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FINANCING UNIVERSITY EDUCATION IN NIGERIA. A PROPOSAL FOR INFRASTRUCTURE REPLENISHMENT IN UNIVERSITIES

The subject of replenishment of infrastructure in Nigerian public universities has been of great concern to stakeholders in the educational system. How to obtain an appropriate replenishment plan that would give the desired infrastructure for a university after a certain period of time is a long-standing problem. We attempt to find a solution to this problem from an engineering perspective based on optimal control theory. The revenue generated through the payment of school fees and the costs of investment in infrastructure are used to construct the objective function. The state variables are the amount budgeted for such an investment and the stock of infrastructure, while the rate of replenishment is used as the control variable. The problem is solved by utilising Pontryagin's principle. The dynamics of the replenishment plan is illustrated with an example. The results show that there should be a steady increase in the amount budgeted, in order to attain the desired infrastructure.

Keywords: *Hamilton–Pontryagin equation, higher education, optimal control theory, replenishment, university education*

1. Introduction

The subject of this paper is financing physical facilities which are necessary for a university to function. These facilities include: well-equipped laboratories, classrooms, lecture theatres, offices, libraries, hostels, health centres, etc. We refer to these physical facilities as infrastructure (or capital stock). Since the facilities are heterogeneous, an appropriate unit of measurement is required. We define the unit of measurement

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for the facilities in monetary terms. We derive formulas for the replenishment of infrastructure for a specific facility in a public university. A specific facility is considered, so as to avoid the problem of aggregation. The study focuses on the public university system in Nigeria, because the government's effort has been on opening many universities without paying much attention to the required infrastructure. This effort by the government has adversely affected the level of productivity of graduates from the universities [1]. Even so, the Academic Staff Union of Universities (ASUU) has expressed its discontentment over the poor state of infrastructure in Nigerian public universities and proposed the need for a replenishment plan within a fixed time horizon. Although the government is willing to develop a blueprint for infrastructural replenishment in the public universities, the budget is a constraint. This is because the replenishment of infrastructure constitutes a cost to the system and drains finances. This is all the more aggravated by the fact that most educational facilities do not only depreciate over time, but also do not have a "second-hand" value.

The aim of this study is to find a suitable framework for the replenishment of infrastructure which could aid the government (or a university in the case of autonomy) in planning its activities. This aim is achieved in the next three sections. Section 2 generally describes the problems of financing and organisation of universities in Nigeria by citing relevant works. Section 3 develops an optimal control model for the replenishment plan. The state variables of the model are the capital stock and the amount budgeted for the build-up of capital. The control variable is the rate of replenishment. In Section 4, three formulas are derived by utilising Pontryagin's principle as a technique to solve the optimal control problem. The first one describes the optimal capital stock in the plan, the second one gives the optimal rate of replenishment and the third defines the optimal amount that should be budgeted for capital build-up in the university. The optimal trajectories arising from these formulas are illustrated with an example in Section 5. Although no real data are used in this study, owing to ethical considerations, the illustrative example is described to gain an insight into the behaviour of the dynamic model. The illustrative example in Section 5 is simply to support the fact that our model can be used to achieve the desired capital stock. Nonetheless, undue importance should not be attached to the hypothetical setting, as the aim of this paper is not to solve a problem faced by a particular public university but to give an insight into the usefulness of the formulas derived in this paper. Section 6 concludes the study.

2. Related works

The beginning of university education in Nigeria is credited to the British colonial government, which established the University College, Ibadan (now University of Ibadan) in 1948 as an affiliate of the University of London. This college was the only university in Nigeria prior to 1960. At present, the number of universities in Nigeria is over 104. The tremendous increase in the number of universities may be attributed to the number of candidates seeking university education and the perception that university education is an important aspect of development [19]. Universities are the main producer of highly specialised manpower. This is achieved through research, knowledge transfer and development of skills. According to Olalekan [14], the goals of higher education (including university education) in Nigeria are:

- to contribute to national development through high level relevant manpower training,
- to develop and inculcate proper values for the survival of the individual and society,
- to develop the intellectual capability of individuals to understand and appreciate their local and external environment,
- to acquire both physical and intellectual skills which will enable individuals to be self reliant and useful members of the society,
- to promote and encourage scholarship and community service,
- to forge and cement national unity, understanding and integration.

