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A BAYESIAN MODEL OF GROUP DECISION-MAKING

A change in the opinion of a group, treated as a network of communicating agents, caused by the accumulation of new information is expected to depend on communication within the group, cooperation and, possibly, a kind of conformity mechanism. We have developed a mathematical model of the creation of a group decision, including this effect. This is based on a Bayesian description of inference and can be used for both conscious and inattentive acts. This model can be used to study the effect of whether a leader exists or not and other group inhomogeneities, as well as establishing the (statistical) significance and quality of a group decision. The proposed evolution equations explain in a straightforward, analytical way some general properties of the general phenomenon of conformity (groupthink). To illustrate this theoretical idea in practice, we created an information technology (IT) tool to study the effect of conformity in a small group. As an example, we present results of an experiment performed using a network of students' tablets, which could not only measure group pressure, but also conduct and control collaborative thinking in the group.

Keywords: *conformity, decision-making, Bayesian inference, groupthink, cooperation*

1. Introduction

A group of individuals is defined by the relation which divides a population into *us* and *them*. The group to which we belong, called us, is distinct from them. In most cases, it is better to be *one of us*. This leads to the concept of protecting *us*, to make *us* stronger, bigger and better than *them*. This leads to cooperation.

On the other hand, cooperation may lead to structure in the population. This has been observed at many levels of the game of evolution from genes and cells to animals

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and men. In the case of humans, we observe complicated structures within a society. The emergence of initial structures is, on one hand, an open question but some mechanisms look indisputable. One of them, discussed by Szathmary [12], is a simple consequence of a nonlinear benefit function which can be interpreted as synergy when the number of cooperators is small. Hunts for mammoth were much more effective when several individuals were engaged. This is an evident example. This effect could lead to group formation among the first hunter-gatherer communities. Even today, many similar examples can be found, but there are also situations which do not follow this scheme, see e.g., [6].

In specific conditions, people in groups formed or caused by such a mechanism can implement defective decision-making processes. Such a phenomenon is called groupthink ([14] and references therein). It is not necessarily the case that the whole is greater than (or at least equal to) the sum of the parts. In principle, individuals can sometimes act better than a group. A group working together on a decision-making problem may sometimes even lead to tragic solutions, as discussed in the case studies presented in [7]. The appeasement of Nazi Germany, Pearl Harbor and the Challenger Space Shuttle disaster are very well-known examples. The reason for such failures is, paradoxically, cooperation. The symptoms defining groupthink as defined by Janis [7] might be initially weak but the first necessary condition for this phenomenon to occur is conformity which, in a sense, takes the place of cooperation. If there is a problem, the group may act to maintain its positive self-esteem and the positive image of the group.

Detailed studies (e.g., [13] and references therein) have provided mixed support for the theory of groupthink. An array of modifications has been proposed for the groupthink model, (see, e.g., [10] and references therein). The importance of groupthink is in its potential consequences. It needs further study, as quantitative as possible, and a well-defined, operational measure of conformity. In this paper, we propose a mathematical model to describe this effect numerically.

Asch's well-known experiments illustrate the importance of conformity, even when judging a simple and obviously clear case [1, 2]. It certainly works in everyday life (e.g., his well-known "elevator experiment"). We would like to believe that scientific judgment is free of it.

2. Groupthink in science

The concept of modern science is based on the objectivity of scientific statements. Experimental observations and measurement results are assumed to be objective. The common belief in the objectivity of science is based on the 17th century construction of a system of empirical knowledge. This is a basic concept of contemporary science in all textbooks. However, it seems that the end of last millennium was also the end of the era when we could say this without any doubts.

First, the Bayesian concept of probability, based on Bayes theorem formulated in the 18th century, is now appearing in state-of-the-art data analysis e.g., in high-energy physics (CERN LHC, searching for the Higgs boson, etc.), as often as the frequentist point of view using classical theory. In the *Review of Particle Physics* in 1993, it was said that *this Bayesian approach is considered unsatisfactory* [9], for two reasons, while nearly ten years later, in the year 2012, the same Particle Data Group concludes that for small data samples and for measurements of a parameter near a physical boundary, the Bayesian approach may yield different results than the frequentist method, and we are forced to make a choice. No general recommendation exists [3].

