Bogdan RĘBIASZ[1]
Bartłomiej GAWEŁ[1]
Iwona SKALNA[1]

VALUING MANAGERIAL FLEXIBILITY. AN APPLICATION OF REAL-OPTION THEORY TO STEEL INDUSTRY INVESTMENTS

In the steel industry which is subject to significant volatility in its output prices and market demands for different ranges of products the diversification of production can generate important value for switch real options. Therefore, a common practice is to invest in various assets, thus generating the possibility of diversification of production and valuable switch options. The incremental benefit of product switch options in steel plant projects has been assessed. Such options are valued using the Monte Carlo simulation and modeling the prices of and demand for steel products as geometric Brownian motion (GBM). Our results show that this option can generate a significant increase in the net present value (NPV) of metallurgical projects.

Keywords: *real options, switch options, stochastic processes, investment decision, Monte Carlo simulation*

1. Introduction

Steel is a highly versatile alloy that can be adapted to suit a wide range of applications throughout industry. The abundance of the raw materials required for its production and its low production cost, give steel an advantage over other comparable materials, and make it the most important alloy in modern society [20]. However, the steel sector is one of the industries that are highly affected by market uncertainties, which can exert great influence on the process of making investment decisions, as well as on the profitability of the resulting investments.

The prices of and demand for steel products are subject to significant volatility. For example, the price of hot rolled plates varied from approximately USD 250.0/ton to

---

[1]AGH University of Science and Technology, ul. Gramatyka 10, 30-067 Krakow, Poland, e-mail addresses: brebiasz@zarz.agh.edu.pl, bgawel@zarz.agh.edu.pl, iskalna@zarz.agh.edu.pl

USD 1200.0/ton between 2000 and 2009 and the apparent consumption (production plus imports minus exports) of hot dip galvanised sheets in Poland varied from approximately 846.0 thousand ton to 1123.0 thousand ton between 2009 and 2015. This volatility stems in part from the fact that different steel products are consumed by distinct sectors of an economy, and even though correlated, can generate valuable product switch options [20, 22]. To take advantage of this flexibility, steel companies invest in various assets.

The flexibility linked to uncertainties regarding the prices of and demands for products cannot be captured by traditional valuation methodologies, such as discounted cash flow [1–3, 5–7, 9, 12, 13, 16, 18–20, 24, 25]. Instead, it can be valued as a real option, which appraises the correct present value of the available output switch option. Although it is intuitive that switch options seem to have significant value, it is important to know how to evaluate them, in order to correctly appraise investment projects [20].

Switch options refer here to changes in the raw materials used, products manufactured, and other production factors or an entire technological process. Such changes aim to adapt to changes in the market situation. They are synchronized with changes in the relation of the prices of raw materials to the prices of products on the market. In order to evaluate the incremental benefit of product switch options in a steel plant project, a Monte Carlo simulation is used in this paper. The behaviour of prices and demands is modelled as geometric Brownian motion (GBM). The valuation of switch options begins with a cash flow simulation, in which various product mixes are tested in diverse scenarios describing product demand and prices, taking into consideration capital expenditures that may be incurred in adapting the production process.

The case study presented here concerns the effect of the flexibility available to a producer of coated sheet that produces organic coated sheets or alternatively decreases production and sells hot dip galvanised sheets, when the price of organic coated sheets generates a negative cash flow or when the price of hot dip galvanised sheets is high enough to give a higher cash flow than from organic coated sheets.

The rest of the paper is organized as follows. In Section 2, a review of the bibliography on real-option valuation of projects is made. Section 3 presents the model and methodology of valuing a product switch option. Section 4 presents a case study together with the data and results obtained. Section 5 contains some concluding remarks.

2. Evaluation of investment projects using real options

The net present value (NPV) is one of the most common concepts used to determine the present value of an investment. It is computed as a sum of discounted net cash flows appearing in consecutive years of the economic life of an investment project. NPV indicates to investors the value added by a project to a company and allows projects to be

prioritized in situations involving several investment opportunities. However, in the presence of a high level of uncertainty and managerial flexibility, traditional valuation rules fail to provide comprehensive indicators for investment decisions. Thus, it is necessary to use additional tools that are able to opitmize managerial choices in an uncertain environment [9, 20]. The most appropriate tool for this purpose is the real options theory [2].

Real option theory was introduced by Myers [2] as a new area of financial research. The origin of real options was based on the idea that real assets (projects) could be evaluated in a similar way to financial options. Real options, in contrary to NPV, allow a project to be modified by subsequent decisions, once it has been undertaken. NPV makes no provision for the flexibility of a project and, consequently, undervalues its benefits.

Much interesting research on the real options theory concerns mining investments [3, 5, 8, 12, 24, 25, 27]. Slade [25] valued the managerial flexibility of investments in copper mining in Canada. He focused on flexible operations, stressing the fact that temporary shutdowns are more commonly observed than the opening or closing of operations. Brennan and Schwartz [5] defined the options of abandoning or temporarily shutting down mining projects. Another relevant contribution was made by Bjerksund and Ekern [3]. They evaluated a bundle of options and their interactions in the analysis of an oil field. Gligoric et al. [12] developed a model on the basis of a full analysis of the discounted cash flow for project regarding an underground zinc mine. Fuzzy interval grey system theory was used to forecast zinc prices, whereas GBM was used to forecast the operating costs over the time frame of the project.

Samis and Davis [24] used Monte Carlo simulation with both discounted cash flow and techniques for pricing real options to evaluate an actual project financing proposal for a small gold mine. Dimitrakopoulos [9] proposed a simulation-based method for assessing a real option value (ROV) that can handle multiple uncertainties, as well as the variability of cash flow that characterizes mining projects. Furthermore, the author presents an example for investigating the impact that ROV may have on the profitability of a project by improving the decision making process.

Wilimowska and Łukaniu [27] presented a binomial model for valuating a real option and its application to valuate defer and abandon options on the basis of an example of a hypothetical mining firm. Dessureault et al. [8] used real option pricing as a method for the flexible valuation of a mining project. They presented methods that can be used for calculating process and project volatility in operations and provides practical applications to mining operations in the USA and Canada.

