OPERATIONS RESEARCH AND DECISIONS

No. 4 2016

DOI: 10.5277/ord160405

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NEW METHOD OF SELECTING EFFICIENT PROJECT PORTFOLIOS IN THE PRESENCE OF HYBRID UNCERTAINTY

A new methods of selecting efficient project portfolios in the presence of hybrid uncertainty has been presented. Pareto optimal solutions have been defined by an algorithm for generating project portfolios. The method presented allows us to select efficient project portfolios taking into account statistical and economic dependencies between projects when some of the parameters used in the calculation of effectiveness can be expressed in the form of an interactive possibility distribution and some in the form of a probability distribution. The procedure for processing such hybrid data combines stochastic simulation with nonlinear programming. The interaction between data are modeled by correlation matrices and the interval regression. Economic dependences are taken into account by the equations balancing the production capacity of the company. The practical example presented indicates that an interaction between projects has a significant impact on the results of calculations.

Keywords: *portfolio selection, data processing, hybrid uncertainty, random fuzzy sets*

1. Introduction

Project selection is a complex decision making process implemented in an uncertain environment. Randomness and imprecise or missing information are two sources of uncertainty [4, 11, 19, 32]. In practice, the most common situation occurs when some parameters of investment projects are specified by a probability distribution, while others are given in the form of fuzzy numbers [11]. Therefore, it is necessary to take both ways of describing uncertainty into account in order to select efficient project portfolios. Ferson and Ginzburg [10] suggested that distinct methods are needed to appropriately

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represent random variability (often referred to as objective uncertainty or aleatory uncertainty) and imprecision (often referred to as subjective uncertainty or epistemic uncertainty).

Many methods for selecting portfolios investment projects which take into account uncertainty of parameters when calculating effectiveness have been presented in the literature [1–3, 8, 18, 23, 24, 28, 29, 34]. These methods for building efficient project portfolios use probability distributions to describe uncertainty regarding parameters in the calculation of effectiveness.

The literature also presents methods for selecting project portfolios based on the assumption that uncertainty regarding parameters is expressed in the form of possibility distributions when calculating effectiveness. Such methods have been proposed by Huang [15], Liu and Iwamura [22], Chan et al. [5] and Tavana et al. [33]. The concepts presented by these authors for selecting efficient project portfolios under conditions of fuzziness do not take into account the fact that statistical dependencies between projects exist. In the selection process, projects are generally treated as economically independent. This limits the practicality of these models by a considerable degree.

We speak of a statistical dependency between investment projects when a correlation exists between the benefits generated by the projects [8, 28, 29, 34]. When a portfolio contains negatively or weakly positively correlated projects, we obtain a low portfolio risk. This correlation is due to the correlation between the parameters used to calculate efficiency. For example, there is a correlation between the prices of an enterprise's products and raw material prices. In addition the size of the markets for different product ranges are correlated [26].

Besides statistical dependencies, there may be economic dependencies between investment projects. Economic dependencies indicate the manner in which a given project affects the benefits generated by other projects or benefits obtained from the activity of the firm to date. Investment projects may therefore be economically independent (when no such effect exists) or economically dependent (when such an effect exists) [8].

Among economically dependent projects, we can define: complementary projects, mutually dependent projects, substitute projects, mutually exclusive projects [8, 29]. In the case of positive economic dependencies, the benefit generated by a given investment project leads to increased benefits generated by another one. At this time, synergy occurs, and such projects are described as complementary projects. However, in the case of mutually dependent projects, either all qualify for implementation or none. On the other hand, negative economic dependencies occur when the benefits obtained from one project decrease as a result of the implementation of another project. Such projects are described as substitute projects. In the case of mutually exclusive projects, only one can qualify for the implementation.

Rębiasz [26] presented a method for selecting of efficient project portfolios, in which uncertainty can be expressed either in the form of probability distributions or in the form of possibility distribution. This method takes into account the statistical and economic dependence between projects. In the literature there are examples of methods of constructing investment portfolios of financial assets, where uncertainty is described by fuzzy random variables [13, 17].

The above brief review indicates that the majority of authors use either probability or possibility distributions in investment analysis. One of the problems limiting the application of hybrid data is the imperfection of the methods of processing such data. However, there are a few studies which describe methods of processing hybrid data [4, 6, 12]. Guyonnet et al. [12] and Cooper et al. [6] propose a method of processing data in the case where both probability and possibility distributions are involved. Some of the proposed methods for hybrid data processing combine stochastic simulation with the arithmetic of fuzzy numbers [6, 12]. Baudrit et al. [4] use probability and possibility distributions in risk analysis. They use a procedure for data processing which also combines stochastic simulation with the arithmetic of fuzzy numbers. In the methods of processing hybrid data developed by the authors mentioned above, it was assumed that data were independent. In the case of selecting investments, data are usually correlated.

Summarizing one can say that there are no methods of selecting efficient project portfolios in the case where hybrid data are available (data partly described by probability distribution and partly by possibility distribution).

This article, at least in part, solves the problem of the lack of such a method. The method presented below is a novel solution for selecting of efficient project portfolios. The proposed method allows the use of hybrid data. In order to process hybrid data an original method is used, which takes into account the correlation between such data. In the course of selecting investment projects, statistical and economic dependences between the projects are taken into account.

The rest of this paper is organized as follows: in Section 2, the basic idea related to Dempster–Shafer (D–S) theory, fuzzy random variables, methods of processing hybrid data, and measures of risk in the case of hybrid data are presented. The definitions and concepts presented here are used in the description of the proposed method of selecting efficient project portfolios. This method is introduced in the Section 3. The formulation of the problem and algorithm for selecting efficient project portfolios are presented. In Section 4, a case study is used to demonstrate the applicability and utility of the proposed method. Finally, in Section 5, the basic conclusions are presented.

2. Preliminaries

2.1. Dempster–Shafer (D–S) theory of evidence

The theory of belief functions (also called evidence theory) was introduced by Shafer [30]. This theory provides mathematical tools to process information which is simultaneously random and imprecise. Contrary to the probability theory, which assigns probability weights to atoms (the reference elements), the theory of evidence may assign such weights to any subsets, called focal sets. Most often we obtain a sample of random intervals. In this case, the information is presented in the form of the intervals $[\underline{a}_i, \overline{a}_j]$ for $I = 1, 2, ..., I$. The probability p_i is assigned to the i-th interval. That is, we obtain a mass distribution p_i on these intervals. The probability mass p_i can be freely re-allocated to points within the interval $[\underline{a}_i, \overline{a}_i]$. However, there is not enough information to do this.

Evidence theory defines two indicators, plausibility *Pl* and belief *Bel*, to quantify the validity of the proposition stating that the value of the variable *X* should lie within the set *A* (a certain interval, for example). The plausibility *Pl* and belief *Bel* measures are defined from the mass distribution [30]

$$p:\mathscr{P}(\Omega)\rightarrow[0,1], \text{ such that } \sum_{E\in\ (\Omega)} p(E)=1 \tag{1}$$

as follows:

$$Bel(A)=\sum_{E,E\subseteq A} p(E) \tag{2}$$

$$Pl(A)=\sum_{E,E\cap A\neq\varphi} p(E)=1-Bel(A^c) \tag{3}$$

where $\mathscr{P}(\Omega)$ – the power set of Ω, E is called a focal element of $\mathscr{P}(\Omega)$ when $p(E)>0$.

Evidence theory encompasses possibility and probability theory [7, 30]. Thus, any the probability or possibility distributions may be interpreted in term of mass functions. The method of creating an appropriate probability mass on the basis of possibility distribution π is described below [4].

Let Y be a possibilistic variable. We denote by π the possibility distribution of Y and by π_α the α cuts of π. The focal elements for Y corresponding to α cuts are nested and denoted by $(\pi_\alpha)_{j=1,\dots,q}$ with $\alpha_0 = \alpha_1 = 1 > \alpha_2 > \dots \alpha_q > \alpha_{q+1} = 0$. We denote by $(p_j = \alpha_j - \alpha_{j+1})_{j=1,\dots,q}$ the mass distribution associated to $(\pi_\alpha)_{j=1,\dots,q}$.