In the Bayesian definition of probability, the subjective aspect of the creation of knowledge is expressed directly. For the last 200 years, attempts to make the classical definition of probability objective have not gone much further than the original Laplace definition. The number of Bayesians is growing continuously.

The next point is related to the fact that modern large-scale scientific experiments are producing an enormous amount of raw data. Collaborating groups of hundreds of scientists are working with each other, trying to reach a solution to particular problems. Symptoms of groupthink are starting to appear in the science knowledge system. Examples can be found. However, it is rather hard to discuss them in scientific journals, where papers are reviewed by specialists who are sometimes involved with the results being discussed, see e.g., [15].

3. Group decision-making

We will use the idea of group decision-making later to model a procedure for parameter fitting and we will show how the effect of the group (groupthink?) can be found, described and analyzed.

3.1. The classical approach – frequentialism

For about the last 200 years, ever since Laplace's *Théorie analytique des probabilités* appeared, probability has been classically defined as the ratio of the number of cases favourable to the event in question, to the number of all cases possible when nothing leads us to expect that any one of these cases should occur more often than any other, which renders them, for us, equally possible.

Certainly this definition is a *circulus in definiendo, idem per idem* definition: the concept of equally probable events is needed before probability is defined. In spite of two centuries of attempts to find a better definition, the classical definition still exists as we gave it above. If we have developed nothing better, then we should accept what we already have.

If theory predicts that the result X could occur in a particular experiment, and we know that in N experiments the result X was observed n times, then we can say that the probability of the result X in a future repetition of this experiment is about n/N. According to classical theory, we cannot say that a theory is true with probability n/N. There is no way to define the probability of a theory: a theory can only be *true* or *false*. We can only estimate the probably of the result X in the next, $(N + 1)$th measurement. This probability is about n/N. If we perform a new series of experiments, the accuracy of estimation is inversely proportional to the square root of the appropriate variance, which is determined by n and N.

Consider one well-known example, the Laplace problem of the chance of the sun rising tomorrow morning. We personally might see about 30 000 sunrises but we could assume that this event has always occurred for at least the last 6000 years (although in the ancient world there were some rumors that some disturbances in the sun's path had been observed, when various gods and heroes had problems). So the chance that the sun will rise tomorrow (at the proper time) is of order (1 – 1/1 500 000). On the other hand, everyone today would be certain that tomorrow will be a normal day. This is, of course, the situation today. In the ancient world both possibilities were taken seriously – people believed in the Phaeton myth. Today we have a theory describing the motion of heavenly bodies. We believe that all the stars in the sky follow the same law of gravity as any object around us. Observing them, we collect data, and we have a strong belief that gravity will also work tomorrow. The estimate of the probability obtained above is not a good solution. Our beliefs are hard to model using classical probability theory.

3.2. The Bayesian approach

One theory which gives a reasonable answer to this problem is the Bayesian theory of probability. It defines probability as a level of certainty related to a potential outcome, and is strongly subjective, since the concept of certainty is subjective. A probability measure depends on the particular individual assessing it, his/her state, the state of his/her mind, so this measure obviously changes with time. The probability measure evolves. This is, in a sense, both a weakness and a strength of the Bayesian approach. Individuals can learn through experience. One can bet that a particular theory is true or false, since one has a particular degree of belief in whether a theory is – or is not – true. This can be called the probability of the theory by Bayesians.

Bayes theorem is a general way of describing the increase in our knowledge about theory T (the evolution of the measure of the probability that theory T is true) by analyzing new data, the result X of a new experiment:

$$P(T \mid X) = \frac{P(T)\,P(X \mid T)}{P(X)} \tag{1}$$

where

- The prior estimate of probability, $P(T)$, is our initial belief about the probability of T being true.
- The posterior estimate $P(T \mid X)$ is the probability of T being true given that X has been observed.
- The likelihood factor, $P(X \mid T)$ is the probability of event X occurring if T is true.
- If we consider a range of possible T's, we can calculate $P(X)$, the total probability of X happening for any T.