The papers mentioned above are very interesting from the point of view of the practical use of real options in the mining industry. However, because of the specifity of the mining industry, they rather focus on defer and abandon options and do not analyze the problem of a switch option.

There is a considerable literature related to the application of the real options theory in the electrical power sector [13, 16]. Among the research carried out on switch options,

it is worth mentioning the work of Bastian-Pinto et al. [1], where the option to switch product output in an ethanol production plant is valued, more specifically, as the option to change the final product – sugar or ethanol. The authors calculate the value of the option, available to this industry, of delivering the product that maximizes revenue at each future period. When valuating such a switch option, they take into account the variability of prices and demand for sugar and ethanol. Prices are modelled using a model of correlated mean reverting processes and options are valuated using the Monte Carlo method. Based on an analysis of this research, it can be stated that the option of switching products has a similar effect in the metallurgical industry. However, the specifity of the latter must be taken into account.

Many interesting articles concern the metallurgical industry, especially the smelting of aluminum [2, 6, 7]. In [2], the authors studied the effect of the flexibility available to a typical smelter (aluminum processing plant) that buys its electricity through long term contracts or alternatively owns a co-generation unit, of stopping production and selling its electricity on the spot market, when the price of its output, aluminum, generates negative or unrewarding cash flows, or when the spot price of electricity is high enough to give a greater cash flow than from its normal production process.

There are also a few studies related to the steel sector and the analysis of real options. Muharam [18] values the real options of investment deferral, expansion and abandonment in the steel industry but does not consider product switch options in this sector. He analyses the risk faced by SMEs in the steel industry, aiming to exploit it in a different perspective. The author argues that ROV is capable of providing solutions to deal with the lack of strategic management practices in SMEs by providing guidelines for general applications of risk assessment to strategic planning. Ozorio et al. [20] value an output switch option in a hypothetical integrated steel plant composed of a blast furnace and a hot laminator. In their analyses, they take into account the variability of prices, but assume a constant demand. The variability of prices is modelled using Geometric Brownian Motion or a mean reverting process. The authors state that there is no consensus as to which stochastic process is more appropriate. They support the thesis of Dixit and Pindyck [10], that the definition of the process depends as much on statistical as on theoretical considerations. Results show that such an option can generate a significant increase in the NPV of an integrated steel plant.

3. Model and methodology

This section introduces the model and methodology for valuating the option of a product switch. Firstly, the modelling of uncertainty using the geometric Brownian motion has been described. Next, a Monte Carlo simulation for estimating the value of such a product switch option has been presented.

3.1. Modeling uncertainty by geometric Brownian motion (GBM)

Geometric Brownian motion (GBM) is usually used to describe stock prices. It is the logarithmic transformation of a continuous-time random walk [10, 21, 23]. In the literature related to capacity planning [11] and commodity prices [1, 3, 17, 19], the GBM has been used to describe demand. GBM is a special case of Brownian motion or a Wiener process. The variable q follows GBM if it satisfies the following diffusion equation [1, 16, 20, 26]:

$$dq_t = \mu q_t\, dt + \sigma q_t\, dw_t \qquad (1)$$

where $dw_t = \varepsilon_t \sqrt{dt}$ is the standard increment of a Wiener process, whereas μ and σ are, respectively, the drift parameter and the standard deviation parameter. The random innovations ε_t are identically distributed and independent (iid) variables from the standard normal distribution.

Based on the simple derivation below, the differences between the q_t after logarithmic transformation follow a normal distribution, which implies that the q_t themselves follow the lognormal distribution [5, 16, 26].

$$\frac{dq_t}{q_t} = \mu dt + \sigma dw_t \qquad (2)$$

$$d \ln\left(q_t \right) = \left(\mu - \frac{\sigma^2}{2} \right) dt + \sigma dw_t \qquad (3)$$

$$\ln\left(q_T \right) - \ln\left(q_t \right) = \ln\left(\frac{q_T}{q_t} \right) \sim N\left[\left(\mu - \frac{\sigma^2}{2} \right)(T-t),\ \sigma^2 (T-t) \right] \qquad (4)$$

For simplicity, let r_t stand for the difference between q_t and q_{t-1} based on a finite interval after logarithmic transformation:

$$r_t = \ln\left(\frac{q_t}{q_{t-1}} \right) \qquad (5)$$

Let Δ denote the length of time between two observations, where the length of time between q_t and q_{t-1} is normalized to be equal to one. The distribution of r_t follows a normal distribution with the parameters defined below:

$$r_t \sim N\left[\left(\mu - \frac{\sigma^2}{2}\right)\varDelta,\ \sigma^2\varDelta\right] \tag{6}$$

Based on the relations constructed above, the drift and variance parameters of GBM can be estimated from the data using the sample mean $\bar{r}$ and the standard deviation s_r. A detailed derivation is presented below [5, 16, 26]:

$$r_t = \ln(q_t) - \ln(q_{t-1}) \tag{7}$$

$$E(r_t) = \left(\mu - \frac{\sigma^2}{2}\right)\varDelta \tag{8}$$

$$V(r_t) = \sigma^2\varDelta \tag{9}$$

$$\bar{r} = \frac{\sum_{t=1}^{n} r_t}{n} \tag{10}$$

$$s_r = \sqrt{\frac{1}{n-1}\sum_{t=1}^{n}(r_t - \bar{r})^2} \tag{11}$$

$$\hat{\sigma} = \frac{s_r}{\sqrt{\varDelta}} \tag{12}$$

$$\hat{\mu} = \frac{\bar{r}}{\varDelta} + \frac{\hat{\sigma}^2}{2} = \frac{\bar{r}}{\varDelta} + \frac{s_r^2}{2} \tag{13}$$

Once the parameters have been estimated, sample paths can be generated based on the results derived from Ito's lemma [23].