University education is provided in Nigeria by the government (federal and state) and private individuals (including religious organisations). The privately owned universities charge school fees to cushion the burden of funding, and thus maintain a certain level of autonomy. The federal universities are financed by the government through the receipt and release of funds by the National Universities Commission (NUC) to the universities. Generally, the sources of funds to the public universities (both federal and state universities) are subvention from the government (about 75% of the total income), grants, consultancy services, donations, endowment funds and the Tertiary Education Trust Fund (TETF) [4]. As at the time of writing this paper, Nigeria is facing an economic challenge owing to the decline in oil revenue. This situation suggests that the public universities may opt for autonomy, as they cannot continue to depend essentially on government subventions for their operations. The payment of tuition fees may have to be introduced. This option had earlier been canvassed [13, 19].

While the demand for university education in Nigeria outstrips the capacity, the government, as a way of responding to this demand, is opening new universities without providing adequate infrastructure. The state of infrastructure in universities in Nigeria and the paucity of funds for higher education have featured prominently in scholarly discourse [4, 7, 13, 19]. These studies have shown that the budgetary allocation to universities is grossly insufficient and the level of infrastructure is largely inadequate. Ebisine [4] attributed the causes of underfunding of public universities in Nigeria to lack of adequate planning, proliferation of universities, and ad hoc expansion of enrolment. The dwindling financing of universities in Nigeria has led to a brain drain, a fall in the quality of education, curtailment of laboratory practice and practical classes, shortage of up-to-date books in the library, insufficient chemicals and basic laboratory equipment, and the long-standing face-off between the government and the Academic Staff Union of

Universities (ASUU). Famurewa [7] stated that the face-off between the government and ASUU, which usually culminates in strikes, had adversely affected the standard of education and that effort by the government to address this issue has not yielded any progressive result.

This paper is designed to provide a plan for the replenishment of infrastructure (or simply, capital stock) from an optimisation perspective. Considerable attention has been placed on the optimal accumulation of capital stock [10, 20]. These works are hinged on the formulation of the problem of capital accumulation as a continuous-time optimal control problem. The theory of optimal control is well-known in the literature [18]. The solution techniques applied to optimal control problems may be derived from: Green's theorem [17], Pontryagin's principle [10], the turnpike theorem [3, 8, 11] and the most rapid approach path [9].

3. Development of the model

This section presents the formulation of a model for the dynamics of capital stock for a facility in a university. We view the problem of capital replenishment as an engineering problem and devise a way out of the problem using optimal control theory. More specifically, this study develops a simple model to describe the replenishment schedule for capital stock for a university where the policy of autonomy is operational. The intent of developing a simple model is credited to Edwards [5]. In this setting, fees are assumed to be the major source of revenue. This assumption is in line with [16]. By the operational mechanism of autonomy in a public university, we assume that such a university will act like a private enterprise, whose motive is profit inclined. We use the stream of profit accruing to the university in a fixed planning period as a measure of performance. We formulate an optimal control model for a facility within a fixed planning period consisting of two state variables (the capital stock and the amount budgeted by the university (institution hereafter)) and a control variable, which is defined to be the fraction of the desired capital stock to be supplied to the facility (or simply the rate of replenishment). Then we derive a replenishment plan for the facility with a view to achieving the desired stock level at the end of the planning period. By a replenishment plan we mean the schedule for supplying the facility. This schedule is formulated in this paper using the rate of replenishment, the budget on capital expenditure for the facility and the pattern of capital build-up in continuous-time. The model formulated here is similar to the one in [10]. Kamien and Schwartz [10] considered a general optimal control model for capital accumulation, where the objective was to maximise the present value of the profit stream over a fixed planning period, $0 \le t \le T$. The profit rate was a function of the productive capital, $x(t)$, and the capital stock was assumed to decay at a constant proportionate rate. This assumption enabled them to write the state transition equation. The control variable was the rate of investment, $u(t)$. Thus the problem was to solve