In the case of the sun rising tomorrow, we obviously have great certainty. This has been established not only by generations of our ancestors, but also by science, which makes the subject in question a phenomenon of nature, which is quite solid and rather stable.

In our discussion, we do not wish to go into details which can be found easily elsewhere. We would like to clearly show the starting point of our ideas.

3.3. The group – a collective of individuals

Equation (1) works for each individual in the group GG$\mathbb{G}$, which contains N similar individuals. Classically, the existence of N individuals does not change anything. Using the Bayesian approach, however, it is slightly more complicated. Each individual i can hold a different belief, i.e. different prior – $P_i(T)$. Thus, the same observation X with the same likelihood $P(X \mid T)$ leads individual i to his/her own, individual, posterior $P_i(T \mid X)$. Recording each of them, we can ask what the average answer of the group $\mathbb{G}$ is.

As a first approximation, we can define this as

$$P_{\mathbb{G}}(T \mid X) == \frac{1}{N}\sum_{i=1}^{N} P_i(T \mid X) \tag{2}$$

From Eq. (1), we obtain

$$P_{\mathbb{G}}(T \mid X) = \frac{1}{N}\,\frac{\sum_{i=1}^{N} P_i(T)P(X \mid T)}{P(X)} = \frac{P(X \mid T)}{P(X)}\,P_{\mathbb{G}}(T) \tag{3}$$

where $P_{\mathbb{G}}(T)$ is the general prior, the average belief that the theory T is correct. By analogy with Eq. (2), this is equal to $\sum_{i=1}^{N} P_i\,(T \mid X)/N.$ Equation (3) shows that the general knowledge of the group changes exactly in the same way as each individual belief. This result is identical to the conventional, frequentist, one. However, this is only valid when all the individuals act independently in their experimental evaluations using the same input X and communicate with the group $\mathbb{G}$ only at the final stage, when averaging the outputs $P_i\,(T \mid X)$.

We can expect different results when groups cooperate. Cooperation may be driven by different reasons, like unconscious, instinctive or intuitive conformity as in the Asch experiment, or intentionally accepted teamwork to get a better or faster result. In both cases, the mechanism and the theoretical description is the same.

In the Asch experiment, participants who were exposed to intense group pressure exhibited absurdly different behaviour than the typical behaviour observed in controlled conditions where there were no confederates. The “decisions” of the confederates reported in the experiment were “wrong” but they played their roles. Their answers, assumed to be “posteriors”, were observed by the one genuine participant and his real decision was influenced by them. It was hardly an independent decision.

Another example is sometimes observed in various fields of science when we deal with a team working with the intent task of getting a particular (expected?) result, e.g. to observe a tiny, subtle effect. If the wisest team member is the first to announce the discovery, then all the team members claim to have seen this effect.

To take this and similar (kinds of groupthink) effects into account, we should modify Eq. (3) which defines $P_{\mathbb{G}}(T \mid X).$ If one has the prior $P_i(T)$ before taking part in a collaborative experiment with output data X, one’s output opinion $P_i'(T \mid X)$ about T is influenced by the rest of the group $\mathbb{G}$. Let us denote the tendency of individual i to correct (to change) his/her opinion as a result of group pressure by the factor $(1 - \beta_i)$

$$P_i'(T \mid X) = \beta_i P_i(T \mid X) + (1-\beta_i) P_{\mathbb{G}-i}(T \mid X) \tag{4}$$

where $P_{\mathbb{G}-i}(T \mid X)$ is the average posterior probability for the group without the i-th individual, defined as in Eq. (2).

