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AN EWMA CONTROL CHART FOR THE EXPONENTIAL DISTRIBUTION USING REPETITIVE SAMPLING

A new EWMA control chart has been proposed under repetitive sampling when a quantitative characteristic follows the exponential distribution. The properties of the proposed chart, including the average run lengths has been is compared with two existing control charts with the help of simulated data. An application of the proposed chart hs been illustrated using a healthcare data set.

Keywords: *control chart, EWMA statistic, exponential distribution, normal distribution, average run length*

1. Introduction

Customers demand high quality products on the market. To meet customers' requirements and maintain a good reputation, a producer must pay full attention to the production process from raw materials to the final product. To achieve the goal of producing a high quality product, the producer has to rely on statistical tools such as control charts and acceptance sampling plans. The former are used for monitoring the manufacturing process and the latter are used for inspection, from the raw materials to the final

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product. Control charts are powerful tools for minimizing the proportion of non-conforming products, as they provide a quick indication of when the process is shifting to an out-of-control state. Such quick indications about shifts in a process help engineers to bring it back into an under-control state.

Shewhart control charts are simple to apply in industry. The major flaw of the Shewhart control chart is that it does not detect small changes in the process. To overcome this problem, researchers have been attempting to introduce various control charts that can detect small changes in a manufacturing process. Roberts introduced an exponentially weighted moving average (EWMA) control chart to detect a small shift in a process [1]. The EWMA statistic in a control chart utilizes current and past information to make a decision about the manufacturing process. The weight of past information is controlled using the smoothing constant. More details about the application and design of EWMA control charts have been published by Lucas and Saucci [2]. More information can be found in [3–6].

Mostly, control charts are designed by assuming that the quantitative characteristic under study follows the normal distribution. Such control charts are not helpful in detecting a shift in the process when the characteristic of interest follows some non-normal distribution. In this situation, the likelihood of false alarms may increase and/or the proportion of non-conforming products may increase. Furthermore, there are many situations where data are not collected in subgroups. As mentioned by [7], the data not collected in a subgroup may follow an exponential distribution. Characteristics such as the lifetime of an item or waiting time of a customer may follow an exponential distribution. A control chart for the exponential distribution is called a t-chart. A detailed study of the design and application of a t-chart can be read in [8].

Hence, t-charts or EWMA t-charts are designed for single sampling. Aslam et al. designed an EWMA t-chart under a single sampling scheme [9]. The use of repetitive group sampling (RGS) in the area of control charts has attracted researchers, due to its simplicity and efficiency compared to sequential or multiple sampling schemes. RGS was originally designed by [10] and its efficiency is well verified compared to single and double sampling schemes in acceptance sampling plans. Recently, some authors designed control charts using an RGS scheme [11–13].

By exploring the literature, it can be noted that results on EWMA control charts using a single sampling scheme are available in the literature. Also, some results are available on control charts using RGS assuming that the quantity of interest follows the normal distribution. According to the best of the authors' knowledge, there is no work available on EWMA t-charts using an RGS scheme. In this paper, we will focus on the design of a t-chart using the EWMA statistic under an RGS scheme. The proposed control chart will be an extension of [9]. The complete structure of the proposed chart will be given. The efficiency of the proposed chart will be compared with [9] using simulated data. An application of the proposed chart in the health service is also given.

2. Design of the proposed control chart

It is assumed that the time between events T follows the exponential distribution with scale (mean) parameter θ whose probability density function (pdf) is given by

$$f\left(t\right)=\frac{1}{\theta}\mathrm{e}^{-t/\theta},\ t>0 \tag{1}$$

If T follows the exponential distribution with mean θ, then $T^{*}=T^{(1/3.6)}$ follows the Weibull distribution with the shape parameter β and scale parameter $\theta^{1/\beta}$ [7]. Nelson suggested that when $\beta = 3.6$, the above Weibull distribution becomes approximately a normal distribution [8].

The proposed control chart can be described as follows, as an extension of the control chart proposed by [9]. It should be noted that there are two pairs of control limits in the proposed control chart.

Step 1. Select an item at random and measure its quantitative characteristic T and calculate the following transformed variable, T^{*}:

$$T^{*} = T^{(1/3.6)}$$

Then, calculate the following EWMA statistic, denoted Z_t, using the smoothing constant λ

$$Z_t = \lambda T^{*} + \left(1-\lambda\right) Z_{t-1}$$

Step 2. Declare the process as out-of-control if $Z_t \geq UCL_1$ or $Z_t \leq LCL_1$. Declare the process as under-control if $LCL_2 \leq Z_t \leq UCL_2$. Otherwise, go to Step 1 and repeat the process.