$$\max_{u(t)} \int_0^T \exp(-rt)\big(P(x(t)) - C(u(t))\big)\,dt \tag{1}$$

subject to the state equation:

$$\frac{dx(t)}{dt} = u(t) - bx(t),\ t \in (0,T) \tag{2}$$

the initial condition: $x(0) = x_0 > 0$, the control constraint: $u(t) \geq 0$, where r is a discount factor, P is the profit rate and C is the cost of investment. This study was earlier based on a formulation of a problem this kind. Later on, we felt that the problem would be more realistic if we introduce the amount budgeted by the institution to sustain capital accumulation as a constraint. The introduction of the institution's budget for capital accumulation within the framework of bounded optimal control makes this study unique.

In this study, we adopt similar notations as in [10]. However, we define $u(t)$ as the fraction of desired capital stock to be supplied to the facility at moment t (or simply the rate of replenishment at moment t), so that the rate of investment for a desired capital stock of worth B is $u(t)B$, $0 < u(t) < 1$, $t \in (0, T)$. The relation $0 < u(t) < 1$ is used to indicate that the institution is willing to provide the desired stock but the funds available are only enough to supply them gradually. Since $u(t)$ lies within the bounds 0 and 1, the problem considered here is a bounded optimal control problem, unlike the one in [10], where the control variable is unbounded. Our focus is on gradual replenishment plans, because for a fixed replenishment plan, there is no guarantee that the choice of $u(t)$ would be unique. The capital stock, $B > x_0$, is measured in monetary terms. The relation $B > x_0$ captures the discontentment arising from the available capital stock at the initial stage.

We assume that the initial state of the capital stock is known and that a desired state is being sought for. We assume that the depreciation of capital stock is proportional to the available capital stock. We assume the absence of free education and that the fees charged by an institution are partly constant and partly vary with the capital stock, as school fees are related to capital stock [15]. We treat the population of students in the institution as an exogenous variable owing to the fact that the population of students is affected by a multiplicity of factors, such as academic performance, admission rate and wastage rate (e.g., voluntary withdrawal, financial insolvency, medical challenges, death, etc.). The income accruing to the institution is assumed to be a linear function of the school fees and we express it as

$$P(x(t)) = \bar{N}(\vartheta_1 + \vartheta_2 x(t)),\ \vartheta_1,\ \vartheta_2 > 0 \tag{3}$$

where $\bar{N}$ is the number of students and the bar indicates that it is an exogenous variable. The constant ϑ_1 is used to capture other exogenous sources of income to the institution, such as government subvention, grants, donations, etc. The constant ϑ_2 is the rate at which the income accruing to the university changes with the school fees. The population size $\bar{N}$ is the sum according to the expected enrolment structure of the grades in the institution [2, 6, 12].

The cost of investment is expressed as

$$C(u(t)) = cu^{\alpha}(t)\,, \quad c > 0, \quad \alpha > 1 \tag{4}$$

where c is the cost of replenishment and α models the diseconomies of scaling up the rate of replenishment [20]. Just as in [10], we use the stream of profit accruing to the institution during the planning period as a measure of performance. Nonetheless, we do not consider inflation within the profit function, so that no discount factor is required, i.e., $r = 0$.

We incorporate the institution's budget into the optimal control problem by assuming that the rate of change of the budget is a linear function of the capital stock. Let $A(t)$ be the amount budgeted by the institution in order to sustain the build-up of capital at moment t. By our assumption, we have

$$\frac{dA(t)}{dt} = k + \beta x(t), \quad k > 0, \quad \beta > 0 \tag{5}$$

where k is the rate at which the budget for capital investment changes regardless of the stock of capital, and β is the growth rate of the maintenance cost of capital. The term $\beta x(t)$ is used to capture the maintenance of capital, $x(t)$. In the light of the foregoing, this paper aims to solve an optimal control problem (OCP) of the following sort:

$$\max_{u(t)} \int_0^T (\bar{N}(\vartheta_1 + \vartheta_2 x(t)) - cu^{\alpha}(t))dt$$

subject to

$$\frac{dx(t)}{dt} = u(t)B - \gamma x(t),\ t \in (0, T)$$

$$\frac{dA(t)}{dt} = k + \beta x(t), \quad k > 0, \quad \beta > 0$$

$$x(0) = x_0, \quad x(T) = B, \; A(0) = A_0 > 0, \quad 0 < u(t) < 1, \; t \in (0, T)$$

