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AIRPORT INTERVAL GAMES AND THEIR SHAPLEY VALUE

This paper deals with the research area of cooperative interval games arising from airport situations with interval data. The major topic of the paper is to present and identify the interval Baker–Thompson rule.

Keywords: *cooperative interval games, concave games, airport games, cost games, interval data*

1. Introduction

In the literature much attention has been paid to airport situations and related games. We refer here to Littlechild and Owen (1973), Littlechild and Thompson (1977) and Driessen (1988). In the sequel we summarize the classical airport situation, the classical airport cost game and the Baker–Thompson rule. Consider the aircraft fee problem of an airport with one runway. Suppose that the planes which are to land are classified into m types. For each $1 \le j \le m$, denote the set of landings of type j planes by N_j and its cardinality by n_j. Then $N = \bigcup_{j=1}^{m} N_j$ represents the set of all landings. Let c_j represent the cost of a runway adequate for planes of type j. We as-

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sume that the types are ordered such that $0 = c_0 < c_1 < ... < c_m$. We consider a runway divided into m consecutive pieces P_j, $1 \le j \le m$, where P_1 is adequate for landings of type 1 planes; P_1 and P_2 together for landings of type 2 planes, and so on. The cost of piece P_j, $1 \le j \le m$, is the marginal cost $c_j - c_{j-1}$. The economists Baker (1965) and Thompson (1971) proposed an appealing rule now called the Baker–Thompson rule, given by $\beta_i = \sum_{k=1}^{j} \left[\sum_{r=k}^{m} n_r\right]^{-1} (c_k - c_{k-1})$ whenever $i \in N_j$. That is, every landing of a type j plane contributes to the cost of piece P_k, $1 \le k \le j$, equally allocated among its users $\cup_{r=k}^{m} N_r$. We denote the marginal costs $c_k - c_{k-1}$ by t_k, $1 \le k \le m$. The classical airport TU game $\langle N, c\rangle$ is given by $c(S) = \max\{c_k \mid 1 \le k \le m, S \cap N_k \ne \phi\}$ for all $S \subset N$. It is well known that airport games are concave and the Shapley value (Shapley (1953)) of a concave game belongs to the core of the game. A game $\langle N, c\rangle$ is called concave (or submodular) if and only if $c(S \cup T) + c(S \cap T) \le c(S) + c(T)$ for all $S, T \in 2^N$. Littlechild and Owen (1973) showed that for this game the Shapley value agrees with the Baker–Thompson rule.

In this paper we consider airport situations where the costs of pieces of the runway are given by intervals. Then, we associate an interval cost game with such a situation as in the classical case and extend the results presented above to airport interval games. The rest of the paper is organized as follows. In Section 2 we recall basic notions and facts from interval calculus, together with the theory of cooperative interval games. Section 3 is devoted to the interval Baker–Thompson rule and airport interval games. In Section 4 we define concave interval games and give some results related to airport interval games. We conclude in Section 5 with some final remarks.

2. Preliminaries

We start with some preliminaries from interval calculus (Alparslan Gök, Branzei and Tijs (2008a)). We denote by $I(R)$ the set of all closed and bounded intervals in R, and by $I(R)^N$ the set of all n-dimensional vectors with elements in $I(R)$.

Let $I, J \in I(R)$ with $I = [\underline{I}, \overline{I}]$, $I = [\underline{J}, \overline{J}]$, $|I| = \overline{I} - \underline{I}$ and $\alpha \in R_+$. Then, $I + J = [\underline{I} + \underline{J}, \overline{I} + \overline{J}]$; $\alpha I = [\alpha \underline{I}, \alpha \overline{I}]$. The partial subtraction operator, $I - J$, is defined, only if $|I| \ge |J|$, by $I - J = [\underline{I} - \underline{J}, \overline{I} - \overline{J}]$. We say that I is weakly better than J, which we denote by $I \succeq J$, if and only if $\underline{I} \ge \underline{J}$ and $\overline{I} \ge \overline{J}$. We also use the reverse notation $J \preceq I$ instead of $I \succeq J$.

