many cases, DMUs may consist of two-stage structures with intermediate factors (also called measures). Seiford and Zhu [15] extended this analysis to two-stage processes by applying standard DEA methodology separately to each stage, without considering the interaction between the two stages. Zhu [17] and Chen and Zhu [7] indicate that the overall efficiency of a DMU requires all the individual stages to be efficient. Kao and Hwang [12] investigate the decomposition of efficiency in a two-stage production process where the outputs of the first stage are the inputs of the second stage. They take the alignment of the two subprocesses in series into account when measuring the efficiencies. Chen et al. [4] presented a model similar to Kao and Hwang's model, but with measurement having an additive form. Chen et al. [4, 6] examined relations and equivalencies between the existing DEA approaches to measuring the performance of two-stage processes. Chen et al. [5] proposed a DEA-based approach to evaluate the efficiency of two stage network systems and to determine the frontier points. Ebrahimnejad et al. [10] introduced a three-stage DEA model with two independent parallel stages linking to a third final stage.

In the studies mentioned above, it was assumed that the outputs of the first stage are purely inputs into the second stage. For instance, consider a supply chain consisting of two stages, supplier and manufacturer. The number of products shipped from the supplier to the manufacturer can be flexible in the sense that some proportions of intermediate products are shipped as final outputs (e.g., these products are placed on sale). The remainder of these intermediate factors are processed further by the manufacturer. Another instance of flexible intermediate materials is found in bank branches in Iran. An important function of Iranian commercial banks is to attract deposits (stage 1) and then to distribute these deposits (stage 2). A portion of the total attracted deposits should be distributed among customers and the remainder should be transferred to the Central Bank of the Republic of Iran. As far as we are aware, there is no DEA-based study considering this issue in the case where there is no information about how the intermediate materials are utilized.

Flexible materials in DEA were initially introduced by Cook and Bala [8] and Cook and Zhu [9]. They applied this concept to materials with an unknown status from the point of view of being an input or output. Cook and Zhu [9] proposed an adaptation of the DEA method for classifying these materials by introducing a fractional programming problem. Toloo [16] claimed that Cook and Zhu's model [9] may produce incorrect efficiency scores, due to a computational problem resulting from the introduction of a large positive number into the model. Amirteimoori and Emrouznejad [1] developed a DEA model to calculate the technical efficiency of DMUs with flexible materials. Amirteimoori et al. [2] proposed a flexible slacks-based measure (FSBM) of efficiency in which each flexible material can play the role of input for some DMUs and the role of output for others, in order to maximise the relative efficiency of the DMU under evaluation. Kordrostami and Jahani Sayyad Noveiri [13] introduced an approach to evaluate the efficiency of DMUs in the presence of flexible and negative data. MA [14] proposed a two-stage DEA model which simultaneously considers the structure of inputs and intermediate materials in evaluating and decomposing efficiency. Taking previous studies into account, this paper proposes a modification of the standard two-stage DEA model to incorporate flexible intermediate materials.

Therefore, the current paper firstly proposes a technology of dynamic two-stage systems taking into account flexible intermediate materials. Then, a non-radial DEA model is proposed to estimate the overall and period efficiency scores of dynamic two stage production systems when flexible materials are present. The proposed approach is applied to evaluate the performance of some banks in Iran.

The paper is organized as follows: Section 2 describes the problem. Next, we introduce our approach to modelling a two-stage production process in Section 3. Section 4 applies the method to Iranian banks. The conclusions appear in Section 5.