4. Solution

Our approach to solving the OCP is hinged on Pontryagin's principle. Firstly, we form the Hamilton–Pontryagin equation (or simply the Hamiltonian), H, for the OCP. That is,

$$H(x(t), A(t), u(t), \lambda_1(t), \lambda_2(t)) = \bar{N}(\vartheta_1 + \vartheta_2 x(t))$$
$$-cu^{\alpha}(t) + \lambda_1(t)(u(t)B - \gamma x(t)) + \lambda_2(t)\,(k + \beta x(t)) \tag{6}$$

This formulation leads us to solve the problem:
Maximise

$$H(x(t), A(t), u(t), \lambda_1(t), \lambda_2(t)) = \bar{N}(\vartheta_1 + \vartheta_2 x(t))$$
$$-cu^{\alpha}(t) + \lambda_1(t)(u(t)B - \gamma x(t)) + \lambda_2(t)\,(k + \beta x(t))$$

subject to $0 < u(t) < 1$.

Secondly, we utilise the conditions for optimality to get

$$\frac{d\lambda_1(t)}{dt} = -\frac{\partial H}{\partial x(t)} = -(\vartheta_2 \bar{N} - \gamma \lambda(t) + \beta \lambda_2(t)) \tag{7}$$

$$\frac{d\lambda_2(t)}{dt} = -\frac{\partial H}{\partial A(t)} = 0 \;\Rightarrow \lambda_2(t) = \Gamma \tag{8}$$

$$\lambda(T) = 0 \tag{9}$$

as it is assumed that $x(t)$ has no salvage value

$$\frac{\partial H}{\partial u(t)} = -c\alpha u^{\alpha-1}(t) + \lambda_1(t)B = 0 \tag{10}$$

$$\frac{dx(t)}{dt} = \frac{\partial H}{\partial \lambda_1(t)} = u(t)B - \gamma x(t) \tag{11}$$

$$\frac{dA(t)}{dt} = \frac{\partial H}{\partial \lambda_1(t)} = k + \beta x(t) \tag{12}$$

We have suppressed the arguments in the Hamiltonian for convenience. From Equations (7) and (8), we have

$$\frac{d\lambda_1(t)}{dt} = -\vartheta_2 \bar{N} + \gamma \lambda_1(t) - \beta \Gamma \tag{13}$$

Equation (13) is a linear equation. Solving Eq. (13) and taking $\lambda_1(T) = 0$, we get

$$\lambda_1(t) = \frac{\vartheta_2 \bar{N} + \beta \Gamma}{\gamma}\left(1 - \exp(\gamma(t-T))\right) \tag{14}$$

Substituting the result in Eq. (14) into Eq. (10), we have

$$-c\alpha u^{\alpha-1}(t) + B\frac{\vartheta_2 \bar{N} + \beta \Gamma}{\gamma}\left(1 - \exp(\gamma(t-T))\right) = 0$$

This simplifies to

$$u(t) = \left(\frac{B(\vartheta_2 \bar{N} + \beta \Gamma)}{c\alpha\gamma}\left(1 - \exp(\gamma(t-T))\right)\right)^{1/(\alpha-1)} \tag{15}$$

