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Narendra Kumar JAIN**  
Bhupender Kumar SOM**

OPTIMIZATION OF AN M/M/1/N FEEDBACK QUEUE WITH RETENTION OF RENEGED CUSTOMERS

Customer impatience has become a threat to the business world. Firms employ various customer retention strategies to retain their impatient (or reneged) customers. Customer retention mechanisms may help to retain some or all impatient customers. Further, due to unsatisfactory service, customers may rejoin a queue immediately after departure. Such cases are referred to as feedback customers. Kumar and Sharma take this situation into account and study an M/M/1/N feedback queuing system with retention of reneged customers. They obtain only a steady-state solution for this model. In this paper, we extend the work of Kumar and Sharma by performing an economic analysis of the model. We develop a model for the costs incurred and perform the appropriate optimization. The optimum system capacity and optimum service rate are obtained.

Keywords: reneging, retention of reneged customers, revenue, queuing system, optimization

1. Introduction

Queues with impatient customers can be observed everywhere, in business or elsewhere. For instance, customers arrive at the office of an insurance company to purchase policies, on arriving at the front of the queue they are served and then they depart. Some customers may not complete the process of initiating a policy due to various reasons (e.g. lack of money to pay for a premium, a better option existing elsewhere). Also, they may get impatient and resign from obtaining a policy. Compa-
nies try to retain such customers. They apply various strategies (counseling, discounts on premiums, etc.) to retain impatient customers. Eventually, they may or may not be able to retain a reneged customer. There are situations when customers are not satisfied with the initial service (unsatisfactory settlement amount, false cheques, etc.). They rejoin the system with some probability and are referred to as feedback customers in the queuing literature. There are lots of other businesses where the same phenomenon can be observed, for example vehicle service stations, restaurants etc. In this process, a customer’s impatience seems to be a very significant threat due to the highly competitive business environment. A popular saying in business is: a customer gone once is a customer gone forever. This paper contributes towards the economic analysis of such situations. It suggests an optimum strategy for a firm to maximize its profit, functioning under the constraints mentioned above.

Kumar et al. [7] analytically study an M/M/1/N feedback queue with retention of reneged customers and obtain the steady-state solution recursively. They simply derive the steady-state solution of the model. They do not perform any economic analysis of the model. In this paper, we extend the work by Kumar et al. [7] through developing a model where the total expected cost, total expected revenue and total expected profit functions are derived.

We present the optimization of various parameters in the model such as system capacity and the service rate. A sensitivity analysis for this model has been carried out with respect to the probability of retention, rate of reneging and arrival rate. The optimum service rate and optimum system capacity are obtained based on various parameters such as the probability of retention of reneged customers, rate of reneging and arrival rate. A comparative analysis has been presented to gain deep insight into expected and optimum costs, revenues and profits.

Pattern search and classical techniques of optimization are used for optimization based on the above mentioned model. Analysis of the model is performed in MS EXCEL and MATLAB. MATLAB programs and Spread Sheets are constructed and executed as and when needed.

2. Literature review

Customer impatience results in loss of business for any firm. Choudhary et al. [2] study customer impatience in multi-server queues. They consider both balking and reneging as functions of the state of the system by taking into consideration situations where the customer is aware of his/her position in the system. Kapodistria [5] studies a single server Markovian queue with impatient customers and considers situations where customers abandon the system simultaneously. She considers two abandonment scenarios. In the former one, all the present customers become impatient and perform
synchronized abandonments, while in the latter scenario, the customer presently being served is excluded from the abandonment procedure. She also extends this analysis to an M/M/c queue under the second abandonment scenario. The phenomenon of customer impatience in single-server queues is discussed in the work of Wu et al. [14] as well. Pan [11] studies a model of an M/M/1/N queue with variable input rates. Jain et al. [4] consider a multi-server queuing system in which additional servers are used when the queue is long, in order to reduce the likelihood of customers balking and reneging. They obtain the equilibrium distribution of the queue size along with other performance measures. Altman et al. [1] study a system operating as an M/M/∞ queue. They discuss the case in which whenever the queue is empty, a server is assigned some other task, say U. While performing this additional task, a new customer arrives, finds the server busy and becomes impatient. They analyze both multiple and U-task scenarios and derive the probability generating function (PGF) of the number of customers present. Dudin et al. [3] analyze a multi-server queuing system with a finite buffer and impatient customers. They give an algorithm for finding the stationary distribution of the state of the system and derive basic performance characteristics. Wu et al. [15] focus on an M/M/s queue with multiple vacations, such that the server works with different service rates rather than no service during a vacation period. They generalize an M/M/1 queue with working vacations. A cost function is formulated to determine the optimal number of servers subject to given stability conditions.