We are interested in the average modified group opinion $P_{\mathbb{G}-i}(T \mid X)$

$$P_{\mathbb{G}}'(T \mid X) = \frac{1}{N} \sum_{i=1}^{N} P_i'(T \mid X) \tag{5}$$

Using the definitions given above, we have

$$P'_{\mathbb{G}}(T \mid X) = \frac{1}{N}\sum_{i+1}^{N}\left[\beta_i P_i(T \mid X) + (1-\beta) P'_{\mathbb{G}-1}(T \mid X)\right]$$

$$= \frac{1}{N}\sum_{i+1}^{N}\left\{\beta_i P_i\left(T \,\middle|\, X\right) + (1-\beta_i)\left[\frac{1}{N-1}\sum_{j=1;\, j\neq i}^{N} P'_j(T \mid X)\right]\right\} = \ldots \tag{6}$$

The procedure given by Eq. (6) recursively diminishes the number of elements in successive sums. Individuals in a group are certainly not exactly the same and do not have the same intellectual, $p_i(T)$, and decision power, β_i.

This is an interesting problem to study, what will be the result if the abilities of the members of the group are not the same: if there are some individuals with significantly different priors (more highly educated!) and different degrees of self-confidence, charisma, allure? This is to some extent the situation when considering a teacher and his/her students. If there is a leader in the group, the effect of his actions and decisions could be pursued by others according to the procedure described by Eq. (4). The role of leadership is crucial to the groupthink effect.

The interpretation of Eq. (6) in the general case is straightforward. It still defines the change in confidence regarding the conclusions from detailed studies based on the opinion of a group, when we take into account the effect of the possible influence of the opinion of the group as a whole on the opinions of its individual members. It is thus obvious that less self-reliant people change their initial individual opinion more easily.

There is an important effect of group collaboration: a decrease in confidence. This is due to the narrowing of the spread of opinions of non-self-reliant individuals. We should, in general, observe this in real cases and it is, in fact, one of the groupthink symptoms related to self-censorship [7]. This behaviour could be responsible for some of the shocking effects of groupthink or the results of Asch-type experiments.

From an educational point of view, collaborative work diminishes the differences between students, so it is very positive, in a sense. The dangerous effects of biasing group opinion in the wrong direction by the erroneous behaviour of strong, self-confident individuals have to be controlled by an educated person - the teacher, a leader who is not involved in “groupthink”. This control is extremely important and is obviously one of the primary duties of a teacher.

All these problems are important and interesting. The model proposed introduces a general framework to study them in detail.

In the present paper, however, we will concentrate on the case of collaboration in a group with almost equal priors and communication factors (β) and we will give a practical example for the use of our model in such a simple case.

If we perform the iteration described in Eq. (6) N times, assuming that all the β_i, as well as all the individual priors – $p_i(T)$, are identical (all the individuals are the same), we eventually obtain

$$P'_G(T|X) = \beta\, P(T|X) + \beta(1-\beta)\, P(T|X) + \ ...$$
$$+\beta(1-\beta)^N P(T|X) = P(T|X)\left[1-(1-\beta)^{N+1}\right] \tag{7}$$

which is different from the average opinion of a group of individuals who do not communicate.

The above equation is only an initial step in our considerations regarding this model, but it already has some interesting consequences. For example:

Remark 1. The factor in brackets […] on the right-hand side of Eq. (7) could be close (or even equal) to one in two cases:

- if the members of the group do not cooperate (communicate) ($\beta = 1$),
- if the number of people in the group (N) is large enough (for $\beta = 0.9$ and $N = 10$, the difference is of order of 10^{-9}, and even for $\beta = 0.5$ and $N = 5$, the difference is still quite small, about 3%).

The effect is thus eventually the same in both cases!

It is interesting to note that we have just analytically obtained in mathematical form a proof of the following famous sentence stated on the occasion of the 30th anniversary of the groupthink idea [11]: *Smart people working collectively can be dumber than the sum of their brains*. It is certainly true!

Equation (7) also leads to other, less obvious but interesting, remarks:

Remark 2. If there is a group of people who have entirely no trust in their abilities ($\beta = 0$), then the group cannot express any opinion.