Substituting the result in Eq. (15) into Eq. (11), we obtain a linear differential equation

$$\frac{dx(t)}{dt} + \gamma x(t) = B\left(\frac{B(\vartheta_2 \bar{N} + \beta \Gamma)}{c\alpha\gamma}\left(1 - \exp(\gamma(t-T))\right)\right)^{1/(\alpha-1)}$$

which yields

$$x(t) = \exp(-\gamma t)\left(x_0 + B\int_0^t \exp(\gamma\zeta)\left(\frac{B(\vartheta_2 \bar{N} + \beta \Gamma)}{c\alpha\gamma}(1 - \exp(\gamma(\zeta - T)))\right)^{1/(\alpha-1)} d\zeta\right)$$

Using the method of substitution, setting $z = 1 - \exp(\gamma(\zeta - T))$, the definite integral is evaluated as

$$x(t) = \frac{B(\alpha - 1)}{\alpha\gamma \exp(-\gamma T)} \left[\frac{B(\vartheta_2 \bar{N} + \beta \Gamma)}{c\alpha\gamma} \right]^{1/(\alpha-1)} \left((1 - \exp(-\gamma T))^{a/(a-1)} \right.$$

$$\left. - (1 - \exp(\gamma(t - T)))^{a/(a-1)} \right) + x_0 \exp(-\gamma\tau) \tag{16}$$

Using the endpoint constraint $x(T) = B$, we have

$$x^{\text{opt}}(t) = x_0 \exp(-\gamma t) + \left(B - x_0 \exp(-\gamma T) \right)$$

$$\times \left(1 - \left(\frac{1 - \exp(\gamma(t - T))}{1 - \exp(-\gamma T)} \right)^{a/(a-1)} \right) \exp(\gamma(T - t)) \tag{17}$$

The superscript opt is used to denote the optimal value. From Eq. (15) and using $x(T) = B$, we obtain

$$\frac{\alpha\gamma}{B(\alpha - 1)} \left(B - x_0 \exp(-\gamma T) \right) (1 - \exp(-\gamma T))^{-a/(a-1)} = \left[\frac{B(\vartheta_2 \bar{N} + \beta\Gamma)}{c\alpha\gamma} \right]^{1/(a-1)}$$

The control variable $u(t)$ is therefore expressed as

$$u^{\text{opt}}(t) = \frac{\alpha\gamma}{\alpha - 1} \left(1 - \frac{x_0}{B} \exp(-\gamma T) \right) \left(\frac{1 - \exp(\gamma(t - T))}{(1 - \exp(-\gamma T))^{\alpha}} \right)^{1/(a-1)} \tag{18}$$

The results in Eqs. (17) and (18) are the same as the results obtained by solving the OCP without the budget constraint. This is because the constrained optimisation problem, wherein the Hamiltonian is maximised subject to the control bounds, does not contain $A(t)$.

Substituting Equation (17) into Eq. (12) and then integrating from 0 to t, we obtain

$$A(t)-A_0=kt+\beta\left(\frac{x_0}{\gamma}(1-\exp(-\gamma t))+(B-x_0\exp(-\gamma T))\right.$$

$$\times\left[\frac{\exp(\gamma T)}{\gamma}(1-\exp(-\gamma t))-\frac{1}{\left(1-\exp(-\gamma T)\right)^{\alpha/(\alpha-1)}}\right.$$

$$\left.\left.\times\int_0^t\left((1-\exp(\gamma(\xi-T)))^{\alpha/(\alpha-1)}\exp(\gamma(T-\xi))\right)d\xi\right]\right) \tag{19}$$

The integral does not have a closed form expression. Using the substitution $z=\exp(\gamma(T-\xi))$ for the integrand in Eq. (19) and then expanding the result using the binomial theorem, we get

$$A(t)-A_0=kt+\beta\left(\frac{x_0}{\gamma}(1-\exp(-\gamma t))+(B-x_0\exp(-\gamma T))\right.$$

$$\times\left[\frac{\exp(\gamma T)}{\gamma}(1-\exp(-\gamma t))+\frac{1}{\gamma\left(1-\exp(-\gamma T)\right)^{\alpha/(\alpha=1)}}\right.$$

$$\left.\left.\times\int_{\exp(\gamma T)}^{\exp(\gamma(T-t))}\left(1-\frac{\alpha}{\alpha-1}\left(\frac{1}{z}\right)+\sum_{s=2}^{\infty}\frac{(-1)^s}{s!}\prod_{j=0}^{s-1}\left(\frac{\alpha}{\alpha-1}-j\right)z^{-s}\right)dz\right]\right) \tag{20}$$