$$\ln(q_t) = \ln(q_0) + (\mu - \frac{\sigma^2}{2})t + \sigma w_t \tag{14}$$

$$q_t = q_0 \exp\left[\left(\mu - \frac{\sigma^2}{2}\right)t + \sigma w_t\right] = q_0 \exp\left[\left(\mu - \frac{\sigma^2}{2}\right)t + \varepsilon_t \sigma \sqrt{t}\right] \tag{15}$$

where

$$w_t - w_0 \sim N(0, t) \tag{16}$$

Hence,

$$q_t = q_{t-1} \exp\left[\left(\mu - \frac{\sigma^2}{2}\right)\Delta + \varepsilon_t \sigma \sqrt{\Delta}\right] \tag{17}$$

3.2. Estimating the risk-premium

When estimating the product switch option value, a Monte Carlo simulation must be carried out under the assumption that the random variables involved follow a risk-neutral GBM. These simulations are performed as follows [1, 4, 14]:

$$q_t = q_{t-1} \exp\left[\left(\mu - \pi - \frac{\sigma^2}{2}\right)\Delta + \varepsilon_t \sigma \sqrt{\Delta}\right] \tag{18}$$

Risk-premium (π) estimation can be done as described in Hull [14] and applied in several studies, such as Irwin [15] and Blank et al. [4]. This approach estimates the risk premium using the correlation between the underlying asset return and market return, together with their volatilities [4, 14, 15, 20], i.e.:

$$\pi_i = \frac{\rho_{im}\delta_i\pi_m}{\delta_m} \tag{19}$$

where: π_i – risk-premium of the price (demand) process of asset i, ρ_{im} – correlation between the market return and asset return, δ_i – volatility parameter for asset i, δ_m – market volatility parameter, π_m – market risk premium.

4. Case study of a product switch option

4.1. Definition of the problem

In this section, the model used for valuating product switch options is defined. Using this model and Monte Carlo simulation, a product switch option in a hypothetical production setup (see Fig. 1) is valued.

In this case, the effectiveness of the project – the construction of a new organic-coated sheet (OC sheet) plant – is analyzed. This plant can produce OC sheet – a product made from hot-dip galvanised sheets (HDG sheet) with greater added value and several uses. For the analyzed production setup, cold-rolled sheets (CR sheets) are the basic feedstock. CR sheets are converted into HDG sheets. HDG sheets are partly sold and partly converted into OC sheets. The latter are all sold. Steel scrap is waste in the production of HDG sheets and OC sheets and is sold.
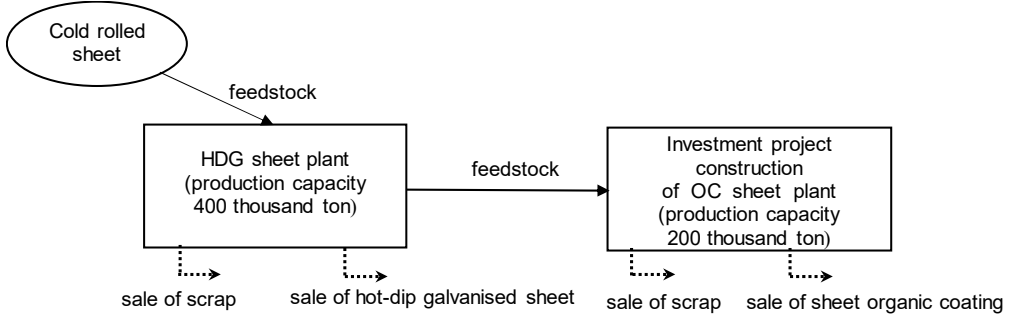

Fig. 1. Description of the analyzed production setup

Firstly, the basic case is analyzed. It consists of a standard valuation of a cash flow from which a static NPV (Eq. (20)) for the projected construction of a new OC sheet plant is obtained. The annual cash flows, used to calculate the static NPV, obtained by investing in the OC plant can be estimated from Eqs. (21)–(41):

$$NPV = \frac{\sum_{t=0}^{T} ICF_t}{\left(1 + r_{ris}\right)^t} - I \tag{20}$$

$$ICF_t = \left(ZNo_t + DAh + DAo + ZKOo_t\right) \\ - \left(ZNh_t + DAh + ZKOh_t\right) \text{ for } t = 0, 1, \ldots, T-1 \tag{21}$$

$$ICF_t = \left(ZNo_t + DAh + DAo + ZKOo_t + RVo_t\right) \\ - \left(ZNh_t + DAh + ZKOh_t + RVh_t\right) \text{ for } t = T \tag{22}$$

$$ZNo_t = SPo_t - Ko_t - \max\left(SPo_t - Ko_t, 0\right)Ta \text{ for } t = 0, 1, \ldots, T \tag{23}$$

$$ZNh_t = SPo_t - Kh_t - \max\left(SPh_t - Kh_t, 0\right)Ta \text{ for } t = 0, 1, \ldots, T \tag{24}$$

$$SPo_t = SRo_t \times So_t + SRh(2)_t \times Sh_t \text{ for } t = 0, 1, \ldots, T \tag{25}$$

$$SPh_t = SRh(1)_t \times Sh_t \text{ for } t = 0, 1, \ldots, T \tag{26}$$

$$SRo_t = \min(SFo_t; CAPo) \text{ for } t = 0, 1, \ldots, T \tag{27}$$

$$SRh(1)_t = \min(SFh_t; CAPh) \text{ for } t = 0, 1, \ldots, T \tag{28}$$

$$SRh(2)_t = \min (SFh(1)_t; (CAPh - \text{Mo} \times SRo_t)) \text{ for } t = 0, 1, \ldots, T \tag{29}$$

$$SFh_t = ACh_t \times MSh \text{ for } t = 0, 1, \ldots, T \tag{30}$$

$$SFo_t = ACo_t \times MSo \text{ for } t = 0, 1, \ldots, T \tag{31}$$

$$Ko_t = SRo_t \times Cc(2)_t - SRh(2)_t \times Cc(1)_t - OPCh \times SRh(2)_t$$
$$- SRo_t \times OPCo - DAh - DAo - GAh - GAo \text{ for } t = 1, 2, \ldots, T \tag{32}$$

$$Kh_t = SRh(1)_t \times Cc(1)_t - OPCh \times SRh(1)_t - DAh - GAh \text{ for } t = 0, 1, \ldots, T \tag{33}$$