As stated in Step 2, the proposed control chart does not make any conclusion on the state of the process if the statistic lies between the inner and outer control limits, in which case repetitive sampling is required.

Suppose first that the process is under control with specified mean θ_0. Because the transformed variable T^{*} follows an approximately normal distribution, so does the EWMA statistic. The mean μ_Z and the variance σ_Z^2 of the Z_t statistic are obtained by

$$\mu_Z = \theta_0^{*}\Gamma\left(1+\frac{1}{3.6}\right)$$

$$\sigma_Z^2 = \theta_0^{*2} \frac{\lambda}{2-\lambda}\left\{1-(1-\lambda)\right\}^{2t}\left[\Gamma\left(1+\frac{2}{3.6}\right)-\Gamma\left(1+\frac{1}{3.6}\right)^2\right]$$

where $\theta_0^* = \theta_0^{1/3.6}$.

Therefore, the two pairs of control limits are given below. First, the outer control limits are

$$LCL_{1(t)} = \mu_Z - k_1\sigma_Z = \theta_0^*\left[\Gamma\left(1+\frac{1}{3.6}\right)\right.$$

$$\left.-k_1\sqrt{\frac{\lambda}{2-\lambda}\left\{1-(1-\lambda)\right\}^{2t}}\sqrt{\Gamma\left(1+\frac{2}{3.6}\right)-\Gamma\left(1+\frac{1}{3.6}\right)^2}\right] \quad (2)$$

$$UCL_{1(t)} = \mu_Z + k_1\sigma_Z = \theta_0^*\left[\Gamma\left(1+\frac{1}{3.6}\right)\right.$$

$$\left.+k_1\sqrt{\frac{\lambda}{2-\lambda}\left\{1-(1-\lambda)\right\}^{2t}}\sqrt{\Gamma\left(1+\frac{2}{3.6}\right)-\Gamma\left(1+\frac{1}{3.6}\right)^2}\right] \quad (3)$$

The inner control limits are

$$LCL_{2(t)} = \mu_Z - k_2\sigma_Z = \theta_0^*\left[\Gamma\left(1+\frac{1}{3.6}\right)\right.$$

$$\left.-k_2\sqrt{\frac{\lambda}{2-\lambda}\left\{1-(1-\lambda)\right\}^{2t}}\sqrt{\Gamma\left(1+\frac{2}{3.6}\right)-\Gamma\left(1+\frac{1}{3.6}\right)^2}\right] \quad (4)$$

$$UCL_{2(t)} = \mu_Z + k_2\sigma_Z = \theta_0^*\left[\Gamma\left(1+\frac{1}{3.6}\right)\right.$$

$$\left.+k_2\sqrt{\frac{\lambda}{2-\lambda}\left\{1-(1-\lambda)\right\}^{2t}}\sqrt{\Gamma\left(1+\frac{2}{3.6}\right)-\Gamma\left(1+\frac{1}{3.6}\right)^2}\right] \quad (5)$$

where the control constants k_1 and k_2 are to be determined.

Let

$$\mathrm{c}_{L1} = \Gamma\left(1+\frac{1}{3.6}\right) - k_1\sqrt{\frac{\lambda}{2-\lambda}\left\{1-(1-\lambda)\right\}^{2t}}\sqrt{\Gamma\left(1+\frac{2}{3.6}\right)-\Gamma\left(1+\frac{1}{3.6}\right)^2}$$

$$c_{U1} = \Gamma\left(1+\frac{1}{3.6}\right) + k_1\sqrt{\frac{\lambda}{2-\lambda}\left\{1-(1-\lambda)\right\}^{2t}}\sqrt{\Gamma\left(1+\frac{2}{3.6}\right)-\Gamma\left(1+\frac{1}{3.6}\right)^2}$$

$$c_{L2} = \Gamma\left(1+\frac{1}{3.6}\right) - k_2\sqrt{\frac{\lambda}{2-\lambda}\left\{1-(1-\lambda)\right\}^{2t}}\sqrt{\Gamma\left(1+\frac{2}{3.6}\right)-\Gamma\left(1+\frac{1}{3.6}\right)^2}$$

$$c_{U2} = \Gamma\left(1+\frac{1}{3.6}\right) - k_2\sqrt{\frac{\lambda}{2-\lambda}\left\{1-(1-\lambda)\right\}^{2t}}\sqrt{\Gamma\left(1+\frac{2}{3.6}\right)-\Gamma\left(1+\frac{1}{3.6}\right)^2}$$