We can interpret [*Bel*, *Pl*] as upper and lower probabilities induced from specific probability families. Namely, a mass distribution p may encode the probability family $\mathscr{P}(p)=\{P,\forall A, Bel(A)\leq P(A)\}=\{P,\forall A, P(A)\leq Pl(A)\}$. In this case, we have $P = Pl$

and $\underline{P} = Bel$. Hence, we can define (*p*-box), based on the upper $\overline{F}(x)$ and lower $\underline{F}(x)$ cumulative distribution function:

$$\overline{F}(x) = Pl(X \in [-\infty, x]) \tag{4}$$

$$\underline{F}(x) = Bel(X \in [-\infty, x]) \tag{5}$$

2.2. Fuzzy random variables

Liu and Liu [20] define fuzzy random variable in the following manner. Let us assume that Z is a set of fuzzy variables. Each element z of the set Z is characterized by a membership function μ_z. Let us assume that (Ω, Σ, P) is a probability space. A fuzzy random variable is then defined as a map $\xi{:}\Omega \rightarrow Z$ such that for each closed subset C of the space $\Re$.

$$\xi^*(C)(\omega) = \Pi\{\xi(\omega) \in C\} = \sup_{x \in C} \mu_{\xi(\omega)}(x) \tag{6}$$

is measurable function ω, where $\mu_{\xi(\omega)}$ is the possibility distribution of the fuzzy variable $\xi(\omega)$. Based on a fuzzy random variable, the upper and lower cumulative distribution functions may be estimated. One can say that each fuzzy variable induces a random set [30]. If there are n fuzzy variables $(X_i)_{i=1, 2..., n}$, with corresponding probabilities $(p_i)_{i=1, 2, ..., n}$, then $(\pi_\alpha)_{j=1,...,q;\, i=1,2,...,n}$ are the focal elements of a random set, while values p_{ij} ($p_{ij} = p_i(\alpha_{ji} - \alpha_{j+1;i})$) constitute probability mass of a random set. Based on the random set defined, one may determine $\overline{F}(x)$ and $\underline{F}(x)$.

2.3. Method of processing hybrid data

In the literature, you can find proposals for methods of processing hybrid data [4, 9]. These methods do not take into account dependencies (correlation) between the data. The method defined by Rębiasz [27] is described below. This method is a modification of the method presented by Baudrit et al. [4] and Dubois et al. [9] and allows us to take into account the correlation between hybrid data. Here the dependences between variables are modeled by correlation matrices and interval regression.

Let us assume that we want to determine the value of $f\left(\hat{X}\right)$ where $\hat{X}=\left[X_1, X_2, ..., X_m\right]$ is a vector of variables burdened with uncertainty. It is assumed that n variables ($n < m$) are random variables $\left[X_1, X_2, ..., X_n\right]$ and $m - n$ variables $[X_{n+1}, X_{n+2}, \ldots, X_m]$ are possibilistic variables, the values of which are limited by the possibility distribution $P_{n+1}, P_{n+2}, ..., P_m$, respectively. Additionally, it is assumed that subsets X^K of interactive variables X_i; $X^K=\left\{X_i, i\in K\right\}, K \in K_s$ may be defined. Here K is the subset of indices of the interactive variables, and K_s is the set of indices of the selected subsets of interactive variables..

The proposed procedure of determining the value $f\left(\hat{X}\right)$ includes two stages. It combines a procedure for stochastic simulation with a procedure for processing of an interactive possibility distribution. In order to process such possibility distributions, nonlinear programming is used. The computational procedure used in this case is the following. The realization $[\mathrm{x}_1, \mathrm{x}_2, ..., \mathrm{x}_n]$ of the random variables are drawn using a procedure which accounts for the correlation between variables. These values and possibility distributions $[P_{n+1}, P_{n+2}, \ldots, P_m]$ allow us to determine $f\left(\mathrm{x}_1, \mathrm{x}_2, ..., \mathrm{x}_n, P_{n+1}, P_{n+2}, ..., P_m\right)$ as a possibility distribution. This can be achieved using the concept of α-levels. The upper (sup) and lower (inf) bounds of the possibility distribution $f\left(\mathrm{x}_1, \mathrm{x}_2, ..., \mathrm{x}_n, P_{n+1}, P_{n+2}, ..., P_m\right)$ α-level may be determined by solving the following nonlinear programming tasks.

When searching for sup, find:

$$f\left(\mathrm{x}_1, \mathrm{x}_2, ..., \mathrm{x}_n, x_{n+1}, x_{n+2}, ..., x_m\right)\rightarrow \max \tag{7}$$

When searching for inf, find:

$$f\left(\mathrm{x}_1, \mathrm{x}_2, ..., \mathrm{x}_n, x_{n+1}, x_{n+2}, ..., x_m\right)\rightarrow \min \tag{8}$$

subject to the following constraints:

$$\inf\left(P_i\right)_\alpha \le x_i \le \sup\left(P_i\right)_\alpha \quad \text{for } I = n + 1, n + 2, \ldots, m \tag{9}$$

$$x_i \ge \inf\left(a_1^{iz}\right)\times x_z + \inf\left(a_2^{iz}\right) \text{ for } i\in K,\ z\in K;\ i\ne z;\ i>n,\ z>n,\ K\in K_s \tag{10}$$

$$x_i \le \sup\left(a_1^{iz}\right)\times x_z + \sup\left(a_z^{iz}\right) \text{ for } i\in K,\ z\in K;\ i\ne z;\ i>n,\ z>n,\ K\in K_s \tag{11}$$

$$x_i \ge \inf\left(a_1^{iz}\right)\times \mathrm{x}_z + \inf\left(a_2^{iz}\right) \text{ for } i\in K,\ z\in K;\ i>n,\ z\le n;\ K\in K_s \tag{12}$$

$$x_i \le \sup\left(a_1^{iz}\right) \times \mathrm{x}_z + \sup\left(a_2^{iz}\right) \text{ for } i \in K,\ z \in K;\ i > n,\ z \le n;\ K \in K_s \tag{13}$$

The values a_1^{iz}, a_2^{iz} are the coefficients of the interval regression equations determining the dependencies between the variables X_i and X_z. These coefficients may be determined using the method proposed by Hladik and Černy (crisp input-crisp output variant) [14].

Drawing realizations [x_1, x_2,..., x_n] and determining f(x_1, x_2, …, x_n, P_{n+1}, P_{n+2}, …, P_m is repeated J times. As a result J possibility distributions (π_1^f, ..., π_J^f) are obtained. In this case, the value $f\left(\hat{X}\right)$ is represented by a random fuzzy variable.

The above hybrid procedure may be described by the following algorithm [27]:

START

Step1. Define α_0, $\wp$, J

Step 2. Set, j = 1

Step 3. Randomly generate a vector [x_1, x_2, x_n] taking into account the correlation between variables

Step 4. Set $\alpha = \alpha_0$

Step 5. Define α-levels $(X_i)_\alpha$ for $i = n + 1, n + 2, m$

Step 6. Define (sup) and (inf) for α-levels for the possibility distribution defining $f\left(\hat{X}\right)$. Find $\overline{f}_{\alpha,j}\left(\mathrm{x}_1, \mathrm{x}_2, ..., \mathrm{x}_n, x_{n+1}, x_{n+2}, ..., x_m\right) \to \max$

$\underline{f}_{\alpha,\ j}\left(\mathrm{x}_1,\ \mathrm{x}_2, ..., \mathrm{x}_n, x_{n+1},\ x_{n+2}, ..., x_m\right) \to \min$

with the problem constraints specified by inequalities (9)−(13)

Step 7. Set $\alpha = \alpha + \wp$

Step 8. If $\alpha \le 1$ goto Step 5

Step 9. Set $j = j + 1$

Step 10. If $j \le J$ goto Step 3

Step 11. Define the set of possibility distributions (π_1^f, ..., π_J^f)

STOP

In this case, the π_j^f for j = 1, …, J are determined by the intervals $[\underline{f}_{\alpha,\ j},\ \overline{f}_{\alpha,j}]$ for $\alpha_o, \alpha_o + \wp, \alpha_o + 2\wp, ..., 1$.