$$Cc(1)_t = Mc \times Sc_t - (Mc - 1) \times Sz_t \text{ for } t = 0, 1, \ldots, T \tag{34}$$

$$Cc(2)_t = Mc \times Mh \times Sc_t - (Mc \times Mh - 1) \times Sz_t \text{ for } t = 0, 1, \ldots, T \tag{35}$$

$$ZKOo_t = KOo_t - KOo_{t-1} \text{ for } t = 0, 1, \ldots, T \tag{36}$$

$$ZKOh_t = KOh_t - KOh_{t-1} \text{ for } t = 0, 1, \ldots, T \tag{37}$$

$$KOo_t = \frac{SPo_t}{Sq} + \frac{SPo_t}{Cna} + \frac{Ko_t - DAo_t - DAh_t}{Cz}$$
$$- \frac{Ko_t - DAo_t - DAh_t}{Czb} \quad \text{for } t = 0, 1, \ldots, T \tag{38}$$

$$KOh_t = \frac{SPh_t}{Sq} + \frac{SPh_t}{Cna} + \frac{Kh_t - DAh}{Cz} - \frac{Kh_t - DAh}{Czb} \quad \text{for } t = 0, 1, \ldots, T \tag{39}$$

$$RVo_t = \frac{SPo_t}{Sq} + 0.7\frac{SPo_t}{Cna} + 0.7\frac{Ko_t - DAo_t - DAh_t}{Cz}$$

$$-\frac{Ko_t - DAo_t - DAh_t}{Czb} \text{ for } t = T \tag{40}$$

$$RVh_T = \frac{SPh_t}{Sq} + 0.7\frac{SPh_t}{Cna} + 0.7\frac{Kh_t - DAh_t}{Cz} - \frac{Kh_t - DAh_t}{Czb} \text{ for } t = T \tag{41}$$

where:

ICF_t – cash flow in year t for the project – construction of a new OC sheet plant,
r_{ris} – weighted average cost of capital,
I – capital expenditure on the project,
ZNo_t – net profit in year t in the scenario where the project is implemented,
ZNh_t – net profit in year t in the scenario where the project is not implemented,
SPo_t – revenue in year t in the scenario where the project is implemented,
SPh_t – revenue in year t in the scenario where the project is not implemented,
Ko_t – total cost in year t in the scenario where the project is implemented,
Kh_t – total cost in year t in the scenario where the project is not implemented,
$CAPh$ – installed capacity of the HDG sheet plant,
$CAPo$ – installed capacity of the OC sheet plant,
SFh_t – sales forecast for HDG sheets in year t,
SFo_t – sales forecast for OC sheets in year t,
MSh – market share for HDG sheets,
MSo – market share for OC sheets,
ACh_t – forecasted apparent consumption of HDG sheets in year t,
ACo_t – forecasted apparent consumption of OC sheets in year t,
SRo_t – sale of OC sheets realized in year t in the scenario where the project is implemented,
$SRh(1)_t$ – sale of HDG sheets realized in year t in the scenario where the project is not implemented,
$SRh(2)_t$ – sale of HDG sheets realized in year t in the scenario where the project is implemented,
Sh_t – price of HDG sheet per ton in year t,
So_t – price of OC sheet per ton in year t,
Sc_t – price of CR sheet per ton in year t,
Sz_t – price of steel scrap per ton in year t,
$Cc(1)_t$ – cost of CR sheet per ton of HDG sheet in year t,
$Cc(2)_t$ – cost of CR sheet per ton of OC sheet in year t,
Mc – per unit consumption of CR sheet when producing HDG sheet,

Mh – per unit consumption of HDG sheet when producing OC sheet,

$OPCh$ – other (with the exception of the cost of CR sheet) annual variable production costs per ton for HDG sheet,

$OPCo$ – other (with the exception of the cost of CR sheet) annual variable production costs per ton for OC sheet,

GAh – annual fixed costs for HDG plant,

GAo – incremental annual fixed costs for OC plant,

DAh – annual amortization for HDG plant,

DAo – annual amortization for OC plant,

KOo_t – net working capital in year t in the scenario where the project is implemented,

KOh_t – net working capital in year t in the scenario where the project is not implemented,

$ZKOo_t$ – change in net working capital in year t in the scenario where the project is implemented,

$ZKOh_t$ – change in net working capital in year t in the scenario where the project is not implemented,

RVo_t – residual value in year t in the scenario where the project is implemented,

RVh_t – residual value in year t in the scenario where the project is not implemented,

Sq – cash in hand turnover,

Cz – inventory turnover,

Czb – debtor turnover,

Cna – receivables turnover,

T – economic life-cycle of project,

Ta – tax

Equations (21) and (22) are used to compute the cash flow for the project – construction of a new OC sheet plant – in successive years of the project's life-cycle. Equation (21) concerns the years 0, 1, ..., $T – 1$, whereas Eq. (22) concerns the last year T. Cash flow is computed according to an incremental relationship, i.e., the cash flow which would arise without taking into account the investment is subtracted from the cash flow which takes into account the construction of a new OC sheet plant.

Equations (23) and (24) show how to compute the net profit in the scenario where the project is implemented and in the scenario where the project is not implemented, respectively. The key problem in computing these profits is determining the revenue and total costs resulting from the analyzed scenarios.

The revenue is computed using Eqs. (25)–(31). The revenue in the scenario where the project is implemented is computed using Eq. (25). Equations (27) (29) and (31) enable computing the level of sales in this case. The revenue in the scenario where the project is not implemented is computed using Eq. (26). Equations (28) and (30) are used to compute the level of sales in this case.

When computing the static NPV, the amount of OC sheets sold (in the scenario where the project is implemented (SRo_t)) is determined as the minimum of the following two values:

- the product of the forecasted apparent consumption of OC sheets and market share,
- the available capacity for producing OC sheets.

The amount of HDG sheets sold in this scenario ($SRh(2)_t$) is determined as the minimum of the following two quantities:

- the product of the forecasted apparent consumption of HDG sheets and market share,
- the available capacity for producing HDG sheets minus the usage of HDG sheets to produce OC sheets.