then the above control limits can be rewritten as

$$LCL_1 = \theta_0^* c_{L1}$$

$$LCU_1 = \theta_0^* c_{U1}$$

$$LCL_2 = \theta_0^* c_{L2}$$

$$LCU_2 = \theta_0^* c_{U2}$$

Therefore, the probability of being declared out-of-control $\left(P_{\text{out}}^0\right)$ when $\theta = \theta_0$ is given as follows:

$$P_{\text{out}}^0 = \frac{P_{\text{out, 1}}^0}{1-P_{\text{rep}}^0} \tag{6}$$

The average run length (*ARL*) when $\theta = \theta_0$ is given as follows:

$$ARL_0 = \frac{1}{P_{\text{out}}^0} \tag{7}$$

The average sample size (ASS_0) for the process when $\theta = \theta_0$ is given by

$$ASS_0 = \frac{1}{1 - P_{\text{rep}}^{0}} \tag{8}$$

Suppose now that the process mean is shifted to $\theta_1 = c\theta_0$, where c is a constant. The probability that the process is inferred to be out-of-control based on a single sample $(P_{\text{out},1}^{1})$ when $\theta = \theta_1$.

Finally, the probability that the process is declared to be out-of-control $(P_{\text{out},1}^{1})$ and the out-of-control ARL (ARL_1)when $\theta = \theta_1$ are given as follows:

$$P_{\text{out}}^{1} = \frac{P_{\text{out},1}^{1}}{1 - P_{\text{rep}}^{1}} \tag{9}$$

and

$$ARL_1 = \frac{1}{P_{\text{out}}^{1}} \tag{10}$$

Algorithm 1. Monte Carlo simulation of an EWMA control chart for the exponential distribution using a repetitive sampling scheme when the process is under-control ($c = 1$).

The following are the steps involved in an R program defining the Monte Carlo simulation:

1. Computation of the proposed EWMA statistic, Z_t.
   1.1. Specify the value of the required under-control ARL, denoted r_0 and λ.
   1.2. Generate T a random sample of size 1 for each subgroup, from the exponential distribution with specified parameter θ_0, i.e. under the assumption that the process is under-control ($c = 1$). Generate 1000 such subgroups.
   1.3. Transform T into $T = T^{*1/\beta}$ and substitute this into the EWMA statistic.
   1.4. Compute the EWMA statistic, Z_t.

2. Computation of the variable control limits.
   2.1. Define appropriate values of the control coefficients, k_1 and k_2.
   2.2. Calculate $LCL_{1(t)}$, $LCL_{2(t)}$, $UCL_{2(t)}$ and $UCL_{1(t)}$ based on 1000 subgroups.

2.3. Keeping in view the operational strategy of the proposed control chart, infer whether the process should be declared as under-control, in repeat mode, or out-of-control. If the process is declared as in-control, repeat Steps 1.1–2.3. If the process is in repeat mode, count the number of repetitions. Otherwise, define the run length to be the number of subgroups, together with the number of repetitions, i.e. the time for which the process was declared to be either under-control or in repeat mode before being declared to be out-of-control.

3. Computation of the average run length (*ARL*).
   3.1. Repeat Steps 1.1–2.3 a sufficient number of times (10 000 say) to calculate the under-control *ARL*. If the under-control *ARL* is equal to the specified ARL_0, then stop the process and go to Step 3.2. Otherwise, modify the values of the control coefficients and repeat Steps 1.1–3.1.
   3.2. Determine k_1 and k_2 such that $ARL_0 \geq r_0$.

4. Computation of the average run length (ARL_1) for the shifted process.
   4.1. Repeat Steps 1–3 for the values of repeat Steps 1.1–3.1.
   4.2. Determine obtained in Step 3.2.

Algorithm 2. Monte Carlo simulation of an EWMA control chart for the exponential distribution using a repetitive sampling scheme for the shifted process $(c \neq 1)$.

The following are the steps involved in an R program defining the Monte Carlo simulation:

1. Computation of the proposed EWMA statistic, Z_t.
   1.1. Specify the value of the smoothing constant λ and shift *c*.
   1.2. Generate *T*, a random sample of size 1 for each subgroup, from the exponential distribution with specified parameter $\theta_1 = c\theta_0$ for a shifted process $(c \neq 1)$. Generate 1000 such subgroups.
   1.3. Transform *T* into $T^* = T^{1/\beta}$ and substitute this into the EWMA statistic.
   1.4. Compute the EWMA statistic, Z_t.