2.4. Measures of risk in the case of hybrid data

As a result of processing hybrid data we obtain a random fuzzy set. A problem arises regarding how to define easily interpretable measures of the risk in this case.

One can use here the method of defining the expected value and variance of a random fuzzy set. The expected value of a random fuzzy set is defined in various ways.

Most frequently, it is defined in the form of a fuzzy set [25]. However, in investment analysis, the expected value should rather be expressed in scalar form [20]. This facilitates interpretation of the results. Methods using such values are generally accepted by practitioners.

Liu [21] proposed a new method for calculating the expected value and the variance of a fuzzy random variable. He expresses these values in a scalar form. The expected value $E(\xi)$ of a normal fuzzy variable ξ defined in the probability space (Ω, Σ, P) is given by the following formula:

$$E(\xi) = \int_{\Omega}\left[\int_0^{\infty} cr\{\xi(\omega) \geq x\}dx - \int_0^{\infty} cr\{\xi(\omega) \leq x\}dx\right]P(d\omega) \tag{14}$$

where $cr\{\xi(\omega)\}$ is the credibility distribution of ξ.

Furthermore, the above authors define the variance of a fuzzy random variable using the formula:

$$Dev(\xi) = E[(\xi - E[\xi])^2] \tag{15}$$

These equations can be used to determine the expected value and standard deviation based on a random fuzzy set which is the result of the processing of hybrid data.

Another method for estimating measures of risk is the use of the upper and lower cumulative distribution function (*p*-box). As was stated in Section 2.2, these functions can be extracted from a random fuzzy set based on the belief and plausibility functions. In the case of discrete upper and lower cumulative distribution functions, as is typically obtained from the algorithm defined in Section 2.3, this comes down to assigning a probability weight $p_{\alpha j} = (1/J)\wp$ to each of a finite set of intervals $[\underline{f}_{\alpha,j}, \overline{f}_{\alpha,j}]$ for $\alpha_o, \alpha_o + \wp, \alpha_o + 2\wp, ..., 1$; $j = 1, 2, \dots, J$.

In order to determine easily interpreted risk measures, *p*-box can be converted into a cumulative distribution function describing the analyzed variable. To perform this conversion, one can use one of the many available concepts. Here several strategies for building a cumulative distribution function based on the upper and lower cumulative distribution are considered.

Dubois and Guyonnet [9] define three approaches to selecting a single probability function on the basis of the *p*-box, namely:

1. Applying the Laplace principle of insufficient reason to each focal set $[\underline{f}_{\alpha,j}, \overline{f}_{\alpha,j}]$, thus changing it into a uniformly distributed probability density function $F_{\alpha j}$ on $[\underline{f}_{\alpha,j}, \overline{f}_{\alpha,j}]$,

and using the function $F_1 = \sum_{\alpha=\alpha_0,\, \alpha_0+\wp,\, \ldots,1;\; j=1,\ldots,J} p_{\alpha,j} F_{\alpha,j}$ to compute the expected value and variance.

2. Replacing each focal set $[\underline{f}_{\alpha,j}, \overline{f}_{\alpha,j}]$ with a value $f([\underline{f}_{\alpha,j}, \overline{f}_{\alpha,j}]) \in [\underline{f}_{\alpha,j}, \overline{f}_{\alpha,j}]$, where f is increasing in both end points; then use the probability distribution function F_2 induced by the probability assignment $f([\underline{f}_{\alpha,j}, \overline{f}_{\alpha,j}])$, $p_{\alpha,j}$, $\alpha = \alpha_0, \alpha_0 + \wp, \ldots, 1$; $j = 1, \ldots, J$; to compute the expected value and variance.

3. Directly selecting a cumulative distribution function F_3 such that $F_3(x) = g(\overline{F}(x), \underline{F}(x)) \in \left[\overline{F}(x), \underline{F}(x)\right]$.

The first method was proposed by Smets [31] under the name of pignistic transformation. The second method was advocated by Jaffray and Fabrice [16]. The third method is more in line with so-called credibility theory developed by Liu [21]. Here a cumulative distribution function is reconstructed from the belief and plausibility function. This approach, proposes to compute a single indicator as a weighted average of the bounds on a focal element. It achieves a trade-off between optimistic and pessimistic estimates according to the equation:

$$F(x) = \lambda \overline{F}(x) + (1-\lambda)\underline{F}(x) \tag{16}$$

3. Proposed method of selecting of efficient project portfolios

3.1. Formulating the problem of selecting efficient project portfolios

The developed method of selecting efficient portfolios of investment projects is adapted to conditions faced by enterprises realizing multi-stage production processes. One such industry is the metallurgical industry. A mathematical model used for selecting efficient project portfolios was presented in [26]. To take into account the presence of hybrid data modifications to the task formulated in this work are necessary. The number and types of modifications depend on the number and types of parameters that are expressed by means of possibilities distribution. In the example presented in Section 4, product prices, raw material prices and indicators of the consumption of raw materials per unit of production were defined using possibility distributions. The modifications relate to these cases.

The model consists of three groups of equations. The first group includes equations defining the production capacity of the company, depending on the portfolios of projects qualified for implementation. This group also includes equations regarding a company’s material balance, and equations determining the conditions for selecting portfolios of investment projects. The second group of equations are financial equations. They enable the development of a company's financial forecasts based on a selected portfolio of investment projects and a given forecast for the parameters used in the calculation of effectiveness. The third group of equations describe the correlations between the prices of product and prices of raw material and determines the constraints on the variables defining per unit consumption indicators for products and raw materials and the variables defining the prices of products and raw materials. These constraints are defined on the basis of appropriate possibility distribution.

To present the model we adopt the following definitions. An investment project is understood to be the modernization or construction of a production department, along with the possible construction or modernization of facilities in auxiliary departments. To simplify the notation, it is assumed that leaving a production department in its current state is one of the possible investment projects. In the algorithm for selecting of efficient project portfolios presented below, it is assumed that any project, which is defined as not changing the current state of the appropriate department, is included in the implementation if no alternative project is selected to be realized. The optimization horizon is the number of years for which the net cash flows of the company are being forecasted. It is equal to the sum of the duration of capital budgeting and the longest life-cycle of the investment projects analyzed. The capital budgeting period is the time interval for which the capital budget is developed.