Remark 3. If we have a group of people who have almost no opinion ($\beta \approx 0$), a large number of them are needed to form any conclusions comparable to the opinion of a single clever, educated individual.

Another point that we should note is the fact that the factor in brackets […] in Eq. (7) is never greater than one. This leads to the following statement:

Remark 4. There is no way to get from a group as a whole any conclusion which is stronger than the opinion of one educated man with strong self-confidence.

The last conclusion is somehow intriguing. It can be reformulated as:

Remark 5. No matter how many people share a particular belief, it could be just as right or wrong even when the number of believers is doubled!

The following question arises again: is there is any “logical” reason, at all, why people form groups working on some particular problems, while groupthink, sometimes works in a dangerous direction and no profit has been seen – yet. But, as we said, it should be remembered that in the general case not all individuals in a group are exactly the same nor do they possess the same intellectual and decision power.

3.4. The case of parameter estimation

The description presented above concerns a Bayesian treatment of the evolution of the general opinion about the theory *T*. More often in science, which is experimental by nature, we have to estimate the unknown value of a particular parameter related to a theory. The nature of such processes complicates the appropriate formulas, but does not change the general concept. We can still use Eq. (7), changing only the meaning of the symbol *t*. We can write Eq. (4) in the form

$$P(T(t) \mid X) = \beta_i P_i(T(t) \mid X) + (1-\beta_i) P_{\mathbb{G}-i}(T(t) \mid X) \tag{8}$$

where $T(t)$ denotes the theory *T* in which the value of the parameter of interest is equal to *t*. The meaning of $P(T \mid X)$ $(\equiv P(T(t) \mid X),$ and $P'(T \mid X)$ $(\equiv P'(T(t) \mid X),$ is still the probability (probability density, in general, for a continuous parameter *t*) of the theory in which the value of the appropriate parameter is equal to *t* (or in the interval $(t, t + dt)$), in short: the probability of *t*.

3.5. Correlation coefficient

Estimation of the set of parameters β_i is possible, in principle, but it requires detailed knowledge of the distributions P_i with respect to *t*, which could be treated, using the Bayesian approach, as a random variable. This knowledge can be based on the information adopted from other external sources, or obtained during the experiment itself. This, however, requires a very rich source of data, which is obviously hard and expensive to realize in practice. We would like to show a way to avoid this. We have further simplified the model to be applicable for the practical purposes.

Let us assume that we have a group of *N* individuals and, for simplicity, assume that the only parameter describing the effect of communication (cooperation) between students is the classical linear coefficient of the correlation between the results reported by a pair of group members:

$$\rho_{ij} = \frac{\left\langle t_i t_j \right\rangle - \left\langle t \right\rangle^2}{\left\langle t^2 \right\rangle - \left\langle t \right\rangle^2} \tag{9}$$

where t_i and t_j are the values of the parameter t obtained by individual i and j, respectively, and $\langle ... \rangle$ is the average value of the variable in parentheses.

Let us assume that all the pairwise correlation coefficients are equal $\rho_{ij} = \rho$ and all the estimates of the distributions of the t_i are the same, i.e. the members of the group have the same intellectual power. The denominator in Eq. (9) is the variance of the estimate of t and is closely related to the variance of the values reported by the participants in the experiment.

The average value of the parameter t obtained by a group is defined as

$$\overline{t} = \frac{1}{N} \sum_{n+1}^{N} t_n \tag{10}$$

and when the experiment is repeated M times (this could be with different groups but, for simplicity, we assume that always with N members), we have a set of M average values of $\overline{t}$: $\overline{t}_1, \overline{t}_2, \overline{t}_3, ..., \overline{t}_M$, one for each group. These values can also be treated as particular realizations of the random variable $\overline{t}$, which has the appropriate probability density function.

The relation of each individual independent estimator of the distribution, $P_i(t)$, to the value of the correlation coefficient ρ is very complicated.