Tadj et al. [12] use a vacation queuing model and develop a set of quantitative performance measures for a two-parameter time allocation policy. Based on renewal cycle analysis, they derive an expression for the average cost and propose a search algorithm to find the optimal time allocation policy that minimizes the average cost. Ke et al. [6] analyze the cost in an M/M/R machine repair problem with balking, reneging and server breakdowns. A cost analysis for a finite M/M/R queuing system with balking, reneging and server breakdown is discussed in Wang et al. [13]. Mishra et al. [10] perform a cost analysis for a machine interference model with balking, reneging and spares. Furthermore, in [16] Yue et al. present an analysis for an M/M/R/N queuing system with balking, reneging and server breakdowns. Yue et al. [17] present an analysis for an M/M/c/N queuing system with balking, reneging and synchronous vacations of partial servers. They formulate a model for the costs to determine the optimal number of servers on vacation. Kumar et al. [8] optimize revenue in an insurance business facing customer impatience. They develop a model of the costs in an M/M/1/N queuing system with retention of reneged customers and balking and then minimize them by applying a pattern search algorithm and classical optimization techniques. They also optimize service rate and system capacity with varying rates of reneging and arrival and probability of retention. Kumar et al. [10] further optimize an M/M/1/N queuing system with retention of reneged customers by developing a model for the costs and using optimization techniques through soft computing.
The literature review discussed above provides a sufficient understanding of queuing models with customer impatience. The modeling of costs and optimization are crucial to the design of optimal queuing systems.

3. Description of the model

The model considered in this paper is based on following assumptions:

- Arrivals occur in a Poisson stream one by one with an average arrival rate of \( \lambda \). The inter-arrival times are independently, identically and exponentially distributed with the parameter \( \lambda \).
- There is only one server and service times are exponentially distributed with the parameter \( \mu \).
- The queue discipline is first-come, first-served (FCFS).
- The capacity of the system is finite (say \( N \)).
- Each customer upon arriving in the queue will wait a certain time (reneging time) for service to begin. If it has not begun by then, with probability \( p \) he becomes impatient and leaves the queue without being served and with probability \( q = 1 - p \) remains in the queue until service is complete. The reneging times follow the exponential distribution with parameter \( \xi \).
- With probability \( p_1 \) any service obtained is incomplete. After getting incomplete service, a customer rejoins the queue. Hence, after being served, with probability \( p_1 \), a customer rejoins the system as a feedback customer to receive another regular service. Otherwise, he leaves the system, i.e. with probability \( q_1 \), where \( p_1 + q_1 = 1 \).

The differential-difference equations for this model are given by:

\[
\frac{dP_0(t)}{dt} = -\lambda P_0(t) + \mu q_1 P_1(t) \tag{1}
\]

\[
\frac{dP_n(t)}{dt} = -\left(\lambda + \mu q_1 + (n-1)\xi p\right)P_n(t) + \left(\mu q_1 + n\xi p\right)P_{n+1}(t) + \lambda P_{n-1}(t), \quad 1 \leq n \leq N-1 \tag{2}
\]

\[
\frac{dP_N(t)}{dt} = \lambda P_{N-1}(t) - \left(\mu q_1 + (N-1)\xi p\right)P_N(t), \quad n = N \tag{3}
\]
4. Model for the costs

In this section, we develop a model for the costs incurred in the queuing system using the following symbols:

- \( \frac{1}{\lambda} \) – mean inter-arrival time
- \( \frac{1}{\mu} \) – mean service time
- \( L_s \) – expected number of customers in the system
- \( R_r \) – average rate of reneging
- \( R_R \) – average rate of retention
- \( C_s \) – cost of service per unit time
- \( C_h \) – unit holding cost per unit time
- \( C_L \) – cost associated with each lost unit
- \( C_r \) – cost associated with each reneged unit
- \( C_R \) – cost of retaining a reneged customer
- \( C_{s1} \) – cost of serving a feedback customer
- \( R \) – revenue earned by providing service to a customer
- \( TEC \) – total expected cost per unit time of the system
- \( TER \) – total expected revenue per unit time of the system
- \( TEP \) – total expected profit per unit time of the system

Here we consider a single server, finite capacity Markovian queuing model as studied by Kumar et al. [7], where the steady state probabilities are given by:

\[
P_n = \prod_{k=1}^{n} \frac{\lambda}{\mu q_1 + (k-1) \xi p} P_0, \quad 1 \leq n \leq N - 1
\]

(4)

also, for \( n = N \), we get

\[
P_N = \prod_{k=1}^{N} \frac{\lambda}{\mu q_1 + (k-1) \xi p} P_0
\]

(5)

Using the normalization condition, \( \sum_{n=0}^{N} P_n = 1 \), we get

\[
P_0 = \frac{1}{\frac{1}{P_0} + \sum_{n=1}^{N} \prod_{k=1}^{n} \frac{\lambda}{\mu q_1 + (k-1) \xi p}}
\]

(6)

and the expected number of customers in the system is:
Here, we derive various functions like the total expected cost per unit time, total expected revenue per unit time and total expected profit per unit time. The total expected profit per unit time is then optimized by using pattern search and classical optimization techniques as mentioned above.

The total expected cost (TEC) per unit time is given by:

$$\text{TEC} = \mu(C_s + p_1C_{s1}) + C_hL_s + C_P\lambda P_N + C_rR_r + C_R R_R$$

where the average reneging rate $R_r$ and the average retention rate $R_R$ are given by:

$$R_r = \sum_{n=1}^{N} (n-1)\xi pP_n$$

$$R_R = \sum_{n=1}^{N} (n-1)\xi qP_n$$

Let $R$ be the revenue earned for providing service to a customer, then $R\mu(1 - P_0)$ is the rate of earning revenue for providing service to customers in the system. Hence, total expected revenue (TER) of the system is given by:

$$\text{TER} = R\mu(1 - P_0)$$

Now, total expected profit (TEP) of the system is defined as:
Thus, we have the TEC, TER and TEP functions in terms of various parameters involved. The economic analysis of the model is performed numerically by using these functions and the results are discussed accordingly. The optimization of the model is also carried out in order to obtain the optimal service rate and optimum system capacity.

5. Optimization of the model

In this section, the optimization of the model is performed. First pattern search algorithm is used to optimize the system capacity. Pattern search optimization technique is a hit and trial technique which states that the function under consideration shall be checked for various values. In this paper, we check the value of the profit function for various values of the system capacity starting from minimum \((N = 2)\) by keeping all other variables fixed. The value of the profit function increases initially and then starts decreasing after a certain value of \(N\). The value after which the value of TEP starts decreasing is considered as optimized value of \(N\). We obtain the optimum value of the service rate at which the total expected profit of the system is maximum. We study the variation in total optimum profit in function of the probability of customer retention associated with a particular customer retention strategy. The total optimum cost and total optimum revenue are also computed. The numerical results are presented for cost-profit analysis of the model.

5.1. Determination of optimal service rate

Computational algorithm

Step 1. Define variables.
Step 2. Write the formula of function TEP in terms of \(\mu\).
Step 3. Obtain critical values for TEP.
Step 4. Find the value of $\mu$ at which TEP is maximum (let it be $\mu^*$).
Step 5. Compute the values of TEC, TER and TEP at $\mu^*$.

Results of the comparative analysis of the average system size ($L_s$) with respect to the rate of reneging (when no retention and when there is a certain probability of retention) are given in Table 1 and Fig. 1.