The following notation is formally used for the model:

N – set of indexes of the individual mutually exclusive projects,
I – set of indexes of products,
I_j – set of indexes of products produced in department j,
J – set of indexes of production departments,
B – set of indexes of raw materials,
W – set of indexes of projects,
WP – set of indexes of projects selected for implementation (project portfolios),
W_j – set of indexes of proposed projects for production department j,
WW_n – set of indexes of the n-th set of mutually exclusive projects,
$\ddot{t}_{jw}$ – duration of the life cycle of project w implemented in department j,
$\dddot{t}$ – time horizon of the optimization,
$\ddot{\tau}$ – capital budgeting period,
v_j^{ς} – manufacturing capacity of department j, in year ς of the life cycle, $\varsigma = t - \tau$
pd^t – tax rate of interest in year t,
ms_i^t – forecasted market share for product i in year t,
g_i^t – forecasted apparent consumption of product i in year t,
kz_{ijw}^{ς} – variable processing cost for product i manufactured in production department j after implementing project w in year ζ of the life cycle, $\varsigma = t - \tau$,

m_{ijzw}^{ς} – amount of product *i* required for producing a unit of product *z* in department *j* after implementing project *w*, in year ς of the life cycle, $\varsigma = t - \tau$,

m_{bijw}^{ς} – amount of raw material *b* required for producing a unit of product *i* in department *j* after implementing project *w* in year ς of the life cycle, $\varsigma = t - \tau$,

c_i^t – selling price of product *i* in year *t*,

c_b^t – price of raw material *b* in year *t*,

η_{jw}^{ς} – forecasted investment outlay in year *t* for project *w* in department *j* whose implementation started in year τ, $\varsigma = t - \tau$,

$s^{t\tau}$ – index representing the value of loan repayments in year *t* for the loan taken out in year τ,

bp^t – current ratio in year *t*,

sq^t – cash ratio in year *t*,

cz^t – inventory turnover ratio in year *t*,

cna^t – days sales outstanding ratio for year *t*,

czb^t – days payable outstanding ratio for year *t*,

a_1^{iz}, a_2^{iz} – coefficients of interval regression equation defining relation between prices of product *i* and *z*,

a_1^{ib}, a_2^{ib} – coefficients of interval regression equation defining relation between the price of product *i* and the price of raw material *b*,

M_{ijzw}^{ς} – variable determining the amount of product *i*, required for producing a unit of product *z* in department *j*, after implementing project *w*, in year ς of the life cycle, $\varsigma = t - \tau$,

M_{bijw}^{ς} – variable determining the amount of raw material *b* required for producing product *i* in department *j*, after implementing project *w*, in year ς of the life cycle, $\varsigma = t - \tau$,

C_i^t – variable determining the selling price of product *i* in year *t*,

$\mathbf{C}_b^t$ – variable determining the price of raw material *b* in year *t*,

$\mathbf{G}_i^t$ – variable determining the sales of product *i* in year *t*,

KRK^t – variable determining the value of short-term credit in year *t*,

KRD^t – variable determining the value of long-term credit in year *t*,

ZKD^t – variable determining the value of the long-term loan taken out in year *t*,

ZKK^t – variable determining the value of the short-term loan taken out in year *t*,

SKK^t – variable determining the value of the short-term loan repayments in year *t*,

KC^t – variable determining the cost of goods sold in year *t*,

ZO^t – variable determining operating profit in year *t*,

ZB^t – variable determining gross profit in year *t*,

ZKO^t – variable determining change in net working capital in year *t*,

ZN^t – variable determining net profit in year *t*,

SP^t – variable determining cash value in year *t*,

$NCF_{\Pr z}^t$ – variable determining a company's net cash flow in year *t*,

$\overline{NPV}_{WP}$ – variable determining the expected value of the *NPV* of investment project portfolio *WP*,

σ_{WP} – variable determining the value of the standard deviation of the *NPV* of investment project portfolio *WP*,

$\xi^t(WP)$ – function assigning a company's fixed costs without amortization in year *t*, to any project portfolio,

$\chi^t(WP)$ – function assigning the value of amortization in year *t*, to any project portfolio.

The equations from the first group are presented below:

- The equations for balancing manufacturing capacities in production departments

$$\sum_{i\in I_j} \mathrm{Pr}_{ijw}^{t\tau} \le v_{jw}^{\varsigma} W_{jw}^{\tau},$$

$$\tau = 0, 1, 2, ..., \ddot{\tau}, \ j \in J, w \in W_j, \ t = \tau, \tau+1, \ \tau+2, ..., \tau + \dddot{t}_{jw} \tag{17}$$

$$W_{jw}^{\tau} = \begin{cases} 1 \text{ for } w \in WP \\ 0 \text{ for } w \in W - WP \end{cases},$$

$$\tau = 0, 1, 2, ..., \ddot{\tau}, \ j \in J, w \in W_j, t = \tau, \tau+1, \tau+2, ..., \tau + \dddot{t}_{jw} \tag{18}$$

$$\sum_{\tau=0}^{\ddot{\tau}} W_{jw}^{\tau} \le 1 \text{ for } j \in J, w \in W_j \tag{19}$$

$$\sum_{j\in WW_n} W_{jw}^{\tau} \le 1 \text{ for } n \in N \tag{20}$$

$$\sum_{j\in J} \sum_{w\in W_j} \sum_{\tau=0}^{t} \eta_{\mathrm{jw}}^{\varsigma} W_{jw}^{\tau} \le \eta_0^t \text{ for } t = 0, 1, 2, ..., \ddot{t} \tag{21}$$

- The equations for balancing the company's materials

$$\sum_{j\in J} \sum_{w\in W_j} \sum_{\tau\le t} \mathrm{Pr}_{ijw}^{t,\tau} - \sum_{j\in J} \sum_{w\in W_j} \sum_{z\in I} \sum_{\tau\le t} M_{izjw}^{\varsigma} \mathrm{Pr}_{zjw}^{t,\tau} - G_i^t = g_i^t \times ms_i^t \text{ for } i \in I;\ t = 0, 1, 2, ..., \ddot{t} \tag{22}$$

Equations (17)–(18) determine the quantity and structure of production in each department in consecutive years of the optimization horizon depending on the portfolio of projects qualified for implementation. The value of the binary variable W_{jw}^{τ} indicates whether a project is qualified for implementation. This variable also determines the year of the budgeting period in which the corresponding project is qualified for implementation. After establishing the values of the binary variables W_{jw}^{τ}, the production capacity is determined for each individual stage of the company's technological cycle. Equations (19)–(21)

determine the feasible sets of projects qualified for implementation. Equation (19) expresses the conditions under which investment projects become mutually exclusive. Equation (20) expresses the condition that each project can only qualify for implementation in one year of the budgeting period. Equation (21) represents the constraint that the outlays on projects qualified for implementation are lower in year *t* than the maximum values η_0^t. Equation (22) denotes the balancing of the company's materials. It determines the distribution of production of specific products for sales and for internal production use.

Equations (17)–(22) determine the feasible sets of projects qualified for implementation, values of the total production of specific departments, values of sales and investment outlays for the implementation of projects.

The second set of equations defining the model are financial equations. They are linear equations, which for all the above mentioned parameters (determined by Eqs. (17)–(22)), determine the individual items of the financial balance sheet, profit and loss statement and cash flow reports of the company. They also ensure the preservation of the appropriate relations between the projected items of a company's financial statements. These relations are determined by given constraints on the values for selected financial ratios used in the financial analysis of a company. Some example equations from this second group are presented below. These equations determine the cost of goods sold, operating profits, gross profit, net profit, changes in net working capital, cash value, long-term and short-term loans, and net cash flow (*NCF*). An equation which ensures that the value of the current ratio does not exceeded its limit has also been presented. These equations express dependencies that are well-known in financial studies. Therefore, a detailed presentation and discussion of all the equations from the second group has been omitted.