Now, we give some basic definitions and useful results for cooperative interval games (Alparslan Gök, Miquel and Tijs (2009), Alparslan Gök, Branzei and Tijs (2008b)). An interval game is given by an ordered pair $\langle N, w\rangle$, where $N = \{1, 2, ..., n\}$ is the set of players and $w: 2^N \to I(R)$ is the characteristic function, such that $w(\phi) = [0, 0]$. For each $S \in 2^N$, the worth set (or worth interval) $w(S)$ of the coalition S in the interval game $\langle N, w\rangle$ is of the form $[\underline{w}(S), \overline{w}(S)]$, where $\underline{w}(S)$ is the minimal reward which coalition S could receive on its own and $\overline{w}(S)$ is the maximal reward which coalition S could get. The family of all interval games with player set N is denoted by IG^N. Some classical cooperative games associated with an interval game $\langle N, w\rangle$ play a key role in the theory of cooperative interval games, namely border games $\langle N, \underline{w}\rangle$, $\langle N, \overline{w}\rangle$, and the length game $\langle N, |w|\rangle$, where $|w|(S) = \overline{w}(S) - \underline{w}(S)$ for each $S \in 2^N$

Let $\langle N, w_1\rangle$ and $\langle N, w_2\rangle$, be interval games. We say that $w_1 \preceq w_2$ if $w_1(S) \preceq w_2(S)$ for each $S \in 2^N$. We define $\langle N, w_1 + w_2\rangle$ by $(w_1 + w_2)(S) = w_1(S) + {}_{w2}(\mathrm{S})$ for each $S \in 2^N$. For $\langle N, w_1\rangle$ and $\langle N, w_2\rangle$ where $|w_1(S)| \geq |w_2(S)|$ for each $S \in 2^N$, $\langle N, w_1 - w_2\rangle$ is defined by $(w_1 - w_2)(S) = w_1(S) - w_2(S)$. Given $\langle N, w\rangle$ and $\lambda \in R_+$, we define $\langle N, \lambda w\rangle$ by $(\lambda w)(S) = \lambda w(S)$ for each $S \in 2^N$. So, we conclude that IG^N endowed with $\preceq$ is a partially ordered set and has a conic structure with respect to addition and multiplication by non-negative scalars as described above.

An interval solution concept $\Im$ on IG^N is a map assigning to each interval game $w \in IG^N$ a set of n-dimensional vectors whose components belong to $I(R)$. We note that the payoff vectors $x = (x_1, x_2, ..., x_n) \in R^n$ from classical cooperative TU game theory are replaced by n-dimensional vectors $(I_1, I_2, ..., I_n) \in I(R)^N$. We denote by $I(R_+)$ the set of all closed and bounded intervals in R_+. Let $I \in I(R_+)$, $T \in 2N\backslash\{\phi\}$, and let $u_T^*: 2^N \to R$ be the classical dual unanimity game based on T. Recall that u_T^* is defined by

$$u_T^*(S) = \begin{cases} 1, & T \cap S \neq \phi \\ 0, & \text{otherwise,} \end{cases}$$

and the Shapley value $\phi(u_T^*)$ is given by

$$\phi(u_T^*) = \begin{cases} 1/T, & i \in T \\ 0, & i \in N \setminus T. \end{cases}$$

The interval game $\langle N, Iu_T^*\rangle$ defined by $(Iu_T^*)(S) = u_T^*(S)I$ for each $S \in 2^N$ will play an important role in this paper. We notice that $\Phi(Iu_T^*)$ for the interval game $\langle N, Iu_T^*\rangle$ is related to the Shapley value $\phi(u_T^*)$ of the classical game $\langle N, u_T^*\rangle$ as follows:

$$\Phi(Iu_T^*) = \phi(u_T^*)I = \begin{cases} 1/|T|, & i \in T \\ [0,0], & i \in N \setminus T. \end{cases} \tag{1}$$

The interval Shapley value is defined on IG^N by additivity and (1).