2. Computation of the variable control limits.
   2.1. Specify the values of control coefficients k_1 and k_2 to be those obtained in Algorithm 1.
   2.2. Calculate $LCL_{1(t)}$, $LCL_{2(t)}$, $UCL_{2(t)}$ and $UCL_{1(t)}$ from 1000 subgroups.
   2.3. Keeping in view the operational strategy of the proposed control chart, infer whether the process should be declared as under-control, in repeat mode or out-

-of-control. If the process is declared as under-control, repeat Steps 1.1–2.3. If the process is inferred to be in repeat mode, count the number of repetitions. Otherwise, define the run length to be the number of subgroups, together with the number of repetitions, i.e. the length of time for which the process was declared to be under-control or in repeat mode before it is declared to be out-of-control.

3. Computation of the average run length (ARL_1) for a shifted process.

3.1. Repeat steps 1.1–2.3 a sufficient number of times (10 000 say) to calculate the ARL_1 for a given shifted process.

The values of ARL_1 and SDRL for various λ, ARL_0, and f are given in Tables 1–3.

Table 1. ARLs and SDRLs for the proposed chart when $ARL_0 \approx 200$

f	$\lambda = 0.2$ $k_1 = 2.28, k_2 = 0.40$		$\lambda = 0.4$ $k_1 = 2.24, k_2 = 0.33$		$\lambda = 0.6$ $k_1 = 2.22, k_2 = 0.27$		$\lambda = 0.8$ $k_1 = 2.165, k_2 = 0.24$		$\lambda = 1.0$ $k_1 = 2.11, k_2 = 0.20$	
	ARL	SDRL	ARL	SDRL	ARL	SDRL	ARL	SDRL	ARL	SDRL
1.00	203.28	247.94	200.40	207.72	204.74	214.03	201.75	219.38	201.14	226.91
0.90	80.94	128.47	117.03	130.98	129.46	145.89	134.34	154.51	137.40	168.18
0.80	24.92	65.21	53.48	71.44	66.59	86.37	75.03	96.86	80.01	110.21
0.70	6.29	25.02	23.29	39.48	31.24	48.79	37.79	58.19	42.42	65.24
0.60	1.63	6.62	9.55	19.43	14.82	25.74	17.90	29.84	20.94	36.27
0.50	1.02	0.61	3.84	8.12	6.51	12.48	8.32	14.07	10.31	18.13
0.40	1.00	0.00	1.86	3.62	3.10	5.38	4.12	6.66	4.97	8.62
0.30	1.00	0.00	1.17	1.20	1.72	2.42	2.22	3.07	2.63	3.72
0.20	1.00	0.00	1.02	0.28	1.19	0.93	1.39	1.37	1.55	1.61
0.10	1.00	0.00	1.00	0.00	1.02	0.27	1.07	0.47	1.13	0.65

Table 2. The ARLs and SDRLs for the proposed chart when $ARL_0 \approx 300$

f	$\lambda = 0.2$ $k_1 = 2.59, k_2 = 0.70$		$\lambda = 0.4$ $k_1 = 2.52, k_2 = 0.50$		$\lambda = 0.6$ $k_1 = 2.45, k_2 = 0.40$		$\lambda = 0.8$ $k_1 = 2.398, k_2 = 0.36$		$\lambda = 1.0$ $k_1 = 2.30, k_2 = 0.25$	
	ARL	SDRL	ARL	SDRL	ARL	SDRL	ARL	SDRL	ARL	SDRL
1.00	303.74	331.66	304.92	309.08	301.57	302.88	302.02	301.57	303.97	314.10
0.90	105.30	143.68	152.60	160.17	166.85	170.87	174.56	179.52	194.27	214.03
0.80	31.22	61.03	65.35	76.00	77.32	86.72	85.87	98.11	110.34	133.51
0.70	8.11	24.15	29.17	42.30	37.79	50.04	43.20	56.85	55.28	77.86
0.60	2.24	8.68	12.66	23.66	18.15	29.53	21.81	32.57	26.43	42.74
0.50	1.05	1.10	5.15	11.28	7.97	14.64	10.11	16.94	12.59	22.01
0.40	1.00	0.00	2.10	3.84	3.71	6.76	5.00	8.16	5.63	9.40
0.30	1.00	0.00	1.26	1.57	1.88	2.76	2.46	3.60	2.89	4.06
0.20	1.00	0.00	1.02	0.31	1.21	1.01	1.47	1.48	1.67	2.00
0.10	1.00	0.00	1.00	0.00	1.02	0.23	1.08	0.51	1.15	0.72