The equations from this second group are presented below:

$$KC^t = \sum_{\tau \le t} \sum_{w \in W_j} \sum_{j \in J} \sum_{i \in I_j} k_{ijw}^{\varsigma} \, \mathrm{Pr}_{ijw}^{t,\tau} + \sum_{b \in B} \sum_{\tau \le t} \sum_{w \in W_j} \sum_{j \in J} \sum_{i \in I} C_b^t M_{bijw}^{\varsigma} \, \mathrm{Pr}_{ijw}^{t,\tau} + \chi^t (WP) + \xi^t (WP) \text{ for } t = 0, 1, \ldots, \dddot{t} \tag{23}$$

$$ZO^t = \sum_{i \in I} C_i^t G_i^t - KC^t \text{ for } t = 0, 1, \ldots, \dddot{t} \tag{24}$$

$$ZB^t = ZO^t + rk^t KRK^t + rd^t KRD^t \text{ for } t = 0, 1, \ldots, \dddot{t} \tag{25}$$

$$ZN^t = \begin{cases} ZB^t \text{ if } & ZB^t \le 0 \\ pd^t ZB^t \text{ if } & ZB^t \ge 0 \end{cases} \text{ for } t = 0, 1, \ldots, \dddot{t} \tag{26}$$

$$ZKO^t = \left(sq^t \left(\frac{czb^t}{360}\left(KC^t - \chi^t(WP)\right) + KRK^t \right) \right.$$

$$\left. - sq^{t-1}\left(\frac{czb^{t-1}}{360}\left(KC^{t-1} - \chi^{t-1}(WP)\right) + KRK^{t-1} \right) \right)$$

$$+ \left(\frac{cna^t}{360}\sum_{i\in I} C_i^t G_i^t - \frac{cna^{t-1}}{360}\sum_{i\in I} C_i^{t-1} G_i^{t-1} \right) + \left(\frac{cz^t}{360} KC^t - \frac{cz^{t-1}}{360} KC^{t-1} \right)$$

$$-\left(\frac{czb^t}{360}\left(KC^t - \chi^t(WP)\right) - \frac{czb^{t-1}}{360}\left(KC^{t-1} - \chi^{t-1}(WP)\right) \right)$$

$$-\left(KRK^t - KRK^{t-1}\right) \text{ for } t = 0, 1, ..., \dddot{t} \tag{27}$$

$$SP^{t-1} + ZN^t + \left(\frac{czb^t}{360}\left(KC^t - \chi^t(WP)\right) - \frac{czb^{t-1}}{360}\left(KC^{t-1} - \chi^{t-1}(WP)\right) \right)$$

$$-\left(\frac{cz^t}{360} KC^t - \frac{cz^{t-1}}{360} KC^{t-1} \right) - \left(\frac{cna^t}{360}\sum_{i\in I} C_i^t G_i^t - \frac{cna^{t-1}}{360}\sum_{i\in I} C_i^{t-1} G_i^{t-1} \right)$$

$$+ZKK^t - SKK^t + ZKD^t - \sum_{t=1,\ \tau<1} s^{t\tau} ZKD^t \tag{28}$$

$$-\sum_{j\in J}\sum_{w\in W_j}\sum_{\tau=0}^{t} \eta_{jw}^{\varsigma} W_{jw}^{\tau} + \chi^t(WP) - SP^t = 0 \quad \text{for } t = 0,1,\cdots,\dddot{t}$$

$$KRK^{t-1} + ZKK^t - SKK^t - KRK^t = 0 \text{ for } t = 0, 1, ..., \dddot{t} \tag{29}$$

$$KRD^{t-1} + ZKD^t - \sum_{t=1,\ \tau<1} s^{t\tau} ZKD^t - KRD^t = 0 \text{ for } t = 0, 1, ..., \dddot{t} \tag{30}$$

$$\frac{cz^t}{360} KC^t + \frac{cna^t}{360}\sum_{i\in I} C_i^t G_i^t + SP^t$$

$$- bp^t \left(\frac{czb^t}{360}\left(KC^t - \chi^t(WP)\right) + KRK^t \right) \geq 0 \text{ for } t = 0, 1, ..., \dddot{t} \tag{31}$$

$$NCF^t_{\Pr z} = \begin{cases} ZN^t + \chi^t(WP) - ZKO^t - \sum_{j\in J}\sum_{w\in W_j}\sum_{\tau=0}^{t}\eta^{\varsigma}_{jw}W^{\tau}_{jw} + ZKK^t \\ - SKK^t + ZKD^t - \sum_{t=1,\tau<1}^{\bar{t}} s^{t\tau}ZKD^t \quad \text{for } t = 0, ..., \ddot{t} - 1 \\ ZN^t + \chi^t(WP) - ZKO^t - \sum_{j\in J}\sum_{w\in W_j}\sum_{\tau=0}^{t}\eta^{\varsigma}_{jw}W^{\tau}_{jw} + ZKK^t \\ - SKK^t + ZKD^t - \sum_{t=1,\tau<1} s^{t\tau}ZKD^t + \frac{cz^t}{360}KC^t + \frac{cna^t}{360}\sum_{i\in I}C^t_iG^t_i \\ + sq^t\left(\frac{czb^t}{360}\left(KC^t - \chi^t(WP)\right) + KRK^t\right) \\ - \frac{czb^t}{360}\left(KC^t - \chi^t(WP)\right) - KRD^t - KRK^t \quad \text{for } t = \ddot{t} \end{cases} \tag{32}$$

The equations from the third group are presented below:

$$\inf\left(c^t_i\right)_{\alpha} \le C^t_i \le \sup(c^t_i)_{\alpha} \quad \text{for } I \in I,\ t = 1, 2, \ldots, \ddot{t} \tag{33}$$

$$\inf\left(c^t_b\right)_{\alpha} \le C^t_b \le \sup\left(c^t_b\right)_{\alpha} \quad \text{for } b \in B,\ t = 1, 2, ..., \ddot{t} \tag{34}$$

$$\inf\left(m^{\varsigma}_{ijzw}\right)_{\alpha} \le M^{\varsigma}_{ijzw} \le \sup\left(m^{\varsigma}_{ijzw}\right)_{\alpha} \text{ for } j\in J,\ i \in I,$$

$$w\in W,\ z \in W,\ t = 1, 2, ..., \ddot{t},\ \tau = 0, 1, 2, ..., \ddot{\tau},\ \varsigma = t - \tau \tag{35}$$

$$\inf\left(m^{\varsigma}_{bijw}\right)_{\alpha} \le M^{\varsigma}_{bijw} \le \sup\left(m^{\varsigma}_{bijw}\right)_{\alpha} \quad \text{for } j\in J,\ i \in I,$$

$$w \in W,\ b \in B,\ t = 1, 2, ...,\ \ddot{t},\ \ \tau = 0, 1, 2, ..., \ddot{\tau},\ \varsigma = t - \tau \tag{36}$$

$$C^t_i \ge \inf\left(a^{iz}_1\right)C^t_z + \inf\left(a^{iz}_2\right) \quad \text{for } i \in I,\ z \in I,\ i \ne z,\ \text{t} = 1, 2, ..., \ddot{t} \tag{37}$$

$$C^t_i \le \sup\left(a^{iz}_1\right)C^t_z + \sup\left(a^{iz}_2\right) \quad \text{for } i \in I,\ z \in I,\ i \ne z,\ \ t = 1, 2, ..., \ddot{t} \tag{38}$$

$$C^t_i \ge \inf\left(a^{ib}_1\right)C^t_b + \inf\left(a^{ib}_2\right) \quad \text{for } i \in I, b \in B,\ t = 1, 2, ..., \ddot{t} \tag{39}$$

$$C^t_i \le \sup\left(a^{ib}_1\right)C^t_b + \sup\left(a^{ib}_2\right) \quad \text{for } i \in I, b \in B,\ t = 1, 2, ..., \ddot{t} \tag{40}$$

The parameters found in these equations: $v^{\varsigma}_{jw}, g^t_i, ms^t_i, \eta^{\varsigma}_{jw}, \eta^t_0, \ddot{t}, \ddot{t}_{jw}, kz^{\varsigma}_{ijw}, rk^t, rd^t$ can be expressed in the form of probability distribution or as crisp value. In our example, the parameters $m^{\varsigma}_{ijzw}, m^{\varsigma}_{bijw}, c^t_i, c^t_b$ are defined in the form of possibility distributions.

The multi-criteria problem of choosing efficient project portfolios is defined as follows: find non-dominated portfolios of investment projects by assuming equations and inequalities (17)–(40) as constraints in the model and the following as criteria:

$$\overline{NPV}_{WP} \to \max \tag{41}$$

$$\sigma_{WP} \to \min \tag{42}$$

The method of estimating $\overline{NPV}_{WP}$ and σ_{WP} is presented in Section 3.2. In the following section, the algorithm for generating efficient project portfolios is presented. For the optimization problem formulated above, we are looking for the Pareto optimal solutions.