3. The interval Baker–Thompson rule and airport interval games

Consider the aircraft fee problem of an airport with one runway. Suppose that the planes which are to land are classified into m types. For each $1 \le j \le m$, denote the set of landings of type j planes by N_j and its cardinality by n_j. Then $N = \cup_{j=1}^m N_j$ represents the set of all landings. Suppose the runway is divided into m consecutive pieces P_j, $1 \le j \le m$, where P_1 is sufficient for landings of type 1 planes; P_1 and P_2 together for landings of type 2 planes, and so on. Let the interval T_j with non-negative finite bounds represent the interval cost of piece P_j, $1 \le j \le m$.

Next, we propose an interval cost allocation rule β, which we call the interval Baker–Thompson rule. For a given airport interval situation $(N, (T_k)_{k=1,\dots,m})$ the Baker–Thompson allocation for each player $i \in N_j$ is given by:

$$\beta_i = \sum_{k=1}^{j} \left(\sum_{r=k}^{m} n_r \right)^{-1} T_k. \tag{2}$$

Note that the users of piece P_k of the runway are $\cup_{r=k}^m N_r$, i.e. there are $\sum_{r=k}^m n_r$ users. So, $\left(\sum_{r=k}^m n_r\right)^{-1} T_k$ is the equal costs share of each user for piece P_k. This means that player i. $i \in N_j$ contributes to the cost of the pieces P_1, ..., P_j. The characteristic cost function d of the airport interval game $\langle N, d\rangle$ is given by $d(\phi) = [0, 0]$ and $d(S) = \sum_{k=1}^{j} T_k$ for all coalitions $S \subset N$ satisfying $S \cap N_j \neq \phi$ and $S \cap N_k = \phi$ for all $j+1 \le k \le m$ (since such a coalition S only needs pieces P_k, $1 \le k \le j$ of the runway). Now we give the description of the airport interval game as follows:

$$d = \sum_{k=1}^{m} T_k u^*_{\cup_{r=k}^m N_r}. \tag{3}$$

In the following proposition we show that the interval Baker–Thompson allocation for the airport situation with interval data coincides with the interval Shapley value of the corresponding airport interval game.

Proposition 3.1. Let $\langle N, d\rangle$ be an airport interval game. The interval Baker–Thompson allocation β of (2) agrees with the interval Shapley value $\Phi(d)$.

Proof: For $i \in N_j$ we have

$$\Phi_i(d) = \Phi_i\left(\sum_{k=1}^{m} T_k u^*_{\cup_{r=k}^{m} N_r}\right) = \sum_{k=1}^{m} \Phi_i(T_k u^*_{\cup_{r=k}^{m} N_r}) = \sum\nolimits_{k=1}^{j}\left(\sum_{r=k}^{m} n_r\right)^{-1} T_k = \beta_i,$$

where the equalities follow from (3), the additivity of the interval Shapley value Φ, (1) and (2), respectively.

□

Note that if we consider the special case $N_1 = \{1\}$, $N_2 = \{2\}$, ..., $N_n = \{n\}$, then $\beta = \left(\frac{T_1}{n}, \frac{T_1}{n} + \frac{T_2}{n-1}, ..., \frac{T_1}{n} + \frac{T_2}{n-1} + ... + \frac{T_n}{1}\right)$. Here, each piece of the runway is completely paid for by its users and all the users of the same piece contribute equally.

Example 3.1. Let $\langle N, d\rangle$ be the three-person airport interval game corresponding to the interval costs $T_1 = [30, 45]$, $T_2 = [20, 40]$ and $T_3 = [100, 120]$. Then, $d(\phi) = [0, 0]$, $d(1) = [30, 45]$, $d(2) = d(1, 2) = [50, 85]$ and $d(3) = d(1, 3) = d(2, 3) = d(N) = [150, 205]$.

Note that $d = [30, 45]u^*_{\{1,2,3\}} + [20,40]u^*_{\{2,3\}} + [100,120]u^*_{\{3\}}$ and $\Phi(d) = ([10, 15], [20, 35], [120, 155])$.