However, the distribution of the variable $\overline{t}$ is well defined and has the following simple formula for its variance:

$$\mathrm{Var}(\overline{t}) = \frac{\sigma_t^2}{N} (1 + (N-1)\rho) \tag{11}$$

In our scenario, the correlation coefficient is naturally non-negative, thus

$$\frac{\sigma_t^2}{N} \le \mathrm{Var}(\overline{t}) \le \sigma_t^2 \tag{12}$$

where the limits relate to the cases of “no communication at all” and “exact one-to-one matching” of individual answers, respectively.

Equation (11) allows us to estimate the correlation coefficient ρ by comparing the variance of independent estimates, σ_t^2, with the observed variance of the group average, $\mathrm{Var}(\bar{t})$.

4. Collaboration in a group. An example

The opinion of a member of a group maybe strongly influenced by the behaviour of the rest of the group – as in the Asch experiment. Together with the danger of groupthink effects, this puts the idea of collective work in the education process in question. If we do not want to produce individuals who only express expected and trained conduct, work in a team with no specially educated leader who does not take part in the group dynamics (e.g. a teacher) is of no use. This is, to some extent, in contradiction with conventional wisdom and educational recipes about partnership and participation.

We have performed an educational experiment to analyse the effect of communication (cooperation, collaboration, conformity) in a group of students solving rather a complicated physical (mathematical, statistical) problem. To make the lesson more attractive, we have used a problem which is presently of great importance to physics – searching for the Higgs boson at CERN. Quite recently, new data have been published with the claim of *at last we have it!* [5]. The existence of *the God particle* appears in the form of a small bump in the observed invariant mass spectrum. There are many decaying channels of the Higgs field and many plots to look at. We have taken data from a well-known picture published by the CMS Collaboration [4]. The task we gave to our students was to find their fit to the Higgs peak on the graph.

We made an application for tablets (compatible with Android 4.0 or higher) with the data points plotted together with the theoretical curves of both the background and the Higgs peak. A view of the screen is presented in Fig. 1. The shape and position of the theoretical Higgs bump is described by five parameters, whose values are controlled by the five sliders beside the data plot. After an introductory lecture about the physics of CERN, LHC and the Higgs boson, lasting about half an hour, students were asked to find their own Higgs particle. It takes up to 5 minutes before they learn how to effectively use the sliders. No special help was necessary, the students are "natives of the digital age" and most of them started to move the curves in the desired directions very quickly. We then asked them to try, as accurately as they can, to place a curve through the measured points summarizing the background and the signal. We did not explain to our students the exact definition of "the best fit". This is, in fact, a rather complicated task and needs time to comprehend the idea of maximum likelihood or minimum χ^2. We only paid attention to common errors, mainly to explain what the background is and where it should be found and to some obvious mistakes usually made by "amateurs", newcomers to statistics. All the time during the students' activity, the tablet application was continuously sending all the current values of the parameters to the host tablet.