3.2. The algorithm for selecting efficient investment project portfolios

The algorithm used to select efficient project portfolios in the case of hybrid data is presented below:

```
        START
Define lp
Define the set of non-dominated solutions PE = ∅
  lp = 0
  Repeat
  Generate WP
  Calculate the expected value of NPV_WP (avg NPV_WP) and standard deviation of
  NPV_WP (σ_WP)
    If avg NPV_WP ≥ 0 and WP is not dominated by any solution in PE
      then
        Add WP to PE
        Remove from PE all the solutions dominated by the solutions added
                                   lp = 0
      else
        lp = lp + 1
      End if
Until: lp = Lp
  Present set PE
        STOP
```

The parameter *Lp* denotes the number of iterations of the algorithm after which the calculation process is terminated if no new solution dominating existing solutions has been found. A genetic algorithm is used to generate *WP* sets. The selection of *WP* sets is made by taking into account the criteria described by formulas (41), (42) and inequalities (18)–(21).

The expected value and standard deviation of the *NP*V are estimated by means of the method of hybrid data processing described in Section 2.3. After randomly drawing the parameters described by probability distributions, to determine the α-level of the fuzzy set defining the *NPV* we should solve a non-linear programming problem. Multiple draws of the parameters described by the probability distributions and calculations of the α-level of the fuzzy set defining the *NP*V enables us to determine the expected value and standard deviation of the *NPV*.

The algorithm for estimating the expected value and standard deviation of the *NPV* is presented below.

START
Define α, $\wp$, N
For n = 1 to N
Repeat
Generate values of the uncertain parameters described by probability distribution
Define α-levels for the uncertain parameters described by possibility distributions
Define the right hand sides of the inequality (17) in accordance with the set *WP*. Solve the non-linear programming problem, whose constrains are specified by equations and inequalities (17), (22–40). The objective function is expressed by:

$$\sum_{t=0}^{\bar{t}} \frac{1}{(1+r_{\text{dis}})^t} NCF^t_{\text{Pr}\,z(WP)}(n) \to \max$$

Define the right hand side of inequalities (17) assuming that no investment project is qualified for execution. Solve the nonlinear programming problem, whose constraints are specified by equations and inequalities (17), (22–40). The objective function is expressed by:

$$\sum_{t=0}^{\bar{t}} \frac{1}{(1+r_{\text{dis}})^t} NCF^t_{\text{Pr}\,z(\varnothing)}(n) \to \max$$

Calculate $NPV_{WP}(n) = \sum_{t=0}^{\bar{t}} \frac{1}{(1+r_{\text{dis}})^t} NCF^t_{WP}(n)$, $NCF^t_{WP}(n)$ is calculated according to formula (43)

$\alpha = \alpha + \wp$
Until $\alpha > 1$
Next n
Define the set of possibility distributions ($\pi_1^{NPV}, ..., \pi_N^{NPV}$)
Calculate the expected value NPV_{WP} ($\overline{NPV}_{WP}$) and standard deviation NPV_{WP} (σ_{WP})
STOP

According to the presented algorithm, realization of each parameter defined by a probability distribution a randomly generated. This process takes into account the correlation between these parameters. Next α-levels are set for each uncertain parameter defined by a possibility distribution. Then, production and sales quantities and also the financial results of a company are optimized. Optimization is carried out for two variants: the former assumes the implementation of a selected portfolio of investment projects and the latter assumes that it is not implemented. For this purpose, a nonlinear optimization problem is solved. The nonlinear optimization problem is defined by equations and inequalities (17), (22–40). In the first variant, the optimization criterion is given by

$$\sum_{t=0}^{\bar{t}} \frac{1}{\left(1+r_{\text{dis}}\right)^t} NCF^t_{\Pr z(WP)} \to \max$$

(where $NCF^t_{\Pr z(WP)}$ denotes the forecasted net cash flows of a company, assuming that investment project *WP* is implemented, r_{dis} denotes the discount rate) and in the second variant,

$$\sum_{t=0}^{\ddot{t}} \frac{1}{\left(1+r_{\text{dis}}\right)^t} NCF^t_{\Pr z((\varnothing)} \to \max$$

(where $NCF^t_{\Pr z((\varnothing)}$ denotes the forecasted net cash flows of a company in the case of this investment projects not being implemented). $NCF^t_{\Pr z((WP)}$ and $NCF^t_{\Pr z((\varnothing)}$ are calculated according to formula (32). The net cash flows connected with the analyzed portfolio (NCF^t_{WP}) are calculated using the formula:

$$NCF^t_{WP} = \begin{cases} \left(\left(1-pd^t\right) ZO^{*t}_{(WP)} + \chi^t\left(WP\right) - ZKO^{*t}_{(WP)} - \sum\limits_{j\in J}\sum\limits_{w\in W_j}\sum\limits_{\tau=0}^{t} \eta^{\varsigma}_{jw} W^{*\tau}_{jw} \right) \\ -\left(\left(1-pd^t\right) ZO^{*t}_{(\varnothing)} + \chi^t\left(\varnothing\right) - ZKO^{*t}_{(\varnothing)}\right) \text{ for } t = 0, \ldots, \ddot{t}-1 \\ \left(\left(1-pd^t\right) ZO^{*t}_{(WP)} + \chi^t\left(WP\right) - ZKO^{*t}_{(WP)} - \sum\limits_{j\in J}\sum\limits_{w\in W_j}\sum\limits_{\tau=0}^{t} \eta^{\varsigma}_{jw} W^{*\tau}_{jw} \right. \\ + sq^t\left(\dfrac{czb^t}{360}\left(KC^{*t}_{(\varnothing)} - \chi^t\left(\varnothing\right)\right) + KRK^{*t} \right) \\ \left. + \ \dfrac{cz^t}{360} KC^{*t}_{(WP)} + \dfrac{cna^t}{360}\sum\limits_{i\in I} C^{*ta}_i G^{*ta}_{i(WP)} - \dfrac{czb^t}{360}\left(KC^{*t}_{(WP)} - \chi^t\left(WP\right)\right) \right) \\ -\left(\left(1-pd^t\right) ZO^{*t}_{(\varnothing)} + \chi^t\left(\varnothing\right) - ZKO^{*t}_{(\varnothing)} \right. \\ + sq^t\left(\dfrac{czb^t}{360} \times \left(KC^{*t}_{(\varnothing)} - \chi^t\left(\varnothing\right)\right) + KRK^{*t} \right) + \dfrac{cz^t}{360} KC^{*t}_{(\varnothing)} \\ \left. + \dfrac{cna'}{360}\sum\limits_{i\in I} C^{*ta}_i G^{*ta}_{i(\varnothing)} - \dfrac{czb^t}{360}\left(KC^{*t}_{(\varnothing)} - \chi^t\left(\varnothing\right)\right)\right) \text{for} \quad t = \ddot{t} \end{cases} \tag{43}$$

The quantities in formula (43) denoted with an asterisk in superscript (*) are the optimal values of the corresponding variables, obtained from solving the optimization tasks mentioned above. The subscripts *WP* and $\varnothing$ indicate the values of variables corresponding to a selected portfolio *WP* of investment projects and the case of it not being implemented, respectively. The *NPV* of a selected portfolio is calculated using the following formula:

$$NPV_{WP} = \sum_{t=0}^{\ddot{t}} \frac{1}{(1+r_{\text{dis}})^t} NCF_{WP}^t \tag{44}$$

The procedure for randomly generating values of the uncertain parameters in the calculation of effectiveness and calculating the *NPV* for different α is repeated *N* times. As a result, the set of possibility distribution for the *NPV* of the analysed investment project portfolio are found. The expected value and standard deviation of the *NPV* are determined based on the cumulative probability distribution function defined by equation (16). We assume $\lambda = 0.5$.

4. Numerical examples

In order to illustrate the applicability and utility of the proposed method, the results of calculations for a simple model problem are presented below. The method was verified on a modification of the example of investment projects for a metallurgical enterprise described in [26]. Efficient project portfolios were determined for the production setup presented in Fig. 1.