Notice also that

$$\Phi(d) = \Phi\left(\sum_{k=1}^{3} T_k u^*_{\cup_{r=k}^{3} N_r}\right) = \Phi(T_1 u^*_{\{1,2,3\}}) + \Phi(T_2 u^*_{\{2,3\}}) + \Phi(T_3 u^*_{\{3\}})$$

$$= \left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right)[30, 45] + \left(0, \frac{1}{2}, \frac{1}{2}\right)[20,40] + (0, 0, 1)[100,120] = \beta.$$

Now, we focus on the interval core membership for the interval Baker–Thompson allocation. Let $\langle N, d\rangle$ be an interval cost game. The interval core $C(d)$ is defined by

$$C(d) = \left\{(I_1, ..., I_n) \in I(R)^N \middle| \sum_{i\in N} I_i = d(N), \sum_{i\in S} I_i \preceq d(S), \forall S \in 2^N \setminus \{\phi\}\right\}.$$

The interval core consists of those interval payoff vectors which assure the distribution of the uncertain cost of the grand coalition, $d(N)$, such that each coalition of players S can expect a weakly better interval cost, $\sum_{i\in S} I_i$, than what that group can expect on its own, implying that no coalition has any incentives to split. We refer to

$\sum_{i \in N} I_i = d(N)$ as the efficiency condition and to $\sum_{i \in S} I_i \preceq d(S)$, $S \in 2^N \setminus \{\phi\}$, as the stability conditions of the interval payoff vectors.

Proposition 3.2. Let $(N, (T_k)_{k=1,\ldots,m})$ be an airport situation with interval data and $\langle N, d\rangle$ be the related airport interval game. The interval Baker–Thompson rule applied to this airport situation provides an allocation which belongs to $C(d)$.

Proof:[1] From Proposition 3.1 the Baker–Thompson allocation is efficient. We need only to check the stability conditions for the interval Baker–Thompson allocation. Consider the airport interval game $\langle N, d\rangle$ and any coalition $S \subset N, S \neq \phi$. Set $d(S) = \sum_{r=1}^{j} T_r$, that is to say $S \cap N_J \neq \phi$ and $S \cap N_p = \phi$ for all $j < p \leq m$. It follows that for $i \in N_k$, $\beta_i = \sum_{r=1}^{k} \frac{T_r}{n_r + \ldots + n_m}$. Thus,

$$\sum_{i \in S} \beta_i = \sum_{k=1}^{j} \left(|S \cap N_k| \sum_{r=1}^{k} \frac{T_r}{n_r + \ldots + n_m} \right) = \sum_{r=1}^{j} \left(\frac{T_r}{n_r + \ldots + n_m} \sum_{k=r}^{j} |S \cap N_k| \right).$$

Note that $\sum_{k=r}^{j} |S \cap N_k| \leq n_r + \ldots + n_j \leq n_r + \ldots + n_m$. From this, we conclude that $\sum_{i \in S} \beta_i \preceq \sum_{r=1}^{j} T_r = d(S)$ by taking care of the ordering of intervals via their lower and upper bounds.

□

4. Concave interval games and related results

We say that an interval game $\langle N, d\rangle$ is submodular if, for all $S, T \in 2^N$,

$$d(S) + d(T) \succeq d(S \cup T) + d(S \cap T).$$

In other words, an interval game $\langle N, d\rangle$ is submodular if and only if its border TU-games $\langle N, \underline{d}\rangle$ and $\langle N, \overline{d}\rangle$ are concave. We say that an interval game $\langle N, d\rangle$ is concave if it is submodular and its length game $\langle N, |d|\rangle$ is concave. Since

[1] This direct proof was provided by one of the anonymous referees; the authors' original proof can be found in Section 4.

$\overline{d} = \underline{d} + |d|$, the concavity conditions for $\langle N, \underline{d}\rangle$ and $\langle N, |d|\rangle$ are sufficient to ensure the concavity of the underlying interval game $\langle N, d\rangle$. We claim the airport interval game to be concave.

Proposition 4.1. Let $\langle N, d\rangle$ be an airport interval game. $\langle N, d\rangle$ is concave.

Proof: It is well known that non-negative multiples of classical dual unanimity games are concave (or submodular). From (3) it follows that $\underline{d} = \sum_{k=1}^{m} \underline{T_k} u_{k,m}^*$ and $|d| = \sum_{k=1}^{m} |T_k| u_{k,m}^*$ are concave, because $\underline{T_k} \geq 0$ and $|T_k| \geq 0$ for each k, implying that $\langle N, d\rangle$ is concave.