Integrating term-by-term, we have

$$A^{\text{opt}}(t)=A_0+kt+\frac{\beta}{\gamma}\left(x_0(1-\exp(-\gamma t))+(B-x_0\ \exp(-\gamma T))\right.$$

$$\left[\exp(\gamma T)(1-\exp(-\gamma t))+\left(1-\exp(-\gamma T)\right)^{-\alpha/(\alpha-1)}\right.$$

$$\left(\frac{\alpha\gamma t}{\alpha-1}+\exp(\gamma(T-t))-\exp(\gamma T)\right.$$

$$\left.\left.\left.+\sum_{s=2}^{\infty}\frac{(-1)^s}{s!(1-s)}\prod_{j=0}^{s-1}\left(\frac{\alpha}{\alpha-1}-j\right)(\exp(-\gamma(1-s)t)-1)\ \exp(\gamma(1-s)\ T)\right)\right]\right) \tag{21}$$

5. An example

Let $t = 1, 2, ..., 10$, $\gamma = 0.02$, $k = 20$ billion naira, $\beta = 0.18$, $x(0) = 2$ billion naira, $x(10) = 15$ billion naira, and $A(0) = 32$ billion naira. The parameters γ, β are chosen to lie in the interval [0, 1], because they are the decay rate and growth rate, respectively. The initial values $x(0)$, $A(0)$ (in billion naira) are representative of the value of physical facilities and budget for a faculty in a university in Nigeria. The terminal value of $x(10) = 15$ billion naira is used to indicate the amount of the desired capital needed to enhance the teaching-learning process in the faculty. We compute $u^{opt}(t)$, $x^{opt}(t)$, $A^{opt}(t)$ for $\alpha = 2, 3, 4, 5$ in the MATLAB environment (see the Appendix). The results are depicted in Figs. 1–3.