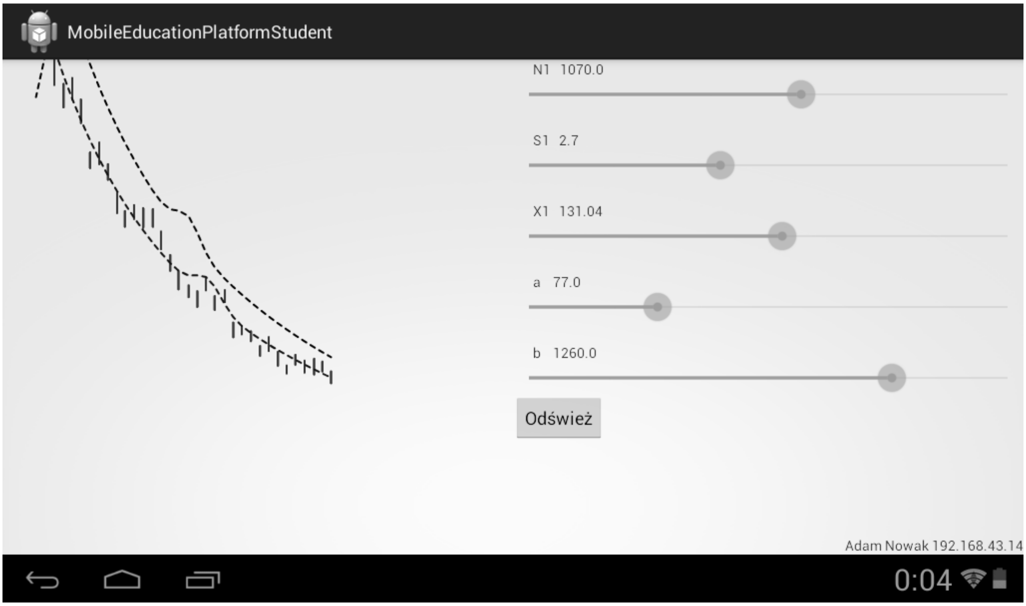

Fig. 1. View of the tablet screen. The CMS data points are represented by the small vertical (in reality, red) bars, an individual fitted curve is given by the black dashed curve (the upper one in this particular screenshot). The average curve is the (blue) dashed one, here the lower one

On each individual tablet screen, another curve, similar to the one students are able to move, is also shown. In the introduction explaining how the application works, we mentioned that this is the curve representing the result of the group as a whole – the current average of all the fits calculated on-line by the host tablet. If one student changes his fit (moving, for example, his curve to a clearly wrong position), the corresponding small change to the average curve appears on the screen of all the others. We asked students to perform the fitting procedure on their own and not to pay attention to the average result. However, it is assumed that they were in principle able, to some extent, (consciously or not) to use this "group average" information, and to compare their particular result with the Higgs peak preferred by others. The analogy to the Asch experiment is obvious. However, in our experiment group stress is set to be very limited and controlled.

We wish to quantitatively estimate this conformity effect. We performed our experiments with a group of 75 high school students. The average of the position of the Higgs particle maximum over all the results was found to be equal to 124.75 GeV/c^2. In the original paper by the CMS Collaboration [4], the position of the Higgs particle for the

$\gamma\gamma$ channel analysed is given as 124.9 GeV/c^2 with an uncertainty of about 1 GeV/c^2. This unexpectedly large uncertainty (looking at the data points when trying to estimate the effect of the "error" of the statistical fitting procedure) is due to the rather unclear physical picture and contamination of other, non-Higgs, reaction channels. The agreement between the results of the students' fitting procedure and the professionals' result is not unexpected. It confirms the ability of the minds of high school students to perform complicated mathematical actions in a natural manner.

We used the mass of the Higgs particle, m, as the parameter t to be studied. Students were divided into 15 groups (M = 15) of 5 students (N = 5) each. Because we expected the effect of conformity to be very small and the number of students in the group is also chosen to be small, much smaller than the total number of participants, we use the distribution of the results of these individual estimates of the mass of the Higgs particle as a good approximation to the distribution of independent fits of m. This allows us to use its variance as the value σ_m^2, present in Eq. (11). This distribution is shown in Fig. 2 by the solid histogram ($\langle m\rangle$ = 124.75 GeV/c^2).

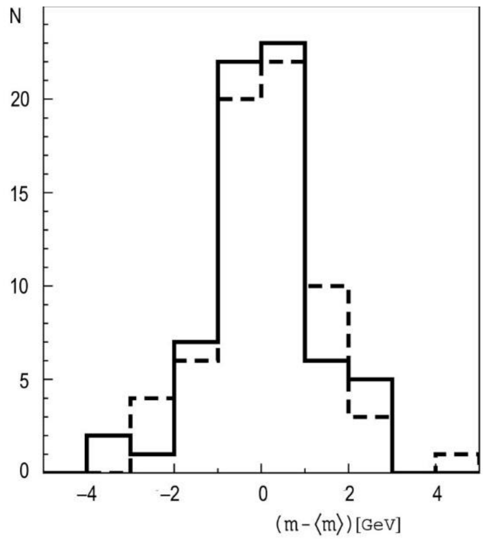

Fig. 2. Distribution of the mass of the Higgs particle with respect to the average mass from all fits (solid histogram – independent fits) and with respect to the respective group average (dashed histogram with conformity effect taken into account)

Figure 2 also shows the distribution of the mass of the Higgs particle estimated in the same way but with respect to the average mass ($\langle m\rangle = \bar{m}_k$) in each group – the dashed histogram. This represents the distribution of the group estimate of $\bar{m}$ with variance $\bar{\sigma}_m^2$.