Table 3. The ARLs and SDRLs for the proposed chart when $ARL_0 \approx 370$

f	$\lambda = 0.2$ $k_1 = 2.70$, $k_2 = 0.92$		$\lambda = 0.4$ $k_1 = 2.75$, $k_2 = 0.86$		$\lambda = 0.6$ $k_1 = 2.70$, $k_2 = 0.81$		$\lambda = 0.8$ $k_1 = 2.64$, $k_2 = 0.75$		$\lambda = 1.0$ $k_1 = 2.64$, $k_2 = 0.75$	
	ARL	SDRL	ARL	SDRL	ARL	SDRL	ARL	SDRL	ARL	SDRL
1.00	371.08	369.66	370.59	342.83	371.49	346.79	374.70	354.12	371.39	356.32
0.90	138.65	175.32	170.79	175.69	179.12	179.04	182.01	182.72	189.95	188.53
0.80	38.10	65.13	62.97	65.57	71.03	72.33	77.14	75.89	86.42	86.02
0.70	10.70	28.95	27.59	33.58	32.69	35.40	36.47	38.06	42.35	44.70
0.60	2.83	10.18	12.78	20.42	16.21	20.92	18.58	23.17	21.23	24.24
0.50	1.16	3.51	5.59	10.52	7.85	11.52	9.59	13.37	11.34	15.03
0.40	1.00	0.18	2.43	4.54	3.86	6.30	5.00	7.40	6.13	8.74
0.30	1.00	0.00	1.31	1.59	2.02	2.84	2.51	3.35	3.11	4.12
0.20	1.00	0.00	1.03	0.38	1.25	1.06	1.51	1.48	1.75	1.94
0.10	1.00	0.00	1.00	0.00	1.03	0.30	1.09	0.52	1.16	0.72

3. Comparative study

In this section, we will discuss the advantages of the proposed chart over the existing t-charts proposed in [14] and [7]. The proposed chart becomes the one in [14] when $\lambda = 1$ and the chart from [7] when $k_1 = k_2 = k$ and $\lambda = 1$. To compare the performance of the proposed control chart with the existing control charts, simulation will be used. Assume that $r_0 = 370$ and $\lambda = 0.20$. The data are generated from the exponential distribution with $\theta_0 = 0.21$. The first 25 samples are generated from the in control process and the next 25 samples are generated from the shifted process with c 1.4. Statistics describing T^* and M_t for these three control charts are reported in Table 4.