□

Note that the interval game $\langle N, d\rangle$ in Example 3.1 is concave from Proposition 4.1. The next proposition provides additional characterizations of concave interval games.

Proposition 4.2. Let $d \in IG^N$ be such that $d \in G^N$ is submodular. The following three assertions are equivalent:

(i) $d \in IG^N$ is concave;

(ii) For all $S_1, S_2, U \in 2^N$ with $S_1 \subset S_2 \subset N \setminus U$ we have

$$d(S_1 \cup U) - d(S_1) \succeq d(S_2 \cup U) - d(S_2);$$

(iii) For all $S_1, S_2 \in 2^N$ and $i \in N$ such that $S_1 \subset S_2 \subset N \setminus \{i\}$ we have

$$d(S_1 \cup \{i\}) - d(S_1) \succeq d(S_2 \cup \{i\}) - d(S_2).$$

Proof: To prove $(i) \Rightarrow (ii)$, $(ii) \Rightarrow (iii)$, $(iii) \Rightarrow (i)$, we simply replace the inequality sign $\preceq$ in the Proof of Theorem 3.1 in Alparslan Gök, Branzei and Tijs (2008b) by the inequality sign $\succeq$.

An alternative proof of the stability of the interval Baker–Thompson allocation is based on the concavity of the airport interval game and the stability of its interval marginal contribution vectors over the set of all permutations $\sigma \in \Pi(N)$. We recall that the interval marginal vector of w with respect to σ, $m^{\sigma}(w)$, corresponds to a situation where the players enter a room one by one in the order $\sigma(1), \sigma(2), ..., \sigma(n)$ and each player is given the marginal contribution he/she creates by entering. We denote the set of predecessors of i in σ by $P_{\sigma}(i) = \{r \in N \mid \sigma^{-1}(r) < \sigma^{-1}(i)\}$, where $\sigma^{-1}(i)$ denotes the entrance number of player i and define $m_i^\sigma(w) = w(P_\sigma(i) \cup \{i\}) - w(P_\sigma(i))$ for each $i \in N$. For concave interval games, all the interval vectors $m^\sigma(w)$ are defined and their average equals the interval Shapley value of the game. The following table shows the interval marginal vectors of the game in Example 3.1, where the rows correspond to orderings of players and the columns correspond to players.

$$\begin{array}{c} 123 \\ 132 \\ 213 \\ 231 \\ 312 \\ 321 \end{array} \begin{bmatrix} [30,45] & [20,40] & [100,120] \\ [30,45] & [0,0] & [120,160] \\ [0,0] & [50,85] & [100,120] \\ [0,0] & [50,85] & [100,120] \\ [0,0] & [0,0] & [150,205] \\ [0,0] & [0,0] & [150,205] \end{bmatrix}.$$

Proof of Proposition 3.2: First, from Proposition 4.1 the airport game $\langle N, d\rangle$ is concave. We prove that $m^\sigma(d) \in C(d)$ for all $\sigma \in \Pi(N)$. Let $\sigma \in \Pi(N)$ and take $m^\sigma(w)$. Clearly, we have $\sum_{k\in N} m_k^\sigma(d) = d(N)$. To prove that $m^\sigma(d) \in C(d)$, we have to show that for $S \in 2^N$, $\sum_{k\in S} m_k^\sigma(d) \preceq d(S)$. Let $S = \{\sigma(i_1), \sigma(i_2), ..., \sigma(i_k)\}$ with $i_1 < i_2 < ... < i_k$. Then,

$$\begin{aligned} d(S) &= d(\sigma(i_1)) - d(\phi) \\ &+ \sum_{r=2}^{k} (d(\sigma(i_1), \sigma(i_2), ..., \sigma(i_r)) - d(\sigma(i_1), \sigma(i_2), ..., \sigma(i_{r-1}))) \\ &\succeq d(\sigma(1), ..., \sigma(i_1)) - d(\sigma(1), ..., \sigma(i_1 - 1)) \\ &+ \sum_{r=2}^{k} (d(\sigma(1), \sigma(2), ..., \sigma(i_r)) - d(\sigma(1), \sigma(2), ..., \sigma(i_r - 1))) \\ &= \sum_{r=1}^{k} m_{\sigma(i_r)}^\sigma(d) = \sum_{k\in S} m_k^\sigma(d), \end{aligned}$$

