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Nr 2 2009

EDITORIAL

This special edition is dedicated to game theory. Applications to economics and financial decision making had a strong influence on the early development of game theory. In recent times the range of applications of game theory has extended rapidly to cover evolutionary biology, politics and a wide range of other social sciences. The articles in this edition reflect the range of diverse fields in which game theoretic concepts are used.

In the opening article, Rossi considers measures of conflict and power in games. The index is initially presented within the framework of non-cooperative games, but its derivation uses concepts from cooperative games and the index can be easily adapted to cooperative games. Conflict occurs since players' utilities cannot be maximised simultaneously. Rossi considers two extremes. The first extreme is given by games of *common interest* where there is at least one profile of actions which simultaneously maximises each of the players' utilities. The second is given by *full conflict games* in which only one player can obtain a greater payoff than the so called status quo payoff and any coalition can ensure that any player from outside the coalition only obtains the status quo payoff. The concept of a *full conflict game* is obviously associated with cooperative game theory. Solutions such as Shapley and Banzhaf indices can be used to describe the power of an individual. In a full conflict game it is clear that each player has the same blocking power. The author uses his approach to interpret the Shapley value as a measure of a person's power when a coalition A uses the best response which minimises the payoff of A^C (a retaliationary action). Hence, power indices can be interpreted as a player's ability to retaliate to the actions of others.

Rossi's article is followed by a sequence of four articles which consider game theoretic problems which have applications in Business and Economics. Gok, Branzei and Tijs consider the Shapley value of airport games with imperfect information regarding costs. In the classic airport game aircraft are classified according to the length of runway they need to use. The Baker–Thompson rule states that the cost of each section of the runway should be split equally over each aircraft using that particular section. It can be shown that this solution corresponds to the Shapley value of the appropriately defined co-operative game describing the construction of the runway. This approach is suitable when the costs are known, e.g. for assigning the costs after the runway has been constructed. However, this approach cannot be used before the construction of the runway when there is imperfect information regarding the construction costs. The authors adapt the Baker–Thompson rule to such a problem and show that this solution corresponds to the Shapley value of the corresponding interval game.

This concept of uncertainty is also considered in Branzei and Dall'Aglio's article on allocation rules. They consider the problem of splitting an estate when there is uncertainty regarding the value of the estate and/or the size of the claim from a party. In the standard bankruptcy problem the sum of value of the values of the claims is greater than the value of the bankrupt firm's assets. Various allocation rules have been put forward for settling the claims when there is perfect information. The simplest one is the proportional rule in which each individual receives the same proportion of his/her claim. The constrained equal awards rule assigns equal amounts to each individual subject to the condition that no-one obtains more than their claim.

The authors first consider problems in which there is only uncertainty regarding the value of the firm's assets. They consider two approaches to defining an allocation in this case. The first is to directly calculate intervals defining the possible allocations for each player. The lower (upper) endpoint of each interval is the allocation given by one of the methods given above based on the minimum (maximum, as appropriate) size of the estate. The second method is to split the minimal estate according to some allocation rule to find the minimal allocation to each individual. The maximum possible excess is then allocated among the creditors in a similar manner. For problems in which there is uncertainty regarding both the size of the estate and the size of the claims, the authors introduce a compromise factor, which turns each fuzzy claim into a non-fuzzy claim. This compromise factor can be either determined by a neutral arbitrator or the allocations averaged according to some joint distribution for these factors.

Kruś also considers the problem of allocating costs. However, he considers a slightly different problem in which the costs/rewards obtained by a player depend not only on the coalition she is part of, but on the other coalitions that have formed. He defines the following payoff functions for such a game: 1) the guaranteed worth of a coalition independent of the behaviour of players outside the coalition, 2) the maximal guaranteed amount that can be gained by a set of players independently of the behaviour of the other players (note that it is possible that individual players in a coalition may sometimes gain by splitting up). The author refines the core defined by Thrall and Lucas by considering a weaker but more natural concept, being based on group rationality, of the dominance of a payoff vector x over a payoff vector y for a given coalition. It is shown that this refinement may lead to the core being empty. The author describes the core of such a partition form game as the core of an appropriately defined standard cooperative game.

In the fifth article Ferrara considers a model of an imperfectly competitive market. A group of agents trade a single commodity. The agents may form coalitions when setting their prices and agree to redistribute the profits in an appropriate manner. This leads to a co-operative version of a Cournot–Bertrand game. At a stable system of prices no coalition can increase its profits by reducing its prices. The set of prices satisfying this condition are called the pseudo-core of the game. He shows that this pseudo-core is non-empty. The core is the subset of the pseudo-core for which each agent obtains a profit. The author gives conditions for the core to be non-empty.

The final article is in the field of evolutionary game theory. The most commonly used solution in this area is the concept of evolutionarily stable strategy (ESS). In defining an ESS of a two-player game it is (often implicitly) assumed that each individual plays a series of N realisations of the game in question with each opponent being picked at random from the population as a whole. However, in war of attrition or parental care games, the time spent playing a realisation of the game depends on the strategy used. For example, a parent who deserts their offspring may well be able to mate more often than a parent who cares for their young, but will have on average a smaller number of surviving offspring from each breeding attempt. Hence, there is a trade-off between the number of times a game is played and the mean payoff obtained in a realisation of the game. Ramsey considers a model of a parental care game as a game against the field in which individuals move between states such as searching for a mate and caring for offspring at rates which depend on their own strategy and the strategy profile used within the population as a whole. He considers various concepts of stability and the difference between a mixed equilibrium (in which all the population use the same mixed strategy) and a stable polymorphism (in which each individual uses a pure strategy, but players can use different strategy).

The articles here illustrate some of the important philosophical and mathematical constructs that underlie the discipline of game theory. The beauty of game theory lies in the fact that it can be appreciated at many different levels, from an intuitive appreciation of why individuals and systems behave as they do to the detailed mathematical arguments involved in deriving the forms of solution. I hope that the articles included in this special edition convey some of this beauty to the reader.

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Co-editor