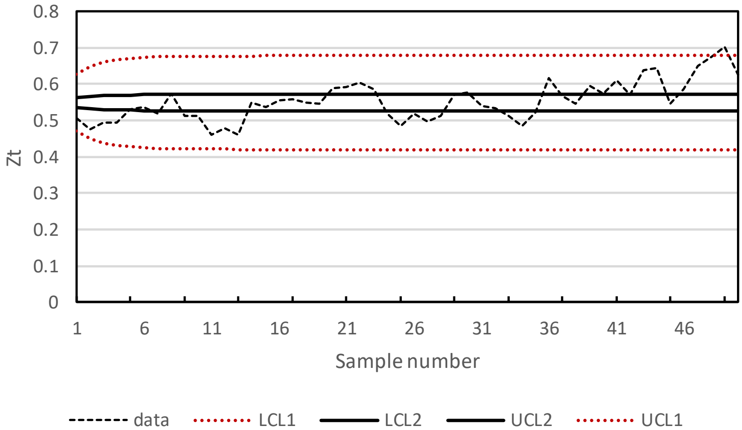

Fig. 1. Proposed control chart for simulated data

Table 4. Simulated data

Proposed chart $\lambda = 0.20$		Chart from [7] $\lambda = 1.0$		Chart from [9] $\lambda = 0.20$		Proposed chart $\lambda = 0.20$		Chart from [7] $\lambda = 1.0$		Chart from [9] $\lambda = 0.20$	
T^*	Z_t	T^*	Z_t	T^*	Z_t	T^*	Z_t	T^*	Z_t	T^*	Z_t
0.4402	0.5063	0.7267	0.7267	0.5447	0.5644	0.6523	0.5181	0.6671	0.6671	0.4459	0.5276
0.3601	0.4771	0.4463	0.4463	0.8661	0.6248	0.4073	0.4960	0.7442	0.7442	0.5075	0.5236
0.5680	0.4952	0.6807	0.6807	0.3026	0.5603	0.5823	0.5132	0.4952	0.4952	0.2149	0.4618
0.4962	0.4954	0.5104	0.5104	0.3848	0.5252	0.8030	0.5712	0.5969	0.5969	0.542	0.4779
0.6687	0.5301	0.3183	0.3183	0.5498	0.5301	0.5966	0.5763	0.7707	0.7707	0.4651	0.4753
0.5634	0.5367	0.5813	0.5813	0.5598	0.5361	0.3888	0.5388	0.4489	0.4489	0.5512	0.4905
0.4517	0.5197	0.5744	0.5744	0.4293	0.5147	0.5208	0.5352	0.6623	0.6623	0.7106	0.5345
0.7850	0.5728	0.6853	0.6853	0.4303	0.4978	0.4253	0.5132	0.5257	0.5257	0.5371	0.535
0.2637	0.5110	0.6210	0.6210	0.4644	0.4911	0.3657	0.4837	0.4737	0.4737	0.6148	0.551
0.5177	0.5123	0.4989	0.4989	0.539	0.5007	0.6728	0.5215	0.5361	0.5361	0.7189	0.5846
0.2576	0.4614	0.5438	0.5438	0.6304	0.5266	0.9897	0.6152	0.5481	0.5481	0.5929	0.5862
0.5444	0.4780	0.7663	0.7663	0.6786	0.557	0.3812	0.5684	0.6275	0.6275	0.2852	0.526
0.3854	0.4594	0.3178	0.3178	0.5505	0.5557	0.4639	0.5475	0.6604	0.6604	0.6003	0.5409
0.9001	0.5476	0.2777	0.2777	0.7011	0.5848	0.7901	0.5960	0.3968	0.3968	0.8309	0.5989
0.4921	0.5365	0.7989	0.7989	0.7093	0.6097	0.4850	0.5738	0.6867	0.6867	0.6885	0.6168
0.6303	0.5552	0.6044	0.6044	0.5669	0.6012	0.7543	0.6099	0.6739	0.6739	0.5489	0.6032
0.5709	0.5584	0.6400	0.6400	0.4906	0.579	0.4082	0.5696	0.8083	0.8083	0.5388	0.5903
0.5159	0.5499	0.3879	0.3879	0.8931	0.6419	0.9154	0.6387	0.2794	0.2794	0.5555	0.5834
0.5252	0.5449	0.5120	0.5120	0.5367	0.6208	0.6583	0.6427	0.5283	0.5283	0.1884	0.5044
0.7641	0.5888	0.6210	0.6210	0.8278	0.6622	0.1612	0.5464	0.7662	0.7662	0.4517	0.4939
0.6121	0.5934	1.0118	1.0118	0.3394	0.5977	0.7501	0.5871	0.9588	0.9588	0.6349	0.5221
0.6479	0.6043	0.6051	0.6051	0.7126	0.6206	0.8957	0.6488	0.6016	0.6016	0.5402	0.5257
0.5071	0.5849	0.6044	0.6044	0.5848	0.6135	0.7774	0.6746	0.5151	0.5151	0.4395	0.5085
0.2580	0.5195	0.4259	0.4259	0.8198	0.6547	0.8131	0.7023	0.7866	0.7866	0.1604	0.4388
0.3450	0.4846	0.8123	0.8123	0.1211	0.548	0.3192	0.6257	0.7745	0.7745	0.5426	0.4596

The four control limits of the proposed chart with $k_1 = 3.09$ and $k_2 = 1.10$ are given as follows: $LCL_1 = 0.3984$, $UCL_1 = 0.7697$, $LCL_2 = 0.5180$, and $UCL_2 = 0.6502$.

The proposed control chart is given in Fig. 1. It can be seen that the 46th sample is declared to be out of control, while the tabulated value of ARL_1 is about 22. Hence, the proposed control has the ability to detect shifts in the process as shown in Fig. 1. Now we see whether the existing control chart proposed by [14] detects shifts or not. The four control limits of the chart given by [9] with $k_1 = 3.09$ and $k_2 = 1.10$ are given as follows: $LCL_1 = 0.0272$, $UCL_1 = 1.1410$, $LCL_2 = 0.3858$ and $UCL_2 = 0.7824$.

The data are also plotted for the chart proposed by [11] in Fig. 2. It can be seen that all the values of M_t are within the control limits. The chart proposed in [11] shows that some values of the plotted statistic are within the repetition area, but did not indicate a shift in the process. Similarly, to prove the increased efficiency of the proposed control chart compared to the chart proposed by [7], the values of the statistic are plotted in Fig. 3 with $k = 3.0$, $LCL = 0.4039$ and $UCL = 0.7643$. From Figure 3, it can be seen that all the values of the simulated data are within the control limits. This chart shows no shift in the manufacturing process.