2. Dynamic two-stage production

Suppose that there are n DMUs and production takes place in periods $t=1,...,T$. Also each DMU consists of two divisions and intermediate products created in stage 1 are partly used as inputs to stage 2 (Fig. 1). In period t, DMU_j, $j=1,...,n$ uses the exogenous input x_{ij}^t, $i=1,...,m$ and the carry over activities $w_h^{t-1,t}$, $h=1,...,H$ which are generated in period $t-1$. The link activity z_{kj}^t, $k=1,...,K$ acts as the vector of intermediate outputs. These K outputs are flexible, in the sense that they can become final outputs or inputs into the second stage. We suppose that all the z_{kj}^t, $k=1,...,K$ are divided into $d_k^t z_{kj}^t$ and $(1-d_k^t)z_{kj}^t$, $k=1,...,K$ where $d_k^t z_{kj}^t$, $k=1,...,K$ is an input into stage 2 and $(1-d_k^t)z_{kj}^t$, $k=1,...,K$ is a final output. The observed split of the output z_j is given by $z_j^{(1)}$ and $z_j^{(2)}$. $z_j^{(1)}$ is used as input into the second stage and $z_j^{(2)}$ is final output from stage 1. Obviously, $z_j^{(1)}+z_j^{(2)}=z$. The outputs from the second stage are y_{rj}^t, $r=1,...,s$ and the carryover products $w_h^{t,t+1}$, $h=1,...,H$ that are used in period $t+1$.

Let T_1 be the production possibility set of the technology under consideration for the first stage. We postulate the following:

P1. Feasibility of observed data. $(x_j^t, z_j^t)\in T_1$ for any $j=1,...,n$.

P2. Unbounded ray. $(x^t, z^t)\in T_1$ implies $\lambda(x^t, z^t)\in T_1$ for any $\lambda\geq 0$.

P3. Convexity. Let $(x''^t, z''^t) \in T_1$ and $(x'''^t, z'''^t) \in T_1$. Then for any $\lambda \in [0,1]$, $\lambda(x''^t, z''^t) + (1-\lambda)(x'''^t, z'''^t) \in T_1$.

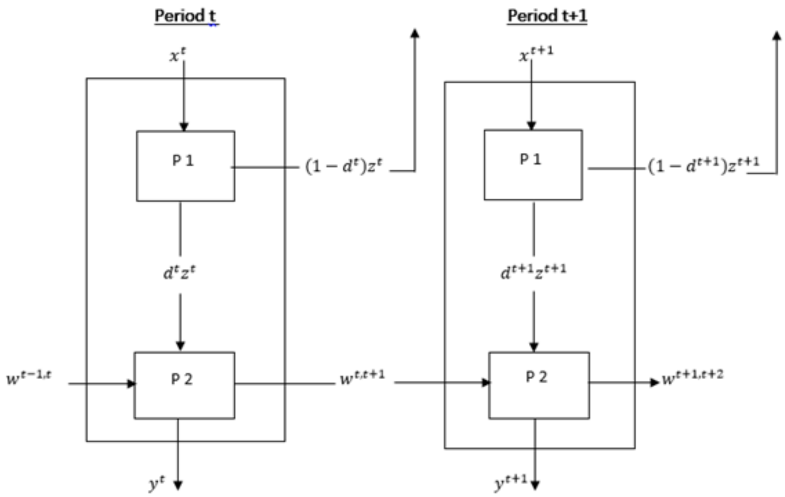

Fig. 1. Dynamic two-stage production with flexible intermediate materials

P4. Free disposability. $(x^t, z^t) \in T_1, x''^t \geq x^t, \ z''^t \leq d^t z^t$ and $z'''^t \leq (1-d^t)z^t$ implies $(x''^t, z''^t + z'''^t) \in T_1$.

P5. Minimal extrapolation. For each T' satisfying the axioms P1–P4, we have $T_1 \subseteq T'$.

Now, an algebraic representation of the production possibility set (PPS) of the technology T_1, which satisfies the axioms P1–P5, is given.

Theorem 1. The PPS T_1, which satisfies the axioms P1–P5, is defined as

$$T_1 = \{(x^t, z^t) \mid x^t \geq \sum_{j=1}^{n} \lambda_j^t x_j^t, d^t z^t \leq \sum_{j=1}^{n} \lambda_j^t z_j^{1t}, (1-d^t)z^t \leq \sum_{j=1}^{n} \lambda_j^t z_j^{2t},$$
$$0 \leq d^t \leq 1, \ \lambda_j^t \geq 0, \ j = 1, \dots, n\} \tag{1}$$

Proof. The proof is clear. □

Also, let T_2 be the production possibility set of the technology under consideration for the second stage. Again, to determine the technology of stage 2, we postulate the following:

P′1. Feasibility of observed data $(w_j^{t-1,t},\ d^t z_j^t, y_j^t, w_j^{t,t+1}) \in T_2$ and $(w_j^{t-1,t},\ z_j^{1t},\ y_j^t, w_j^{t,t+1}) \in T_2$ for any $j = 1, ..., n.$.