Table 1. Average system size when no retention strategy is followed and when some customer retention strategy for reneged customers in applied

<table>
<thead>
<tr>
<th>$\xi$</th>
<th>$L_s$ at $q = 0$</th>
<th>$L_s$ at $q = 0.6$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.05</td>
<td>1.3812</td>
<td>1.4053</td>
</tr>
<tr>
<td>0.06</td>
<td>1.3734</td>
<td>1.4020</td>
</tr>
<tr>
<td>0.07</td>
<td>1.3657</td>
<td>1.3987</td>
</tr>
<tr>
<td>0.08</td>
<td>1.3582</td>
<td>1.3955</td>
</tr>
<tr>
<td>0.09</td>
<td>1.3508</td>
<td>1.3923</td>
</tr>
<tr>
<td>0.10</td>
<td>1.3435</td>
<td>1.3891</td>
</tr>
<tr>
<td>0.11</td>
<td>1.3364</td>
<td>1.3859</td>
</tr>
<tr>
<td>0.12</td>
<td>1.3293</td>
<td>1.3827</td>
</tr>
<tr>
<td>0.13</td>
<td>1.3224</td>
<td>1.3796</td>
</tr>
<tr>
<td>0.14</td>
<td>1.3155</td>
<td>1.3765</td>
</tr>
<tr>
<td>0.15</td>
<td>1.3088</td>
<td>1.3734</td>
</tr>
</tbody>
</table>

$\lambda = 4, \mu = 3, N = 4, q_1 = 0.9.$

![Graph](image-url)  

*Fig. 1. $L_s$ at $q = 0$ and $q = 0.6$ in function of $\xi$*

It can be observed that the average system size remains high when the retention of reneged customers takes place with certain probability say, $q = 0.6$ in comparison to
the system size when no retention of reneged customers take place. This affects the total profit and revenue of the firm as increase in system size results in more and more customers in the system and the revenue made goes high.

5.2. Optimization of the system capacity and service rate

In Table 2, first we optimize system capacity \( N^* \) by using search technique and then calculate optimal service rate \( \mu^* \) for the obtained optimal system capacity by using classical optimization algorithm in MATLAB.

<table>
<thead>
<tr>
<th>( N )</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6*</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>TEP</td>
<td>136.0567</td>
<td>194.7702</td>
<td>219.3722</td>
<td>230.6481</td>
<td>235.5184</td>
<td>236.9310</td>
<td>236.3410</td>
<td>234.5400</td>
<td>231.9905</td>
<td>228.9773</td>
</tr>
</tbody>
</table>

\( \lambda = 4, \ \xi = 0.20, \ \mu = 3, \ C_s = 4, \ C_{s1} = 2, \ C_h = 3, \ q = 0.6, q_1 = 0.9, \ C_r = 8, \ C_R = 25, \ C_L = 12. \)

It can be observed from the table that the TEP is maximum at \( N = 6 \) and then decreases successively, hence it can be identified that optimum system capacity in this case is at \( N = 6 \) and is represented as \( N^* = 6 \). To obtain optimum service rate \( \mu^* \) for optimum system capacity \( N^* \) thus obtained is obtained by using a MATLAB program in which classical optimization technique has been followed. The algorithm gives us optimum service rate \( \mu^* = 9.0674 \) at which the TEP at \( N^* = 6 \) increases to 395.4931 from 236.9310 while all other parameters kept constant as they were.

![Fig. 2. Total expected profit in function of \( N \)](image)

Figure 2 depicts that total expected profit is maximum at \( N = 6 \), therefore it can be observed that \( N^* = 6 \) for above mentioned data set and for \( N^* = 6 \) the optimum service rate is \( \mu^* = 9.0674 \).
Finding optimum triplet \((N^*, q^*, \mu^*)\)

Now we optimize the total expected revenue (TER), total expected cost (TEC) and total expected profit (TEP). MATLAB programming is used to obtain optimum service rate for each value of \(q\).