The distribution of the group estimate is, of course, narrower than the variance of the individually estimated one. This difference is not very large, which confirms the assumption about the weak conformity effect. The difference between these variances gives us the variance of the group average $\bar{m}$.

$$\mathrm{Var}(\bar{t}) = \sigma_t^2 - \tilde{\sigma}_t^2 \tag{13}$$

Substituting the values of the widths of the respective histograms shown in Fig. 2 into Eqs. (13) and (11), we obtain an estimate of the correlation coefficient ρ, which describes the level of group conformity. In our experiment, its value is equal to 0.06. This is very small but we wish to know whether this result is statistically significant. It is important to note that the distributions of the estimates are strongly non-Gaussian (see the *J-curve hypothesis* of Allport [8]).

The relatively substantial and long tails could be the results of the lack of experience of students but also of the limited time that we gave them to learn the new tool and, in general, the quite novel scientific material presented to them. Thus neither Fisher –Snedecor type tests nor ANOVA tables of critical values are really applicable here. We have performed Monte Carlo simulations to check the probability of a correlation coefficient of 0.06 or greater appearing by chance based on the statistics we observed. We found that this probability is about 0.3, so the observed value is clearly insignificant.

We can say that in the experiment we performed, no effect of conformity (cooperation) was observed. We do not have any substantial evidence that the results of the group as a whole had an influence on the fits made by individual students.

5. Conclusions

We have proposed a model of the evolution of the beliefs held by the scientific community. Any group analysing new information is expected to improve knowledge of a subject. We used a Bayesian approach to statistical inference to quantitatively describe the process of how the opinion of each individual member of the group changes, taking into account the influence of other group members. This model formulates in an analytic way the expectation that this (conformity/nonconformity) effect modifies the opinion of the group as a whole, both consciously and unconsciously. Our general model can be used to study the role of a leader and the effects of any other inhomogeneities within a group, and to establish the (statistical) significance and quality of the group's decision. The evolution equations proposed explain in a straightforward, analytical way some of the general properties of the general phenomenon of conformity (groupthink).

Since we were working with limited statistics, we simplified the general procedure and eventually applied the approach we also used in [16]. Introducing a single parameter which describes the pairwise correlation of individual results – the common correlation coefficient, this process could describe the group judgment quantitatively, and the parameters of the model could be estimated experimentally. We have created a tool to study the effect of group work in classes, but also to conduct and control the collaborative work of students. The tablet application developed for the experiment considered here creates an automatic wi-fi connection with all the users, turning the teacher's tablet into a network server. Thanks to the use of a wi-fi connection, this application can be used in classrooms without any range or connection problems. This method could be used further to create tools for educational purposes, especially for interactive tests and much more complicated problem solving tests, training courses, etc. Continuous recording of a student's actions could be used to control whether a student is working independently, and to send immediately an alert to the teacher, when necessary.

We used the method described to measure the effect of conformity in a small student group. If the effect of group behaviour on an individual's behaviour were of similar strength to the one observed by Ash some time ago, we would be able to measure it easily. We did not expect to. Our measurements show no significant effect of conformity. The groups were formed *ad hoc*, so almost no cohesion effects are expected. No special pressure or additional stress were introduced, we even asked students to ignore the "average result" which was displayed. Groupthink was expected to be absent, in accordance to the results in [13]. One important point is that we have a tool to measure and study the conformity effect in different conditions. We can use it, for example, for groups of the same size as a class and use the results in educational research. For practical reasons, it is always better to know which educational actions could be made more effective with, or without, the influence of any kind of communication between students, whether conscious or not.

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