P′2. Unbounded ray. $(w^{t-1,t},\ d^t z^t,\ y^t, w^{t,t+1}) \in T_2$ implies $\lambda^t (w^{t-1,t},\ d^t z^t,\ y^t,\ w^{t,t+1}) \in T_2$ for any $\lambda^t \geq 0$.

P′3. Convexity. Let $(w'^{t-1,t}, d^t z'^t,\ y'^t,\ w'^{t,t+1}) \in T_2$ and $(w''^{t-1,t},\ d^t z''^t,\ y''^t,\ w''^{t,t+1}) \in T_2$. Then for any $\lambda \in [0,1]$, $\lambda(w'^{t-1,t},\ d^t z'^t,\ y'^t,\ w'^{t,t+1}) + (1-\lambda)(w''^{t-1,t},\ d^t z''^t,\ y''^t,\ w''^{t,t+1}) \in T_2$.

P′4. Free disposability. $(w^{t-1,t},\ d^t z^t,\ y^t,\ w^{t,t+1}) \in T_2$, $w'^{t-1,t} \geq w^{t-1,t}$, $z'^t \geq d^t z^t$, $y'^t \leq y^t$ and $w'^{t,t+1} \leq w^{t,t+1}$ implies $(w'^{t-1,t},\ d^t z'^t,\ y'^t,\ w'^{t,t+1}) \in T_2$.

P′5. Minimal extrapolation. For each T' satisfying the axioms P′1– P′4, we have $T_2 \subseteq T'$.

Now, an algebraic representation of the production possibility set (PPS) of the technology T_2, which satisfies the axioms P1-P5, is given.

Theorem 2. The PPS T_2, which satisfies the axioms P′1–P′5, is defined as

$$\begin{aligned} T_2 = \Big(w^{t-1,t},\ d^t z^t,\ y^t, w^{t,t+1} \Big) \sum_{j=1}^{n} \gamma_j^{t-1,t} w_j^{t-1,t} \geq w^{t-1,t},\ d^t z^t \\ \geq \sum_{j=1}^{n} \gamma_j^t z_j^{1t}, \sum_{j=1}^{n} \gamma_j^{t,t+1} w_j^{t,t+1} \leq w^{t,t+1}, \\ \sum_{j=1}^{n} \gamma_j^t y_j^t \leq y^t;\ 0 \leq d^t \leq 1, \gamma_j^{t-1,t},\ \gamma_j^t, \gamma_j^{t,t+1} \geq 0;\ j = 1, ..., n\} \end{aligned} \tag{2}$$

Proof. The proof is clear. □

In the definition of T_1 and T_2, it is assumed that the intermediate materials z are flexible and some of the intermediate products are consumed in the second stage and the remainder is exported as final outputs. In the proposed model, this measure describes how the intermediate products are freely distributed between the stages and gives the optimal set of appropriate proportions for the intermediate materials.

3. Proposed two-stage DEA method

In this section, the additive model is extended to evaluate the efficiency of dynamic two-stage systems when flexible intermediate materials are present. Using the constant returns to scale expressions (1) and (2) to describe the first and second stages of technology, we introduce unit invariant additive model (3) to evaluate the efficiency of the DMU under consideration o:

$$\rho_o = \max \sum_{t=1}^{T} \left(\sum_{i=1}^{m} s_i^t + \sum_{r=1}^{s} s_r^t + \sum_{k=1}^{K} s_{k1}^{-,t} + \sum_{k=1}^{K} s_{k1}^{+,t} + \sum_{k=1}^{K} s_{k2}^{+,t} + \sum_{h=1}^{H} s_h^{+,t} + \sum_{h=1}^{H} s_h^{-,t} \right)$$

s.t.