In this case, the greatest possible sales of OC sheets are realized according to production capacity and market conditions. The sales of HDG sheets stem from the market conditions, production capacities and the amount of HDG used to produce OC sheets. On the other hand, the level of sales of HDG sheets in the scenario where the project is not implemented ($SRh(1)_t$), is determined as the minimum of the two following quantities:

- the product of the forecasted apparent consumption of HDG sheets and market share,
- the available capacity for producing HDG sheets.

The remaining quantities required to calculate the NPV are computed based on the revenue resulting from these sales.

Equations (32) (33) are used to compute the total costs, and Eqs. (34) and (35) to compute the cost of materials, for both of the scenarios analyzed.

Equations (37)–(39) enable assessing the change in the level of working capital in each year of the analyzed scenarios. The level of working capital is computed as a function of the levels of cash in hand turnover, inventory turnover, debtor turnover and receivables turnover.

Equations (40) and (41) are used to compute the residual value for the analyzed scenarios. The residual value is computed according to the Wilcox formula, which determines that the residual value is equal to [28]:

- 100% of the value of means of payment,
- 70% of the book value of supplies,
- 70% of the book value of debts,
- –100% of the value of liabilities.

The static NPV is calculated N times using Monte Carlo simulation according to Eq. (20). The result gives an estimate of the probability distribution of the static NPV. The GBM stochastic process defined by Eq. (17) is used to define the underlying uncertainty. The uncertainty of demand for HDG sheets, OC sheets and the uncertainty of prices for CR sheets, HDG sheets, OC sheets and scrap are taken into consideration. Such a calculation takes into account the correlations between the prices of the follow-

ing products: steel scrap, CR sheets, HDG sheets and OC sheets. The correlation be-
tween the apparent consumption of HDG sheets and OC sheets is also taken into account
in the calculating procedure.

Next, the switch option is valued. The analysis of the static NPV of the project
does not take into account the managerial flexibility of being able to switch the output
product. In some periods, the production of HDG sheet may be a more interesting and
profitable alternative to the company than the production of OC sheet. Therefore, in
such cases the switch option is realized. The largest possible sales of HDG sheets are
realized according to the available production capacities and market conditions. The
sales of OC sheets stem from market conditions, production capacities and the avail-
ability of feedstock, i.e., HDG sheets. When the product switch option is realized, the
level of sales of OC sheets ($SRo(3)_t$) is determined as the minimum of the following
quantities:

- the product of the forecasted apparent consumption of OC sheet and market share,
- available capacity for producing OC sheets,
- available feedstock in the form of HDG sheets.

The sales of HDG sheets in this case is determined as the minimum of the following
quantities:

- the product of the forecasted apparent consumption of HDG sheets and market
share,
- capacity for producing HDG sheets, i.e. it is assumed to equal $SRh(1)_t$.

The values of product switch options can be obtained by simulating the incremental
cash flows defined for the level of sales of OC sheets and HDG sheets discussed above
in relation to the cash flow defined according to the conditions assumed in the calcula-
tion of the static NPV. The following equations are used to compute the value of the
switch option:

$$OPT_0 = \sum_{t=1}^{T} \frac{ICF_t^*}{\left(1+r_f\right)^t} \tag{42}$$

OPT_0 − value of the switch option at time 0, r_f − risk free rate, ICF_t^* − incremental cash
flow related to the product switch option in year t,

$$ICF_t^* = \max\left(\left(ZNo(1)_t + DAo + DAh + ZKOo(1)_t\right.\right.$$
$$\left.\left. - \left(ZNo_t + DAo + DAh + ZKOo_t\right); 0\right)\text{for } t = 1, 2, \ldots, T-1 \tag{43}$$

$$ICF_t^* = \max\left(\left(ZNo(1)_t + DAo + DAh + ZKOo(1)_t + RVo(1)_t\right)\right.$$

$$\left.-\left(ZNo_t + DAo + DAh + ZKOo_t + RVo(1)_t\right); 0\right) \text{for } t = T \tag{44}$$

The values of $ZNo(1)_t$, $ZKOo(1)_t$, $RVo(1)_t$ correspond respectively to the net profit, change in working capital and residual value after realizing the product switch option. They are computed based on the values of SPo_t, and Ko_t which are given by the following formulas:

$$SPo_t = SRo(3)_t \times So_t + SRh(1)_t \times Sh_t \tag{45}$$

$$Ko_t = SRo(3)_t \times Cc(2)_t - SRh(1)_t \times Cc(1)_t - OPCh \times SRh(1)_t$$

$$- SRo(3)_t \times OPCo - DAh - DAo - GAh - Gao \tag{45}$$

$$SRo(3)_t = \min\left[SFo_t; CAPo; CAPh - \frac{SRh(1)_t}{Mh}\right] \tag{47}$$

The remaining parameters necessary to calculate the ICF_t^* are defined according to the equations shown above in the calculation of the static NPV.

Nevertheless, both the simulations and discount rate must assume risk neutrality, as when valuing options, the level of risk will change when these options are exercised. Thus, we must use the risk-free rate for discounting the incremental cash flows when the option is exercised, but these must be simulated using a risk-neutral expectation. Therefore, the GBM stochastic process defined by Eq. (18) is used here to model the underlying uncertainty.

It is important to mention that it is assumed that the flexibility of switching output back and forth does not involve any operating cost for the analyzed plant. Also, the status of the output product at any moment in time is only dependent on the free cash flow at that time. This enables flexibility to be valued as a bundle of European call options, for which a Monte Carlo simulation approach is applicable.