where the inequality follows from Proposition 4.2 (iii) applied to $i = \sigma(i_r)$ and $S_1 = \{\sigma(i_1), \sigma(i_2), ..., \sigma(i_{r-1})\} \subset S_2 = \{\sigma(1), \sigma(2), ..., \sigma(i_r - 1)\}$ for $r \in \{1, 2, ..., k\}$.

Furthermore, since the interval Shapley value of d is the average of all the marginal interval vectors of d and from the convexity of $C(d)$, we obtain $\Phi(d) \in C(d)$. Now we apply Proposition 3.1.

□

5. Final remarks

In this paper we studied airport situations with interval data and related games. We notice that the interval Baker–Thompson rule is useful at an ex ante stage to inform users about what they can expect to pay, between two bounds, for the construction of the runway. At an ex post stage when all the costs are known with certainty, the classical Baker–Thompson rule can be applied to derive the effective costs $x_i \in \beta_i$ for each $i \in N$, such that $\sum_{i \in N} x_i$ equals the realization $\widetilde{d} \in [\underline{d}(N), \overline{d}(N)]$. To the best of our knowledge, no axiomatic characterization for the classical Baker–Thompson rule exists in the literature. It remains as a topic for further research to try to find an axiomatic characterization for the interval Baker–Thompson rule.

Other economic and operations research problems with interval data and related interval games have also been studied, such as bankruptcy situations (Branzei and Alparslan Gök (2008)), sequencing situations (Alparslan Gök et al. (2008)) and minimum cost spanning tree situations (Montemanni (2006) and Moretti et al. (2008)). We notice that other OR situations and combinatorial optimization problems with interval data, including flow problems, linear production problems and holding problems, might also give rise to interesting interval games.

An interesting topic for further research could be to relate airport interval games and concave interval games with their corresponding set games (Sun (2003)). Recall that a set game is a triple (N, v, U), where N is a finite set of players, U denotes an abstract set, called universe, and v is a mapping $v: 2^N \to 2^U$. By choosing the universe $U = R$, the worth interval $w(S)$ of any coalition S in any interval game $\langle N, w\rangle$ may be interpreted as a subset of U, implying that cooperative interval games form a special subclass of cooperative set games. In our opinion, no overlaps between these two theories exist mainly because the role of the (weakly) better than operator to compare intervals is played by the inclusion operator in set game theory.

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Lotniskowe gry przedziałowe i ich wartość Shapleya

Praca dotyczy gier kooperacyjnych opisujących sytuacje występujące na lotniskach, gdzie dane są przedziałowe. Głównym celem pracy jest przedstawienie i identyfikacja przedziałowych zasad Bakera–Thompsona, służących do rozwiązywania problemu opłat lotniskowych dla samolotów lądujących na lotniskach z jednym pasem startowym, gdzie koszt użycia pasa startowego jest nieustalony. Dla takich gier lotniskowych podany jest dowód zgodności pomiędzy alokacją przedziałową Bakera–Thompsona a wartością Shapleya. Pokazano, że zasady przedziałowe Bakera–Thompsona zastosowane do dowolnej sytuacji lotniskowej z danymi przedziałowymi prowadzą do alokacji, należącej do przedziałowego jądra odpowiedniej przedziałowej gry lotniskowej. Zdefiniowano oraz podano niektóre charakterystyki takich wklęsłych gier przedziałowych. Dowiedziono wklęsłości lotniskowych gier przedziałowych oraz użyto tej właściwości do alternatywnego dowodu stabilności przedziałowej alokacji Bakera–Thompsona.

Słowa kluczowe: *kooperatywne gry przedziałowe, gry wklęsłe, gry lotniskowe, gry kosztowe, dane przedziałowe*