Stage 1 constraints:

$$\sum_{j=1}^{n} \lambda_j^t x_{ij}^t + s_i^t = x_{io}^t; i = 1, ..., m, t = 1, ..., T \tag{3.1}$$

$$\sum_{j=1}^{n} \lambda_j^t z_{kj}^{1t} - s_{k1}^{+,t} = d_k^t (z_{ko}^{1t} + z_{ko}^{2t}), k = 1, ..., K, t = 1, ..., T \tag{3.2}$$

$$\sum_{j=1}^{n} \lambda_j^t z_{kj}^{2t} - s_{k2}^{+,t} = (1 - d_k^t)(z_{ko}^{1t} + z_{ko}^{2t}), k = 1, ..., K, t = 1, ..., T \tag{3.3}$$

Stage 2 constraints:

$$\sum_{j=1}^{n} \gamma_j^t z_{kj}^{1t} + s_{k1}^{-,t} = d_k^t (z_{ko}^{1t} + z_{ko}^{2t}), k = 1, ..., K, t = 1, ..., T \tag{3.4}$$

$$\sum_{j=1}^{n} \gamma_j^t y_{rj}^t - s_r^t = y_{ro}^t; r = 1, ..., s; t = 1, ..., T \tag{3.5}$$

Carry-over constraints

$$\sum_{j=1}^{n} \gamma_j^{t-1,t} w_{hj}^{t-1,t} + s_h^{-,t} = w_{ho}^{t-1,t}, \ h = 1, ..., H, \ t = 1, ..., T \tag{3.6}$$

$$\sum_{j=1}^{n} \gamma_j^{t,t+1} w_{hj}^{t,t+1} - s_h^{+,t} = w_{ho}^{t,t+1}, \ h = 1, ..., H, t = 1, ..., T \tag{3.7}$$

$$0 \le d_k^t \le 1; k = 1, ..., K \tag{3.8}$$

$$s_i^t, s_k^{+,t}, s_k^{-,t}, s_h^{-,t}, s_h^{+,t}, s_r^t, \gamma_j^t, \lambda_j^t, \gamma_j^{t-1,t}, \gamma_j^{t,t+1} \ge 0, \forall i, k, h, r, t \tag{3.9}$$

Since we know that any software has limitations regarding the admissible number of constraints and variables, the possibility of decomposing such a problem will be beneficial to practitioners who use a standard software package for measuring the efficiency of networks involving a large number of processes. Taking this fact into account, we suggest solving the following smaller linear programming problems.

$$\rho_o^t = \max \sum_{i=1}^{m} s_i^t + \sum_{r=1}^{s} s_r^t + \sum_{k=1}^{K} s_{k1}^{-,t} + \sum_{k=1}^{K} s_{k1}^{+,t} + \sum_{k=1}^{K} s_{k2}^{+,t} + \sum_{h=1}^{H} s_h^{+,t} + \sum_{h=1}^{H} s_h^{-,t}$$

s.t.

Stage 1 constraints:

$$\sum_{j=1}^{n} \lambda_j^t x_{ij}^t + s_i^t = x_{io}^t, i = 1, ..., m \tag{4.1}$$

$$\sum_{j=1}^{n} \lambda_j^t z_{kj}^{1t} - s_{k1}^{+,t} = d_k^t (z_{ko}^{1t} + z_{ko}^{2t}), k = 1, ..., K \tag{4.2}$$

$$\sum_{j=1}^{n} \lambda_j^t z_{kj}^{2t} - s_{k2}^{+,t} = (1 - d_k^t)(z_{ko}^{1t} + z_{ko}^{2t}), k = 1, ..., K \tag{4.3}$$

Stage 2 constraints:

$$\sum_{j=1}^{n} \gamma_j^t z_{kj}^{1t} + s_{k1}^{-,t} = d_k^t (z_{ko}^{1t} + z_{ko}^{2t}), k = 1, ..., K \tag{4.4}$$

$$\sum_{j=1}^{n} \gamma_j^t y_{rj}^t - s_r^t = y_{ro}^t, r = 1, ..., s \tag{4.5}$$