4.2. Data used in the calculation

The data used in the analysis are presented in Figs. 2–3 and Table 1. Figure 2 shows the prices (in real terms) of CR sheet, HDG sheet, OC sheet and scrap. Figure 3 shows the apparent consumption of HDG sheet and OC sheet. Table 1 presents the correlation matrix for the prices of the analyzed products.
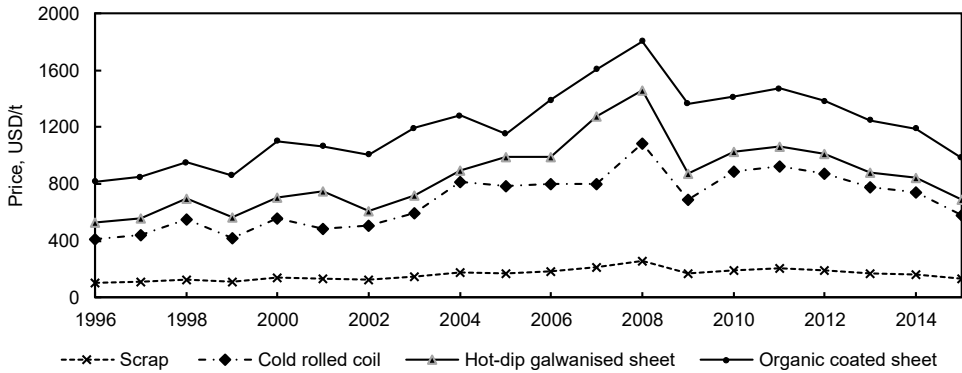
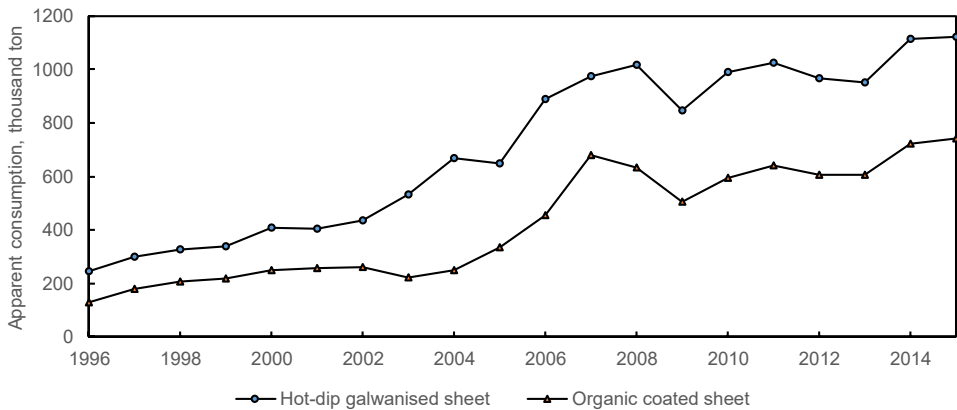Fig. 2. Prices of the analyzed products (in real terms)

Fig. 3. Apparent consumption of HDG sheet and OC sheet

Table 1. Correlation matrix for the prices of the analyzed products

	Scrap	*CR* sheet	HDG sheet	OC sheet
Scrap	1.000	0.930	0.952	0.936
CR sheet	0.930	1.000	0.839	0.809
HDG sheet	0.952	0.839	1.000	0.828
OC sheet	0.936	0.809	0.828	1.000

The correlation coefficient for the levels of apparent consumption of HDG sheet and OC sheet is 0.501. The weighted average cost of capital, considering the financial leverage usual in this industry, for this type of steel plant is estimated to be 10% per year as a real rate and the risk free rate is 5% [20].

Analysis of the historical time series of the price of OC sheet (from 1996 to 2015 in real terms – Fig. 1) shows an average growth rate of 0.98% per year. For the prices of other products, the average growth rate is as follows: HDG sheet – 1.60%, CR sheet

– 1.86%, scrap – 1.74%. Based on this, we estimate an annual price growth rate of 1.5% in common for all of the products, as it is assumed that this increase was partly a result of the structural changes which occurred in the sector during the economic boom, which occurred in 2006–2007.

Analysis of the historical series of the apparent consumption of OC sheet and HDG sheet (from 1996 to 2015, Fig. 2) shows an average growth rate of 8.00% per year for HDG sheet and 9.23% for OC sheet. Based on this, we use 7.00% as the annual increase in demand for all products, as it is assumed (as above) that this increase was partly a result of the structural changes which occurred in the sector during the economic boom, which occurred in 2006–2007.

The volatility parameter σ was estimated by the standard deviation of the log return of the price series and demand series shown, respectively, in Figs. 1 and 2 according to Eqs. (10)–(12). These values are equal to:

for prices:

scrap	16.37%,
CR sheet	21.09%,
HDG sheet	19.92%,
OC sheet	13.70%;

for apparent consumption:

HDG sheet	11.97%,
OC sheet	16.64%.

The other variable production costs (energy, manpower and maintenance) amount to approximately USD 114.0 per ton of HDG sheet and USD 174.0 per ton of OC sheet. The fixed costs for a HDG sheet plant are estimated as USD 31.08 million per year. The incremental fixed costs for an OC sheet plant are estimated as USD 12.95 million per year. The initial investment for an OC sheet plant amounts to approximately USD 40 million. The market share for HDG sheet was estimated to be 30% and for OC sheet to be 25%.

Using the methodology described above (Eq. (19)), the premiums (π) were estimated as follows:

for prices:

scrap	0.81%,
CR sheet	0.79%,
HDG sheet	1.01%,
OC sheet	0.75%;

for apparent consumption:

HDG sheet	1.32%,
OC sheet	2.11%.

On the basis of these assumptions, the simulations were realized and the static *NPV* and the value of a product switch option for the project were calculated.