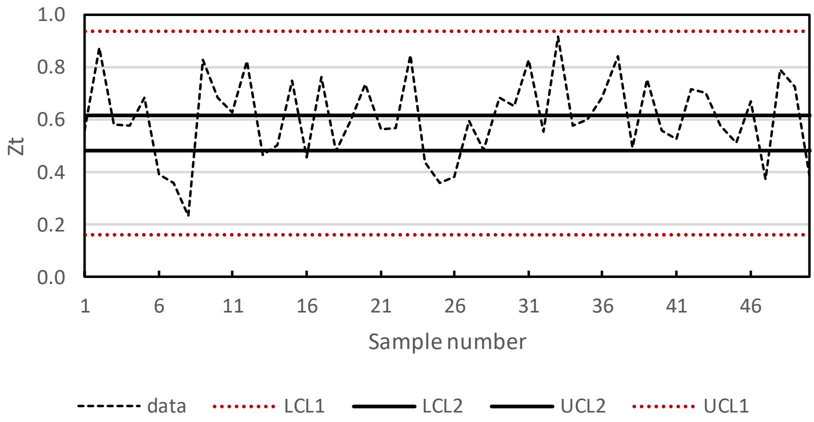

Fig. 2. Chart for simulated data based on the procedure from [9]

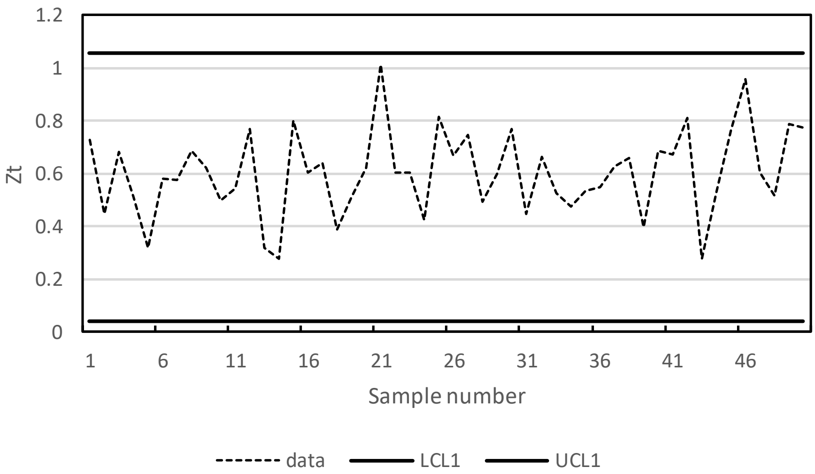

Fig. 3. Chart for simulated data based on the procedure from [7]

Hence, from Figures 1–3, it can be seen that the proposed control chart performs better than the two existing control charts in providing a quick indication of a shift in the process. Thus appropriate introduction of the proposed control chart will lead to a fall in the frequency of faulty goods. It has been noted that the chart proposed by [11] is better than the one proposed in [7], so we will only compare the proposed control chart with the one from [11].