Carry-over constraints:

$$\sum_{j=1}^{n} \gamma_j^{t-1,t} w_{hj}^{t-1,t} + s_h^{-,t} = w_{ho}^{t-1,t}, h = 1, ..., H \tag{4.6}$$

$$\sum_{j=1}^{n} \gamma_j^{t,t+1} w_{hj}^{t,t+1} - s_h^{+,t} = w_{ho}^{t,t+1}, \ h = 1, ..., H \tag{4.7}$$

$$0 \le d_k^t \le 1, k = 1, ..., K \tag{4.8}$$

$$s_i^t, s_k^{+,t}, s_k^{-,t}, s_h^{-,t}, s_h^{+,t}, s_r^t, \gamma_j^t, \lambda_j^t, \gamma_j^{t-1,t}, \gamma_j^{t,t+1} \ge 0, \forall i, k, h, r, t \tag{4.9}$$

Theorem 3. The optimal objective value of (3), ρ_o, equals the sum of the optimal divisional slacks represented by ρ_o^t for $t = 1, ..., T,$ that is, $\rho_o = \sum_{t=1}^{T} \rho_o^t.$

Proof. This is clear. □

Theorem 3 states that DMU_o is efficient if and only if $\rho_o^t = 0$ for all $t = 1, ..., T$. Moreover, it shows that problem (3) can be divided into T independent problems in the form of (4). The proof of Theorem 3 is easy and hence omitted.

Definition 1. The efficiency of any two-stage system in a given period can be obtained by

$$E_o^t = \frac{1 - \dfrac{1}{m+K+H}\left(\displaystyle\sum_{i=1}^{m} \frac{s_i^t}{x_i^t} + \sum_{k=1}^{K} \frac{s_k^{-,t}}{z_k^{1t}} + \sum_{h=1}^{H} \frac{s_h^{-,t}}{w_h^{t-1,t}}\right)}{1 + \dfrac{1}{S+2K+H}\left(\displaystyle\sum_{r=1}^{s} \frac{s_r^t}{y_r^t} + \sum_{k=1}^{K} \frac{s_{k1}^{+,t}}{z_k^{1t}} + \sum_{k=1}^{K} \frac{s_{k2}^{+,t}}{z_k^{2t}} + \sum_{h=1}^{H} \frac{s_h^{+,t}}{w_h^{t,t+1}}\right)}$$

and the overall efficiency is equal to

$$E_o = \frac{1}{T}\sum_{t=1}^{T} E_o^t$$

The system *o* under evaluation is efficient for period *t* if and only if $E_o^t = 1$, otherwise it is inefficient. Furthermore, DMU_o is totally efficient if and only if $E_o = 1$.

4. Numerical example

We apply the model for a dynamic network to 10 Iranian banks operating during two periods, *t* and *t* + 1. There exists some disagreement on whether deposits should be treated as an input or an output. An important function of Iranian commercial banks is to attract deposits (stage 1) and then to distribute these deposits (stage 2). A portion of the total deposits attracted should be distributed among customers and the remainder should be transferred to the Central Bank of the Republic of Iran.

Similar to previous work on the evaluation of banks' performance, bank's processes are divided into two stages: the deposit process and loan process. There are two inputs in the first stage: fund from customers (x_1^t) and the number of cheque accounts (x_2^t). The two outputs of this stage are the deposits distributed among customers (z_1^t) and deposits transferred to the Central Bank (z_2^t). Some portions of these deposits are distributed among customers in stage 2, and the remainder should be transferred to the Central Bank. The additional inputs for the second stage are characterized as unused assets in period $t-1$ $(w^{t-1,t})$. The final outputs of the second stage are recorded as the number of transactions (y_1^t), loans (y_2^t), profits (y_3^t) and unused assets in period t $(w^{t,t+1})$. The data related to period t and period $t+1$ are provided in Tables 1 and 2, respectively.