4.3. Results and discussion

Figures 4 and 5, respectively, show the probability distribution of the static *NPV* and the probability distribution of the value of a product switch option for the project.
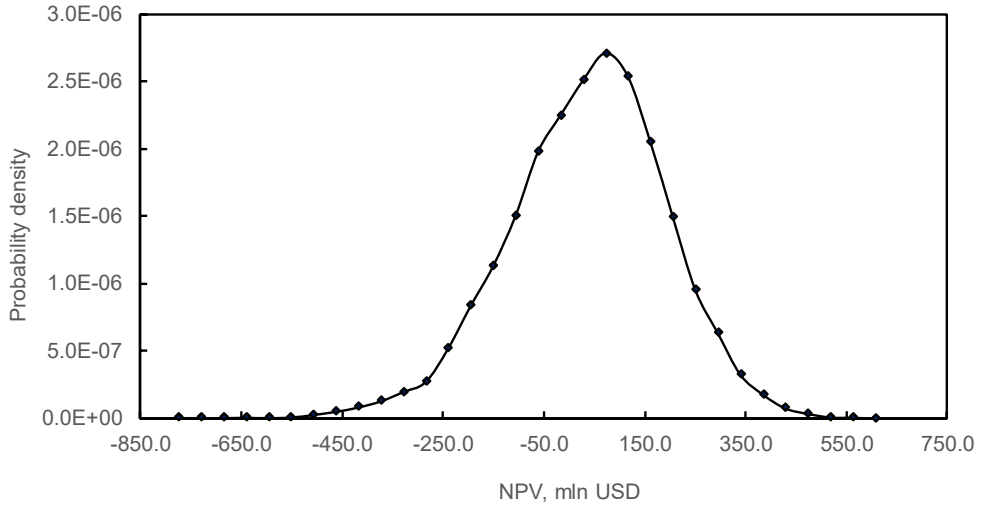

Fig. 4. Static NPV of the project *Construction of a new OC plant*; mean: USD 13 605.0 thousand
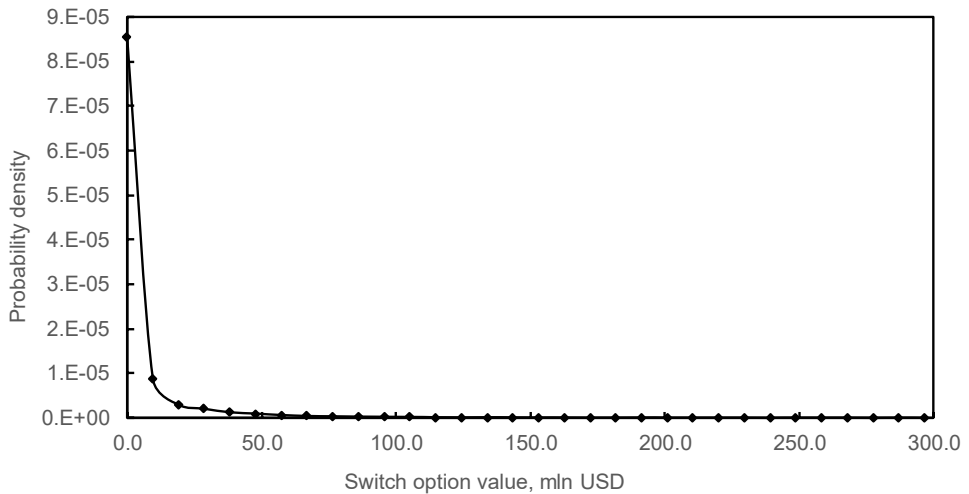

Fig. 5. The value of a product switch option of the project *Construction of a new OC plant*; mean: USD 4648.8 thousand

Table 2 summarizes the values found for the project when a product switch option is taken into account.

Table 2. The value of a product switch option and static NPV
for the project – construction of a new OC sheet plant

OC plant static NPV, thousand USD	13 605.0
OC plant extended NPV, thousand USD	18 253.8
Product switch option value, thousand USD	4648.8
Increase in comparison to OC static NPV, %	34.17%

Tables 3–6 demonstrate the sensitivity of the value of the option to variations in the following parameters: the drifts and volatilities. Table 3 demonstrates the sensitivity of the value of the switch option assuming various values of the drift parameters for the GBM process describing price variation. Table 4 demonstrates the sensitivity of the value of the switch option assuming various values of the volatility parameters for the GBM process describing price variation. Table 5 demonstrates the sensitivity of the value of the switch option assuming various values of the drift parameters for the GBM process describing the variation in demand. Table 6 demonstrates the sensitivity of the value of the switch option assuming various values of the volatility parameters for the GBM process describing the variation in demand.

Table 3. Sensitivity of the value of a product switch option
(thousand USD) assuming various values of the drift parameters
for the GBM process describing price variation

HDG sheet price drift	OC sheet price drift		
	0.5%	1.5%	2.5%
0.5%	4504.4	1176.5	248.6
1.5%	5182.7	3641.6	1232.4
2.5%	7261.0	5218.8	2664.5

Table 4. Sensitivity of the value of a product switch option
(thousand USD) assuming various values of the volatility parameters
for the GBM process describing price variation

HDG sheet price volatility	OC sheet price volatility		
	12.0%	14.0%	16.0%
18%	3 890,9	4 678,1	5 972,8
20%	4 426,7	4 888,1	6 373,4
22%	4 732,4	5 106,0	7 183,8

Table 5. Sensitivity of the value of a product switch option
(thousand USD) assuming various values of the drift parameters
for the GBM process describing variation in demand

HDG sheet demand drift	OC sheet demand drift		
	1.0%	2.0%	3.0%
1.0%	4124.8	4024,7	3905,5
2.0%	4749.1	4639.6	4512.5
3.0%	5184.1	5004.1	4708.0

Table 6. Sensitivity of the value of a product switch option
(thousand USD) assuming various values of the volatility parameters
for the GBM process describing variation in demand

HDG sheet demand volatility	OC sheet demand volatility		
	7.0%	9.0%	11.0%
6.0%	1201,9	2417,6	4192,9
8.0%	1218,8	2756,9	4715,8
10.0%	1221,3	3160,3	5379,9

As can be seen in Table 2, assuming that the prices of the analyzed products follow GBM, the value of a product switch option represents 34.17% of the static NPV. In Tables 3 and 5, it can be observed that the value of a product switch option increases as the drift of the price of and demand for HDG sheet increases relative to the drift of the price of and demand for OC sheets.

Tables 4 and 6 show that the value of the switch option increases as the volatility of the prices of and demand for the analyzed products increases.

The sensitivity analysis shows that product switch options have a significant relevance to the evaluation of a project in the steel industry for a wide range of values of the parameters describing the underlying stochastic processes.

5. Conclusions

Investment decisions in industrial plants are taken in an uncertain environment, where the variability of prices and the quantities demanded have the capacity to significantly affect the economic performance of projects.

The proposed model of a product switch option was tested on a hypothetical example, which is similar to many real case studies in the metallurgical industry and the experimental results verify the rationality and effectiveness of the method.