Table 3. Values of TEC, TER and TEP for various probabilities of customer retention, \(q\)

<table>
<thead>
<tr>
<th>(q)</th>
<th>(C_R)</th>
<th>(\mu = 3)</th>
<th>(\mu = \mu^*)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>TER</td>
<td>TEC</td>
</tr>
<tr>
<td>0.2</td>
<td>8</td>
<td>291.5763</td>
<td>44.4871</td>
</tr>
<tr>
<td>0.3</td>
<td>12</td>
<td>290.8458</td>
<td>45.7609</td>
</tr>
<tr>
<td>0.4</td>
<td>14</td>
<td>289.3779</td>
<td>45.9429</td>
</tr>
<tr>
<td>0.5*</td>
<td>20</td>
<td>290.1051</td>
<td>49.8856</td>
</tr>
<tr>
<td>0.6</td>
<td>25</td>
<td>287.4766</td>
<td>50.5456</td>
</tr>
<tr>
<td>0.7</td>
<td>32</td>
<td>288.1573</td>
<td>55.8861</td>
</tr>
<tr>
<td>0.8</td>
<td>36</td>
<td>283.6454</td>
<td>54.5200</td>
</tr>
<tr>
<td>0.9</td>
<td>40</td>
<td>284.2787</td>
<td>58.6724</td>
</tr>
<tr>
<td>1.0</td>
<td>45</td>
<td>276.4612</td>
<td>55.3030</td>
</tr>
</tbody>
</table>

\(\lambda = 4, \mu = 3, q_1 = 0.9, \xi = 0.2, C_s = 4, C_{s1} = 2, C_h = 3, C_L = 12, C_r = 8, R = 100\)

When no optimum strategy is followed, the TEP is lower and when the optimum strategy is followed the profit is higher. We obtain an optimum triplet \(q^* = 0.5, N^* = 7\) and \(\mu^* = 8.6304\) (Table 3). For the system capacity 7, retention strategy is applied in such a way that 50\% of reneged customers are retained and the customers are provided a service at the rate of 8.6304, the profit obtained is maximum. From Figure 3, it can be observed easily that the profit obtained is much higher if the optimum policy is
followed and is maximum at optimum triplet i.e., at $N^*, q^*, \mu^*$ while keeping all other parameters constant.

**Finding optimum triplet ($N^*, q^*, \mu^*$)**

Table 4. Variation in total optimum profit in function of $\xi$

<table>
<thead>
<tr>
<th>$\xi$</th>
<th>$\mu = 3$</th>
<th>$\mu^*$</th>
<th>$N^*$</th>
<th>$\mu^*$</th>
<th>TEP</th>
<th>TEP*</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1*</td>
<td>TER 292.1964</td>
<td>TEC 49.8478</td>
<td>TEP 242.3485</td>
<td>7</td>
<td>8.2299</td>
<td>439.4631</td>
</tr>
<tr>
<td>0.2</td>
<td>TER 287.4766</td>
<td>TEC 50.5456</td>
<td>TEP 236.9310</td>
<td>6</td>
<td>8.7302</td>
<td>437.1208</td>
</tr>
<tr>
<td>0.3</td>
<td>TER 286.0972</td>
<td>TEC 54.1401</td>
<td>TEP 231.9571</td>
<td>6</td>
<td>8.9857</td>
<td>436.2713</td>
</tr>
<tr>
<td>0.4</td>
<td>TER 279.8442</td>
<td>TEC 52.5270</td>
<td>TEP 227.3173</td>
<td>5</td>
<td>9.6575</td>
<td>428.4839</td>
</tr>
<tr>
<td>0.5</td>
<td>TER 278.5869</td>
<td>TEC 55.1180</td>
<td>TEP 223.4688</td>
<td>5</td>
<td>9.8459</td>
<td>433.9795</td>
</tr>
<tr>
<td>0.6</td>
<td>TER 277.3391</td>
<td>TEC 57.5592</td>
<td>TEP 219.7798</td>
<td>5</td>
<td>10.0271</td>
<td>433.4630</td>
</tr>
<tr>
<td>0.7</td>
<td>TER 276.1031</td>
<td>TEC 59.8593</td>
<td>TEP 216.2438</td>
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<td>10.2014</td>
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</tr>
<tr>
<td>0.8</td>
<td>TER 274.8810</td>
<td>TEC 62.0269</td>
<td>TEP 212.8540</td>
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<td>10.3691</td>
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</tr>
<tr>
<td>0.9</td>
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<td>TEC 57.0897</td>
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<td>4</td>
<td>10.9868</td>
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</tr>
<tr>
<td>1</td>
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<td>TEC 58.7161</td>
<td>TEP 207.2642</td>
<td>4</td>
<td>11.1120</td>
<td>429.9423</td>
</tr>
</tbody>
</table>