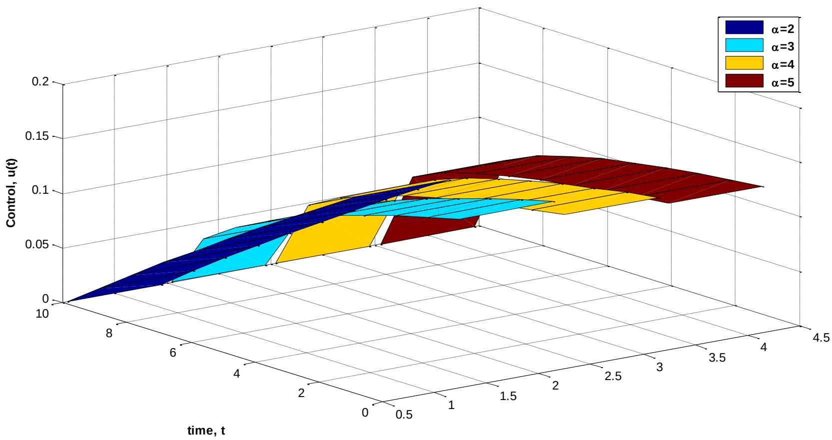

Fig. 1. The dynamics of replenishment for a ten-year period

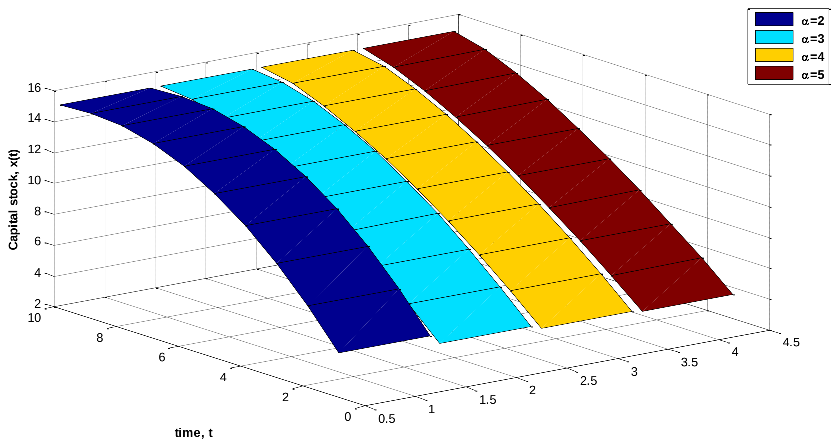

Fig. 2. The dynamics of capital stock for a ten-year period

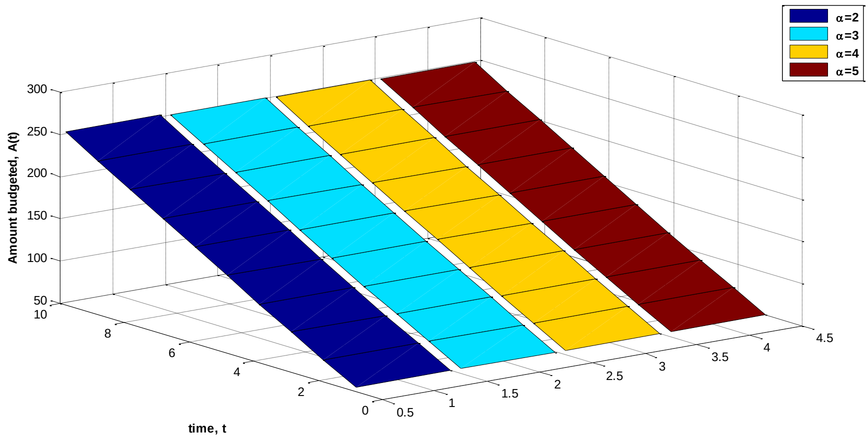

Fig. 3. The dynamics of amount budgeted for a ten-year period

Figure 1 shows the dynamics of the optimal rate of replenishment for the facility at moment $t \in [0, 10]$. Figure 2 shows the corresponding pattern of the optimal capital stock given the rate of replenishment for the facility at moment $t \in [0, 10]$. From Figure 1, the rate of replenishment decreases gradually with time, whereas in Figure 2, the capital stock increases steadily with time. The increase in capital stock over time indicates an increase in the value of capital stock towards the desired state, and the decrease in the rate of replenishment indicates a reduction in investment as the time period elapses. Figure 3 shows that there is a steady increase in the amount budgeted. These results lead us to conclude that the institution enjoys economies of scale as the time period elapses and that the steady increase in the amount budgeted is necessary for the desired stock to be attained in the institution.

Our results agree reasonably well with what the stakeholders (ASUU, parents, etc.) in the university system expect, as both the amount budgeted for the build-up of capital and the stock of capital continue to grow over the period under consideration with a decreasing rate of replenishment. Thus the assumptions and the results of this study are enough motivation for university planners to adopt our formulas as a direction of policy change regarding the replenishment of infrastructure.

6. Conclusion

This study has proposed a replenishment plan that could achieve the desired stock for a facility in a university. The replenishment plan is derived by treating the problem of infrastructure replenishment as an optimal control problem with special reference to the budget constraint. It may be said that without analysing real data, it is difficult to develop adequate equations for the system. It is in this light that the study adopted an already existing model which is well suited to the transition dynamics of capital stock [10], rather than developing a new one. Since the transition equation for capital stock is well-known, there is no need to perform analysis on its adequacy and validity. The model formulated in this paper was centred on an autonomous university system and on a gradual replenishment plan. Pontryagin's principle was adopted as a technique to solve the OCP. Given the desired stock, estimating the parameters of the model gives the fitted replenishment plan and this could help guide educational planners towards steering up the value of capital stock in public universities for a fixed time period. Nonetheless, this study did not consider the opportunity cost of the physical infrastructure, e.g., investing money in a staff development program. This shortcoming, coupled with the relaxation of some of the assumptions in this study, is a future research direction.