This paper estimated the projected cash flows, which are dependent on fluctuations in demand and prices, and calculated the value of the option available to this industry of delivering the product that maximizes revenue in each future period. Using a GBM approach, the authors concluded that this option significantly increases the NPV of an OC sheet production plant and considerably enhances the value created by metallurgical projects for shareholders.

Furthermore, it is possible to observe that the value of the product switch option increases when the drift of the price of and demand for HDG plate is greater than the drift of the price of and demand for OC plates and as the volatility of demand and prices increases. The sensitivity analysis shows that product switch options have a significant relevance to the evaluation of a project in the metallurgical industry for a wide range of values of the parameters describing the underlying stochastic processes.

References

[1] Bastian-Pinto C., Brandão T., Hahn W.J., *Flexibility as a source of value in the production of alternative fuels. The ethanol case*, Energy Economics, 2009, 31 (3), 411–422.

[2] Bastian-Pinto C., Eduardo L., Brandão T., Raphael R., Ozorio L.M., *Financial Valuation of Operational Flexibilities in the Aluminum Industry Using Real Option Theory*, [in:] Proc. 17th Annual International Conference Real Options: Theory Meets Practice, Tokyo 2013, 1–23.

[3] Bjerksund P., Ekern S., *Managing investment opportunities under price uncertainty. From last chance to wait and see strategies*, Fin. Manage., 1990, 19 (3), 65–83.

[4] Blank F.F., Baydia T.K.N., Dias M.A., *Private infrastructure investment through public private partnership. An application to a toll road highway concession in Brazil*, 13th Annual International Conference on Real Options, Braga, Portugal, Santiago, Spain, 2009, 1–21.

[5] Brennan M.J., Schwartz E.S., *Evaluating natural resource investments*, J. Business, 1985, 58 (2), 135–157.

[6] Byko M., *TMS Plenary Symposium. Energy Reduction in the Aluminum Industry*, J. Miner., Met. Mater. Soc., 2002, 54 (5), 39–40.

[7] Das S., Long W., Hayden H., Green J., Hunt W., *Energy implications of the changing world of aluminum metal supply*, J. Miner., Met. Mater. Soc., 2004, 56 (8), 14–17.

[8] Dessureault S., Kazakidis V.N., Mayer Z., *Flexibility valuation in operating mine decisions using real options pricing*, Int. J. Risk As. Manage., 2007, 7 (5), 656–674.

[9] Dimitrakopoulos R., Sabour S.A.A., *Evaluating mine plans under uncertainty. Can the real options make a difference?*, Res. Pol., 2007, 32 (3), 116–125.

[10] Dixit A.K., Pindyck R.S., *Investment under Uncertainty*, Princeton University Press, 1994.

[11] Duley B.D.L.J., Johnson B.E., *As Good as It Gets. Optimal Fab Design and Deployment*, IEEE Trans. Sem. Manuf., 1999, 12 (3), 281–287.

[12] Gligoric Z., Kricak L., Beljic C., Lutovac S., Milojevic J., *Evaluation of underground zinc mine investment based on fuzzy-interval grey system theory and geometric Brownian motion*, J. Appl. Math., 2014, article ID 914643, 1–12.

[13] Herbelot O., *Option valuation of flexible investments. the case of environmental investments in the electric power industry*, Department of Nuclear Engineering Massachusetts Institute of Technology, 1992 (https://dspace.mit.edu/bitstream/handle/1721.1/50208/35720932.pdf?sequence = 1).

[14] Hull J.C., *Options, futures and other derivatives securities*, 6th Ed., Prentice Hall, Englewood Cliffs 2006.

[15] Irwin T., *Public Money for Private Infrastructure. Deciding When to Offer Guarantees, Output-Based Subsidies, and Other Fiscal Support*, World Bank Working Paper, 2003 (https://openknowledge.worldbank.org/handle/10986/15117 License: CC BY 3.0 IGO).

[16] Marathe R., Ryan S.M., *On the validity of the geometric Brownian motion assumption*, Eng. Econ., 2005, 50 (2), 1–40.

[17] Moreira A., Rocha K., David P., *Thermopower generation investment in Brazil. Economic conditions*, En. Policy, 2004, 32 (1), 91–100.

[18] Muharam F.M., *Assessing risk for strategy formulation in steel industry through real options analysis*, J. Global Strat. Manage., 2011, 5 (2), 5–15.

[19] Myers S., *Determinants of corporate borrowing*, J. Fin. Econ., 1977, 5 (2), 147–175.

[20] Ozorio L.M., Bastian-Pinto C., Baidya T.K.N., Brandão T., *Investment decision in integrated steel plants under uncertainty*, Int. Rev. Fin. Anal., 2013, 27 (3), 55–64.

[21] Pindyck R.S., Rubinfeld D., *Econometric Models and Economic Forecasts*, McGraw-Hill, 1998.

[22] Rebiasz B., *Polish steel consumption 1974–2008*, Res. Pol., 2006, 31 (1), 37–49.

[23] Ruey S.T., *Analysis of Financial Time Series*, Wiley, New York 2002.

[24] SAMIS M., DAVIS G.A., *Using Monte Carlo simulation with DCF and real options risk pricing techniques to analyse a mine financing proposal*, Int. J. Fin. Eng. Risk Manage., 2014, 1 (3), 264–281.

[25] SLADE M., *Valuing managerial flexibility. An application of real-option theory to mining investments*, J. Environ. Econ. Manage., 2001, 41 (2), 193–233.

[26] WATTANARAT V., PHIMPHAVONG P., MATSUMARU M., *Demand and Price Forecasting Models for Strategic and Planning Decisions in a Supply Chain*, Proc. of the School of Science of Tokai University, 2010, 3 (2), 37–42.

[27] WILIMOWSKA Z., ŁUKANIUK M., *Binomial model of valuation of property options*, Bad. Oper. Dec., 2005 (1), 71–83 (in Polish).

[28] WILCOX J.W., *The P/B-ROE valuation model*, Fin. Anal. J., 1984, 40 (1), 58–66.