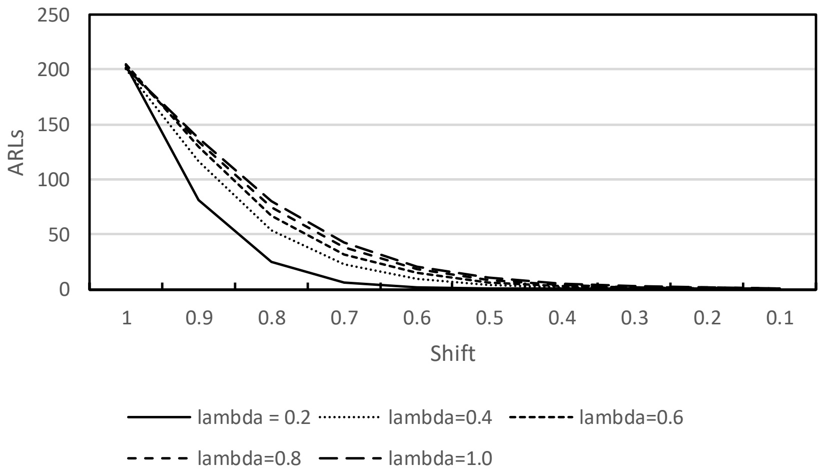

Fig. 4. Plots of ARLs for various values of λ when $ARL_0 \approx 200$

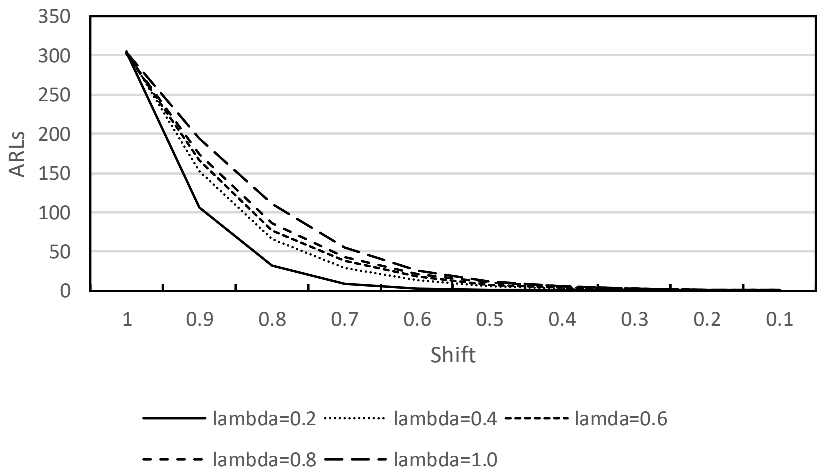

Fig. 5. Plots of ARLs for various values of λ when $ARL_0 \approx 300$

For this purpose, we compare the ARLs for various specified parameters, illustrated in Figs. 4–6. It can be seen from these curves that, in each case, the proposed control chart gives smaller values of the ARLs as compared to the chart defined in [11]. For example, from Fig. 4, it can be seen that the curve corresponding to the proposed chart is below the curves corresponding to the two other control charts, which shows that the proposed chart is better than the existing charts.

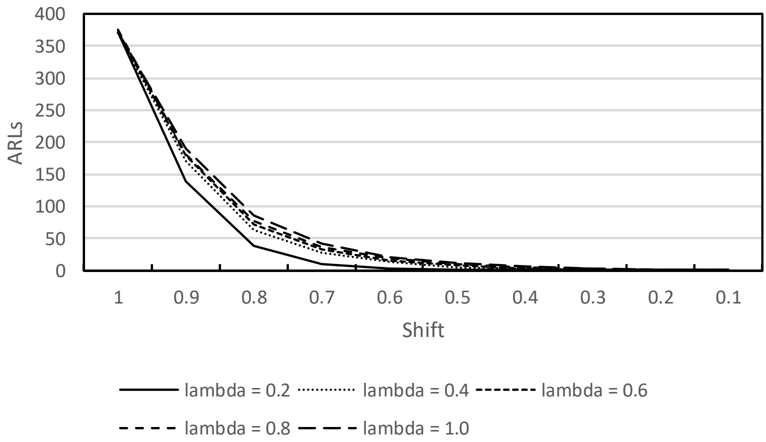

Fig. 6. Plots of ARLs for various values of λ when $ARL_0 \approx 370$

4. Industrial application

In this section, we will present the application of the proposed chart in healthcare by using data regarding urinary tract infections (UTIs). The same data were used by [7] and [9]. The use of control charts for monitoring long-term health are described in [15] and [16]. The data are taken from [7]. Let $r_0 = 370$. The values of the statistic M_t for the proposed control chart are plotted in Fig. 7. Note that several points UCL_2 and some points are below LCL_2, while the control chart defined in [7] shows that the process is under control.

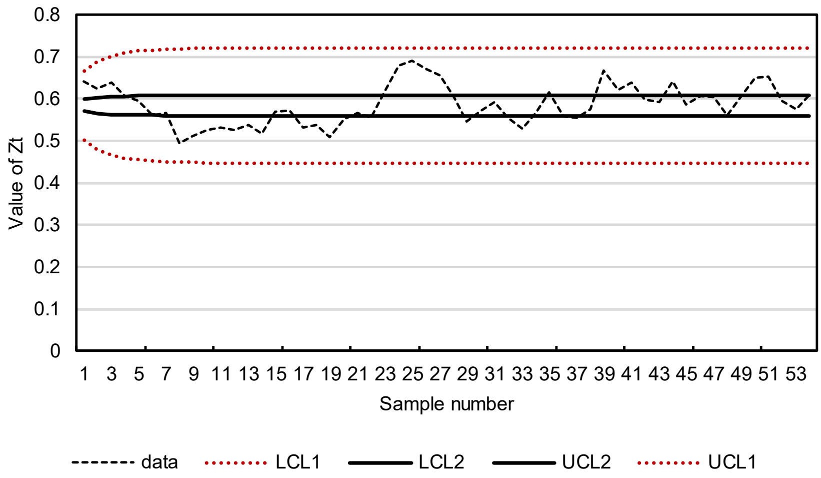

Fig. 7. Proposed chart for data describing a urinary tract infection

5. Concluding remarks

The proposed EWMA chart is designed to utilize repetitive sampling when the quantitative characteristic of interest follows the exponential distribution. The *ARLs* of the proposed chart are derived for under-control and out-of-control processes. The advantages of the proposed chart over two existing control charts are described. The proposed chart performs better than these existing charts in terms of *ARLs*, i.e. it provides swift indication about shifts in a process. An application of the proposed chart is illustrated with the help of real healthcare data for monitoring *UTIs*. The ability of the proposed chart to detect shifts in a process is also compared with other existing control charts on the basis of simulated data.

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