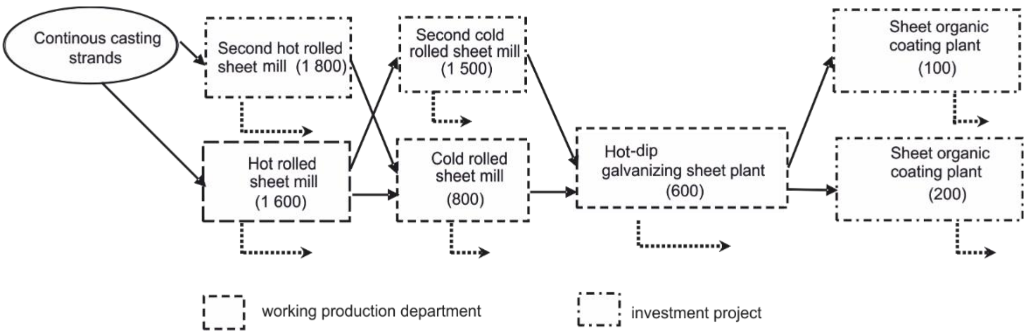


Fig. 1. Diagram of the analyzed production setup;
production capacity in thousand tonnes is given in parentheses

For the analysed company, continuous casting semi-products (CC semi-products) are the basic production material. The CC semi-products are converted into hot-rolled sheets (HR sheets). The HR sheets are partly converted into cold-rolled sheets (CR sheets) and partly sold. The CR sheets are partly sold and partly converted into hot-dip galvanized sheets (HDG sheets). The HDG sheets are partly sold and partly converted into organic-coated sheets (OC sheets). The latter are all sold.

The computations took into consideration the uncertainty regarding selected parameters. The apparent consumption of the company's products was defined by normal probability density functions. The prices of these products, prices of CC semi-products and consumptions of materials per production unit were defined by possibility distributions. It was assumed, that the remaining parameters in the calculations of the effectiveness were fixed constants. Table 1 presents the considered investment projects.

Table 1. Investment projects being taken under consideration

No.	Investment project	Investment outlay, thousand USD
1	Construction of a second HR sheet mill	135 000.0
2	Construction of a second CR sheet mill	170 000.0
3	Construction of a new department of OC sheet, production capacity 100 thousand tonnes	12 000.0
4	Construction of a new department of OC sheet, production capacity 200 thousand tonnes	18 000.0

The construction of a new department of OC sheet, production capacity 100 thousand tonnes and the construction of a new department of OC sheet, production capacity 200 thousand tonnes were mutually exclusive projects. A budgeting period of three years was assumed when constructing the model. We assume that projects 1 and 2 could be qualified for implementation in any year of the budgeting period and projects 3 and 4 in the second and third years of the budgeting period. A four-year life cycle is assumed for each project. This assumption required using 10 binary variables. The individual binary variables determine whether or not to implement the projects listed in Table 1 in the appropriate year of the capital budgeting period. Besides these binary variables, there were also 284 continuous variables used in the model. There were 342 constraints defined by the model.

Table 2. Parameters of the probability density functions representing forecasts for the apparent consumption of particular product ranges [10^3 t]

Year	HR sheets (m; σ)	CR sheets (m; σ)	HDG sheets (m; σ)	OC sheets (m; σ
0	(2 905.4; 123.4)	(1 242.3; 72.5)	(1 182.7; 69.8)	(805.4; 41.5)
1	(2 971.9; 127.5)	(1 288.0; 73.3)	(1 187.5; 72.0)	(828.2; 42.2)
2	(2 982.4; 131.4)	(1 291.4; 75.2)	(1 197.0; 74.0)	(846.9; 43.2)
3	(2 984.5; 131.8)	(1 292.0; 78.2)	(1 216.1; 75.2)	(855.2; 43.7)
4	(3 071.8; 137.3)	(1 332.1; 79.8)	(1 256.8; 78.5)	(883.1; 44.1)
5	(3 110.9; 139.3)	(1 366.2; 82.5)	(1 278.5; 79.0)	(908.9; 44.6)
6	(3 234.2; 140.0)	(1 397.1; 82.7)	(1 285.2; 79.1)	(913.4; 46.0)
7	(3 317.2; 142.4)	(1 407.1; 84.2)	(1 291.3; 82.5)	(918.0; 46.3)

Table 2 presents the parameters of the probability density function representing apparent steel consumption for the company's products under analysis. Table 3 presents the relevant trapezoidal fuzzy numbers representing forecasts of the prices of products and of CC semi-products. We assume fixed prices throughout the period of budgeting. Table 4 presents trapezoidal fuzzy numbers reflecting indicators of material consumption for the individual stages of processing under analysis.

Table 3. Trapezoidal fuzzy numbers representing the prices of the company's products and continuous casting semi-products

Price [USD/t]	Trapezoidal fuzzy numbers
CC semi-products	(410.0; 450.0; 500.0; 550.0)
HR sheets	(465.0; 515.0; 565.0; 615.0)
CR sheets	(565.0; 615.0; 900.0; 715.0)
HDG sheets	(785.0; 840.0; 1080.0; 1250.0)
OC sheets	(955.0; 1 120.0; 1250.0; 1 345.0)

Table 4. Trapezoidal fuzzy numbers representing indicators of material consumption

Material consumption indicator [t/t]	Trapezoidal fuzzy numbers
CC semi-products–HR sheets	(1.058; 1.064.0; 1.075; 1.078)
HR sheets–CR sheets	(1.105; 1.111; 1.124; 1.130)
CR sheets–HDG sheets	(1.010; 1.020; 1.026; 1.031)
HDG sheets–OC sheets	(0.998; 0.999; 1.000; 1.001)

The prices of individual assortments of steel products and CC semi-products are strongly correlated. Similarly, the apparent consumption of particular product ranges are correlated. Table 5 presents the correlation matrix for the apparent consumption of individual product ranges. Table 6 presents the coefficients of the interval regression equations depicting the interrelations between the prices of individual products and of CC semi-products.

The adjusted unit variable processing costs for particular product ranges were also defined to be the values given below:

Adjusted unit variable processing cost, USD/tnne	CC semi-products	HR sheet	CR sheets	HDG sheets	OC sheets
	26.5	28.4	28.9	117.8	174.0

The following market share for particular product ranges were adopted in the calculations:

Market share for particular product ranges	HR sheets	CR sheets	HDG sheets	OC sheets
	42.5%	40.0%	46.0%	45.0%

Table 5. Correlation matrix for the apparent consumption
of metallurgical products manufactured by the analyzed company

	HR strip	CR sheets	HDG sheets	OC sheets
HR sheets	1.000	0.878	0.911	0.863
CR sheets	0.878	1.000	0.915	0.888
HDG sheets	0.911	0.915	1.000	0.966
OC sheets	0.863	0.888	0.966	1.000

Table 6. Coefficients of the interval regression equations depicting the interrelations
between the prices of particular product ranges and the prices of CC semi-products

Independent variable		Dependent variable				
		CC semi–products	HR sheet	CR sheets	HDG sheets	OC sheets
CC semi--products	a_1		[0.702; 0.880]	[0.396; 0.560]	[0.353; 0.455]	[0.312; 0.421]
	a_2		[3.666; 4.595]	[101.812; 143.900]	[101.805; 131.017]	[49.311; 66.675]
HR sheet	a_1	[1.050; 1.337]		[0.597; 0.790]	[0.481; 0.664]	[0.394; 0.663]
	a_2	[22.011; 28.014]		[112.164; 148.390]	[112.607; 155.430]	[31.683; 53.319]
CR sheets	a_1	[0.497; 0.697]	[0.868; 1.404]		[0.753; 0.917]	[0.570; 0.955]
	a_2	[128.781; 180.537]	[–10.590; –17.121]		[–2.176; –2.650]	[–95.300; –159.728]
HDG sheets	a_1	[0.445; 0.568]	[0.964; 1.504]	[0.980; 1.220]		[0.596; 1.068]
	a_2	[127.513; 162.700]	[45.667; 71.292]	[57.877; 72.083]		[–48.893; –87.593]
OC sheets	a_1	[0.347; 0.539]	[0.354; 0.866]	[0.808; 1.313]	[0.633; 1.123]	
	a_2	[69.208; 107.604]	[385.568; 941.971]	[248.176; 403.481]	[236.971; 420.889]	