Table 1. Data set for period *t*

Branch	x_1^t	x_2^t	z_1^t	z_2^t	$w^{t-1,t}$	y_1^t	y_2^t	y_3^t	$w^{t,t+1}$	z^t
1	0.948	0.838	0.894	0.362	0.603	0.221	0.111	0.211	0.133	1.256
2	1.33	1.233	0.678	0.188	0.982	0.232	0.212	0.210	0.073	0.866
3	0.621	0.321	0.836	0.207	0.979	0.423	0.123	0.153	0.053	1.043
4	1.783	1.483	0.869	0.516	0.720	0.514	0.214	0.114	0.054	1.385
5	1.892	1.592	0.693	0.407	0.595	0.351	0.321	0.221	0.072	1.1
6	0.990	0.790	0.966	0.269	0.936	0.021	0.121	0.221	0.094	1.235
7	0.151	0.451	0.647	0.257	0.906	0.312	0.412	0.332	0.084	0.904
8	0.108	0.408	0.756	0.103	0.574	0.723	0.323	0.423	0.104	0.859
9	1.364	1.864	1.191	0.402	0.713	0.833	0.233	0.333	0.023	1.593
10	1.922	1.222	0.792	0.187	0.715	0.133	0.333	0.235	0.087	0.979

Table 2. Data set for period *t* + 1

Branch	x_1^{t+1}	x_2^{t+1}	z_1^{t+1}	z_2^{t+1}	$w^{t,t+1}$	y_1^{t+1}	y_2^{t+1}	y_3^{t+1}	$w^{t+1,t+2}$	z^{t+1}
1	0.848	0.938	0.694	0.462	0.403	0.121	0.211	0.311	0.233	1.156
2	2.33	1.133	0.578	0.288	0.782	0.432	0.312	0.410	0.173	0.866
3	0.521	0.421	0.846	0.217	0.879	0.323	0.122	0.143	0.043	1.063
4	1.883	1.482	0.859	0.506	0.820	0.513	0.224	0.214	0.034	1.365
5	1.992	1.492	0.593	0.417	0.695	0.352	0.311	0.211	0.172	1.01
6	0.790	0.690	0.866	0.369	0.946	0.031	0.221	0.121	0.054	1.235
7	0.451	0.351	0.657	0.247	0.926	0.212	0.422	0.312	0.184	0.904
8	0.208	0.508	0.754	0.113	0.474	0.623	0.423	0.523	0.103	0.867
9	1.564	1.764	1.181	0.412	0.813	0.834	0.223	0.333	0.033	1.593
10	1.822	1.122	0.791	0.186	0.815	0.132	0.323	0.335	0.057	0.977

Tables 3 and 4 report the results from the proposed model for periods *t* and *t* + 1, respectively. The second to tenth columns report the inefficiency slacks and the eleventh column presents the sum of the inefficiency slacks. For DMU_8, all the slacks are equal to 0. Analysis of the slack variables reveals the status of excesses in input resources and output shortfalls and indicates possible improvements in the ways of utilizing the intermediate materials. The twelfth column reports the values of d^t that indicate the optimal proportions of intermediate materials to be used as inputs in the second stage.

Consider branch 3 for example. As seen in Table 1, $z_1^t = 0.836$ and $z_2^t = 0.207$ with $z^t = 1.043$. Based on the results displayed in Table 3, the optimal proportion d^t is equal to 0.8. This means that there is a need to change the relative values of the deposits distributed among customers and those transferred to the Central Bank. The appropriate value of deposits distributed among the customers should be $d^{t*}(z^t) = 0.834$ and $(1 - d^{t*})(z^t) = 0.2086$ should be transferred to the Central Bank. From the results in Tables 3 and 4, we notice that both DMU_7 and DMU_8 are efficient in the period $t + 1$, but only DMU_8 is efficient on aggregate.