Appendix.
The MATLAB codes for the illustrative example

```
    clear
clc
g = 0.02;x = 2;B = 15;T = 10;k = 20;b = 0.18;Ai = 32;
fort = 1:10;
    a1 = 2;
    u1(t) = ((a1*g*(1-(x/B)*exp(-g*T)))/(a1-1))*((1-exp(g*(t-T)))/(1-exp(-g*T))^a1)^(1/(a1-1));
    x1(t) = exp(-g*t)*(x+(B*exp(g*T)-x)*(1-((1-exp(g*(t-T)))/(1-exp(-g*T)))^(a1/(a1-1))));
    A1(t) = Ai+k*t+b*int('exp(-0.02*x)*(2+(15*exp(0.02*10)-2)*(1-((1-exp(0.02*(x-10)))/(1-exp(-
    0.02* 10)))^(2/(2-1))))',0,t);
  A1 = double(A1);
  a2 = 3;
    u2(t) = ((a2*g*(1-(x/B)*exp(-g*T)))/(a2-1))*((1-exp(g*(t-T)))/(1-exp(-g*T))^a2)^(1/(a2-1));
    x2(t) = exp(-g*t)*(x+(B*exp(g*T)-x)*(1-((1-exp(g*(t-T)))/(1-exp(-g*T)))^(a2/(a2-1))));
  A2(t) = Ai+k*t+b*int('exp(-0.02*x)*(2+(15*exp(0.02*10)-2)*(1-((1-exp(0.02*(x-10)))/(1-exp(-
  0.02*10)))^(3/(3-1))))',0,t);
  A2 = double(A2);
    a3 = 4;
    u3(t) = ((a3*g*(1-(x/B)*exp(-g*T)))/(a3-1))*((1-exp(g*(t-T)))/(1-exp(-g*T))^a3)^(1/(a3-1));
    x3(t) = exp(-g*t)*(x+(B*exp(g*T)-x)*(1-((1-exp(g*(t-T)))/(1-exp(-g*T)))^(a3/(a3-1))));
A3(t) = Ai+k*t+b*int('exp(-0.02*x)*(2+(15*exp(0.02*10)-2)*(1-((1-exp(0.02*(x-10)))/(1-exp(-
0.02*10)))^(4/(4-1))))',0,t);
  A3 = double(A3);
    a4 = 5;
    u4(t) = ((a4*g*(1-(x/B)*exp(-g*T)))/(a4-1))*((1-exp(g*(t-T)))/(1-exp(-g*T))^a4)^(1/(a4-1));
    x4(t) = exp(-g*t)*(x+(B*exp(g*T)-x)*(1-((1-exp(g*(t-T)))/(1-exp(-g*T)))^(a4/(a4-1))));
```

```
A4(t) = Ai+k*t+b*int('exp(-0.02*x)*(2+(15*exp(0.02*10)-2)*(1-((1-exp(0.02*(x-10)))/(1-exp(-
0.02*10))) ^(5/(5-1))))',0,t);
   A4 = double(A4);
end

clf
subplot(1,3,1)
t = 1:10;
ribbon(t',[u1' u2' u3' u4'],0.9), grid on
ylabel('\bf time, t')
zlabel('\bf Control, u(t)')
legend('\bf \alpha = 2','\bf \alpha = 3','\bf \alpha = 4','\bf \alpha = 5')
title('\bf Fig. 1')

subplot(1,3,2)
ribbon(t',[x1' x2' x3' x4'],0.9), grid on
ylabel('\bf time, t')
zlabel('\bf Capital stock, x(t)')
legend('\bf \alpha = 2','\bf \alpha = 3','\bf \alpha = 4','\bf \alpha = 5')
title('\bf Fig. 2')

subplot(1,3,3)
ribbon(t',[A1' A2' A3' A4'],0.9), grid on
ylabel('\bf time, t')
zlabel('\bf Amount budgeted, A(t)')
legend('\bf \alpha = 2','\bf \alpha = 3','\bf \alpha = 4','\bf \alpha = 5')
title('\bf Fig. 3')
```

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