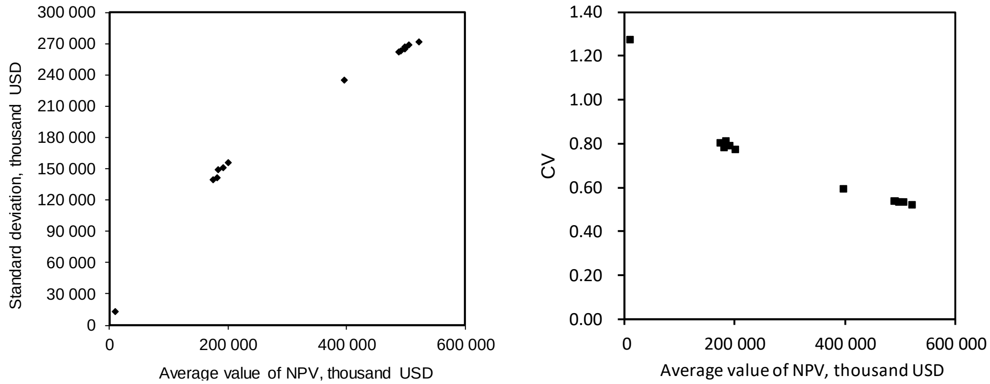
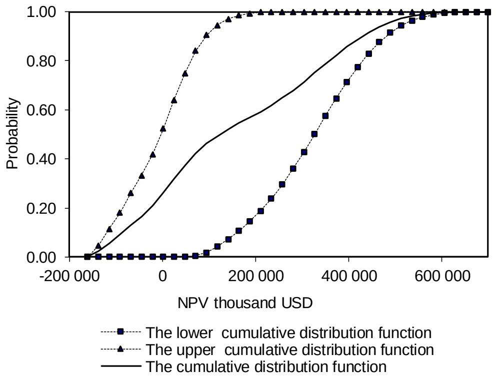
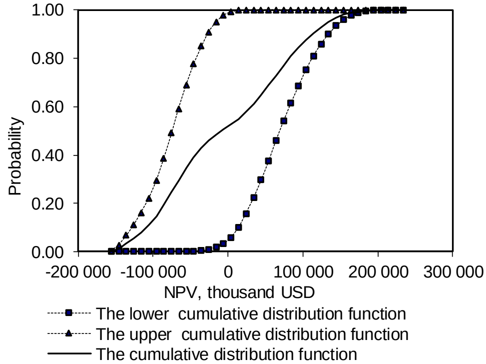


Fig. 2. The non-dominated solution

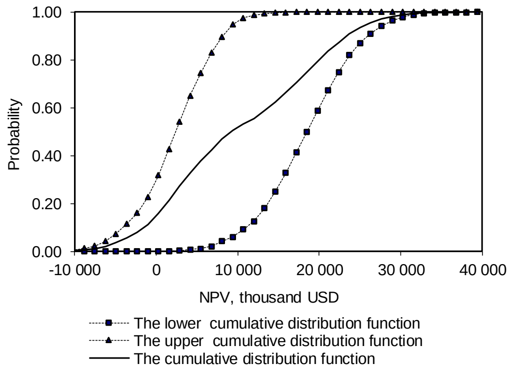
Figure 2 presents the set of non-dominated portfolios found using the presented algorithm to select efficient project portfolios. The coefficient of variation *CV* was calculated as the ratio between the standard deviation of the *NPV* and average value of the *NPV*.



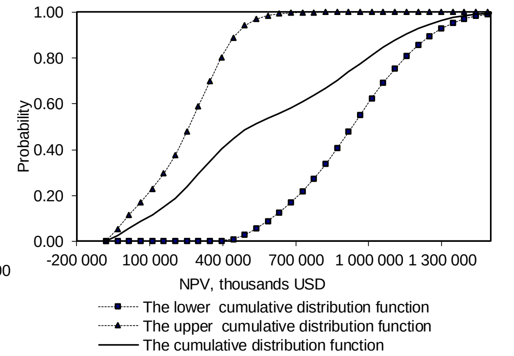
Construction of the second hot rolled sheet mill – commencement of construction: year 0; average value of NPV – 114 544.8 thousand USD, standard deviation of NPV – 131 299.8 thousand USD



Construction of the second cold rolled sheet mill – commencement of construction: year 1; average value of NPV – 2 449.8 thousand USD, standard deviation of NPV –62 195.7 thousand USD



Construction of a new department of organic sheet coating (production capacity 100 thousand tonnes) – commencement of construction: year 2; average value of NPV – 9 929.5 thousand USD, standard deviation of NPV – 6 423.7 thousand USD



An investment portfolio; average value of NPV – 521 745.3 thousand USD, standard deviation of NPV – 395 746.2 thousand USD

Fig. 3. Upper and lower cumulative distribution functions for selected projects and the investment portfolio of these projects

Figure 3 presents the upper and lower cumulative distribution functions, $\overline{F}(x)$ and $\underline{F}(x)$, respectively, and cumulative distribution function ($F(x)$ calculated according to formula (16)) for the following projects: construction of a second HR sheet mill – commencement of construction: year 0, construction of a second CR sheet mill – commencement of construction: year 1, construction of a new department of OC sheet (production capacity 100 thousand tonnes) – commencement of construction: year 2, and these results correspond to a project portfolio which includes the above-mentioned projects.

The data presented in Fig. 2 allow us to state that despite the existence of a strong correlation between the prices of steel products and their apparent consumption volumes, the effects of diversifying the portfolio are clearly noticeable. An increase in the expected values of the *NPV* of the investment portfolio, is accompanied by an increase in the standard deviations. However, the dynamics of change in the expected values and standard deviations of the *NPV* vary. As a result, the ratio *CV* varies in the range of 0.52–1.27.

The data presented in Fig. 3 illustrate well the effects of the economic dependencies between projects. The sum of the expected values of the *NPV* of the projects under analysis is equal to 122 024.5 thousand USD. The *NPV* of the portfolio, taking into account the effects of the economic dependencies between the investment projects, is almost 4.3 times greater (521 745.3 thousand USD) than the sum of *NPV* of the projects analysed separately. We can say that the projects included in the portfolio are complementary projects.

5. Conclusion

In the last dozen years or so, the treatment of uncertainty in risk assessments and portfolio selection has witnessed a shift of paradigm with an increasing awareness of the difference between stochastic and epistemic uncertainties. The lack of any distinction between these two types of uncertainties has been one of the shortcomings in portfolio selection and risk assessment. Random variability can be represented by probability distribution, imprecision or incomplete information is better accounted by possibility distributions (or the families of probability distributions). Therefore new methods combining these two modes of representing uncertainty are needed for the selection of efficient project portfolios.

A new method for selecting efficient project portfolios has been introduced in this paper. This method advances previous works concerning the selection of efficient project portfolios. The presented method allows us to select efficient project portfolios taking into account statistical and economic dependencies between projects in the situation where the data are expressed in the form of interactive hybrid data. The selection of efficient project portfolios is formulated as a multi-criteria optimization problem. Therefore, we seek a set of non-dominated investment portfolios.

The example calculations presented indicate that economic dependencies significantly affect the effectiveness and risk of investment projects. In view of this, selecting efficient portfolios without considering such relations would be burdened with a significant error.

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Received 31 July 2016
Accepted 18 December 2016