Table 3. Results related to period t

Branch	Inefficiency slacks										ρ_o^t	d^t
	s_{i1}^t	s_{i2}^t	$s_{k1}^{+,t}$	$s_{k2}^{+,t}$	$s_{k1}^{-,t}$	s_{r1}^t	s_{r2}^t	s_{r3}^t	$s_h^{-,t}$	$s_h^{+,t}$		
1	0.73	0	0.30	0.21	0.36	0	0	0	0	0	1.60	1
2	1	0	1.42	0.31	0	0.6	0.16	0.27	0.32	0.05	4.13	1
3	0	0	0	0	0	0.38	0.23	0.31	0.34	0.06	1.33	0.8
4	1.39	0	1.36	0.37	0.44	0.39	0.19	0.42	0	0.08	4.64	1
5	1.47	0	1.85	0.40	0.32	0.4	0.01	0.22	0	0.04	4.70	1
6	0.78	0	0.23	0.20	0	1.16	0.41	0.47	0	0.08	3.32	1
7	0	0	0	0.07	0	0.39	0	0.13	0.11	0.03	0.73	0.94
8	0	0	0	0	0	0	0	0	0	0	0.00	0.88
9	0.87	0	1.86	0.47	0.65	0.07	0.17	0.19	0	0.11	4.39	1
10	1.60	0	1.29	0.31	0.04	0.77	0.07	0.29	0	0.04	4.40	1

Table 4. Results related to period $t + 1$

Branch	Inefficiency slacks										ρ_o^{t+1}	d^{t+1}
	s_{i1}^{t+1}	s_{i2}^{t+1}	$s_{k1}^{+,t+1}$	$s_{k2}^{+,t+1}$	$s_{k1}^{-,t+1}$	s_{r1}^{t+1}	s_{r2}^{t+1}	s_{r3}^{t+1}	$s_h^{-,t+1}$	$s_h^{+,t+1}$		
1	0.46	0	0.24	0.21	0.46	0	0	0	0	0	1.37	1
2	1.87	0	0.82	0.25	0	0.25	0.17	0.19	0	0	3.55	1
3	0	0	0	0	0	0.38	0.35	0.44	0.35	0.07	1.59	0.8
4	1.28	0	0.83	0.33	0.06	0.56	0.51	0.69	0	0.14	4.41	1
5	1.38	0	1.20	0.33	0	0.41	0.23	0.47	0	0	4.02	1
6	0.03	0	0	0.42	0	0.99	0.47	0.74	0.17	0.11	4.93	1
7	0	0	0	0	0	0	0	0	0	0	0.00	0.73
8	0	0	0	0	0	0	0	0	0	0	0.00	0.87
9	0.84	0	1.03	0.39	0.30	0.23	0.50	0.56	0	0.14	4.00	1
10	1.36	0	0.69	0.25	0	0.68	0.23	0.34	0.20	0.08	3.82	1

Also, Table 5 presents the efficiency scores based on definition 1. Column 2 indicates the efficiency of the branches in period t. The efficiencies of branches in period $t+1$ are presented in column 3. As shown, two branches 7 and 8 are efficient in period $t+1$ while branch 8 is the only efficient unit in period t. The overall efficiency of the branches is displayed in column 4. As can be seen, branch 8 is efficient in each period and overall. Therefore, it seems that branch 8 performs the best in comparison to the other branches. Also, branch 6 has the lowest efficiency score in each period. Consequently, it appears that managers and decision makers should review their performance and the use of their resources to improve efficiency.

Table 5. The efficency scores

Branch	E_o^t	E_o^{t+1}	E_o
1	0.60	0.60	0.60
2	0.25	0.41	0.33
3	0.46	0.36	0.41
4	0.21	0.21	0.21
5	0.33	0.35	0.34
6	0.07	0.11	0.09
7	0.68	1.00	0.84
8	1.00	1.00	1.00
9	0.25	0.24	0.245
10	0.25	0.24	0.245

Now, to compare the results obtained from the proposed approach with those from existing models, we assume that the intermediate materials are purely used as inputs to the second stage. In this case, the constraints (4.2) and (4.4) in model (4) are substituted by the following constraints, respectively:

$$\sum_{j=1}^{n} \lambda_j^t z_{kj}^t - s_k^{+,t} = z_{ko}^t,\ k = 1, ..., K$$

$$\sum_{j=1}^{n} \gamma_j^t z_{kj}^t + s_k^{-,t} = z_{ko}^t,\ k = 1, ..., K$$

Notice that the constraint (4.3) is omitted in this case and the objective function in model (4) is replaced by

$$\rho_o'^t = \max \sum_{i=1}^{m} s_i^t + \sum_{r=1}^{s} s_r^+ + \sum_{k=1}^{K} s_k^{-,t} + \sum_{k=1}^{K} s_k^{+,t} + \sum_{h=1}^{H} s_h^{+,t} + \sum_{h=1}^{H} s_h^{-,t}$$

Based on these substitutions, efficiencies are defined as follows: the efficiency of each DMU in each period can be calculated from

$$E_o''^t = \frac{1 - \dfrac{1}{m+K+H}\left(\sum_{i=1}^{m}\dfrac{s_i^t}{x_i^t} + \sum_{k=1}^{K}\dfrac{s_k^{-,t}}{z_k^t} + \sum_{h=1}^{H}\dfrac{s_h^{-,t}}{w_h^{t-1,t}}\right)}{1 + \dfrac{1}{S+K+H}\left(\sum_{r=1}^{s}\dfrac{s_r^t}{y_r^t} + \sum_{k=1}^{K}\dfrac{s_k^{+,t}}{z_k^t} + \sum_{h=1}^{H}\dfrac{s_h^{+,t}}{w_h^{t,t+1}}\right)}$$

and the overall efficiency is equal to

$$E_o' = \frac{1}{T}\sum_{t=1}^{T} E_o''^t$$

The results are shown in Tables 6 and 7.

Table 6. Sums of slack variables

Branch	$\rho_o''^t$	$\rho_o''^{t+1}$
1	1.42	0.83
2	1.96	4.17
3	2.57	2.47
4	3.94	4.15
5	3.83	3.98
6	3.53	3.36
7	1.68	2.14
8	0	0
9	3.30	3.46
10	4.03	4.17

It is assumed that intermediate materials are purely used as inputs into stage 2.

Table 7. The efficency scores

Branch	$E_o''^t$	$E_o''^{t+1}$	E_o'
1	0.60	0.76	0.68
2	0.34	0.36	0.35
3	0.21	0.22	0.215
4	0.24	0.18	0.21
5	0.36	0.32	0.34
6	0.05	0.07	0.06
7	0.40	0.38	0.39
8	1	1	1
9	0.26	0.23	0.245
10	0.22	0.19	0.205

It is assumed that intermediate materials are purely used as inputs into stage 2.

According to both approaches, branch 8 is the most efficient overall and branch 6 is the least efficient. Nevertheless, the results found by considering intermediate materials are different from those obtained from the proposed approach in a number of respects. According to the proposed approach, two branches, 7 and 8, were determined to be efficient in period $t + 1$, while only one branch 8 is inferred to be efficient considering z as the vector of intermediate materials in this period. Furthermore, the efficiency scores according to these two methods have significant differences for some branches, such as branch 7. The overall efficiency score of branch 7 is 0.84 when the utilization of deposits is regarded as flexible, whilst this value is 0.39 when the utilization of deposits is considered to be fixed. Furthermore, using the proposed approach, the measures of branch 1's efficiency for the two periods were the same. However, the results obtained from the model which assumes that the utilization of deposits is fixed indicate that branch 1 has improved its performance in period $t + 1$ compared to period t. Therefore, determining the role of factors is a significant aspect of calculating accurate efficiency scores and rational decision making.

5. Conclusions

A dynamic DEA-based model has been proposed to analyse two-stage processes with flexible intermediate materials. In many real-life situations, only a certain proportion of the intermediate materials are used as inputs in the second stage, for example, in the banking example referenced in this paper. Basic two-stage DEA models cannot provide a good evaluation of the performance of such structures. The proposed model has identified the status of intermediate materials and argued how intermediate products should be utilized to increase the system's overall efficiency. This paper has also incorporated the time factor into evaluating processes. The approach was illustrated using a data set based on 10 Iranian banks for two consecutive periods.

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