$\lambda = 4, q = 0.6, q_1 = 0.9, C_{s1} = 2, C_s = 4, C_h = 3, C_r = 25, C_L = 12, C_r = 8, R = 100$

In the case of variation of $\xi$ and no optimum strategy followed, the total expected profit is lower as compared to the case when the optimum strategy is followed (Table 4). We obtain an optimum triplet $\xi^* = 0.1, N^* = 7$ and $\mu^* = 8.2299$. It is observed that if the system capacity is 7, 10% of the customers are reneging due to whatever reason and the customers are provided with a service at the rate 8.2299, the total expected profit obtained is maximum for the values taken in this scenario.
From Figure 4 it can be observed easily that the profit obtained is much higher if the optimum policy is followed and is maximum at optimum triplet i.e., at $N^*, \xi^*, \mu^*$ while keeping all other parameters constant.

**Finding optimum triplet ($N^*, \lambda^*, \mu^*$):**

Table 5. Variation in total optimum profit w. r. t. $\lambda$

<table>
<thead>
<tr>
<th>$\lambda$</th>
<th>$\mu = 3$</th>
<th>$N = N^*$</th>
<th>$\mu^*$</th>
<th>$\mu = \mu^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>TER</td>
<td>TEC</td>
<td>TEP</td>
<td>TER*</td>
</tr>
<tr>
<td>3.5</td>
<td>282.948</td>
<td>46.2032</td>
<td>236.7456</td>
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</tr>
<tr>
<td>3.6</td>
<td>284.9410</td>
<td>47.8883</td>
<td>237.0526</td>
<td>7</td>
</tr>
<tr>
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<td>286.6998</td>
<td>49.5599</td>
<td>237.1400</td>
<td>7</td>
</tr>
<tr>
<td>3.8</td>
<td>288.2506</td>
<td>51.2160</td>
<td>237.0346</td>
<td>7</td>
</tr>
<tr>
<td>3.9</td>
<td>286.1037</td>
<td>49.0582</td>
<td>237.0455</td>
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</tr>
<tr>
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<td>50.5456</td>
<td>236.9310</td>
<td>6</td>
</tr>
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<td>4.1</td>
<td>288.7063</td>
<td>52.0247</td>
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</tr>
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<td>289.8076</td>
<td>53.4949</td>
<td>236.3127</td>
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<td>4.3</td>
<td>290.7941</td>
<td>54.9560</td>
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</tr>
<tr>
<td>4.4</td>
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<td>56.4078</td>
<td>235.2701</td>
<td>6</td>
</tr>
<tr>
<td>4.5*</td>
<td>288.3666</td>
<td>53.6270</td>
<td>234.7397</td>
<td>5</td>
</tr>
</tbody>
</table>

$\xi = 0.2, q = 0.6, q_1 = 0.9, C_{t1} = 2, C_t = 4, C_h = 3, C_R = 25, C_L = 12, C_r = 8, R = 100$

In the case of variation of $\lambda$ and no optimum strategy is followed, the total expected profit is lower as compared to the case when the optimum strategy is followed (Table 5). We obtain an optimum triplet $\lambda^* = 4.5, N^* = 5$ and $\mu^* = 10.3369$. If the system capacity is 5, arrival rate – 4.5 customers per unit time and the customers are provided with a service at rate 10.3369, the profit obtained is maximum for the values taken in this scenario.
From Figure 5 it can be observed easily that the profit obtained is much higher if the optimum policy is followed and is maximum at optimum triplet i.e., at \( N^*, \lambda^*, \mu^* \) while keeping all other parameters constant.

6. Conclusions

Economic analysis of an M/M/1/N feedback queuing system has been performed with retention of reneged customers. Average system sizes with and without retention of reneged customers have been analyzed, the cost model developed and various parameters of the system optimized such as capacity of the system and average service rate.

Three optimum triplets \((N^*, q^*, \mu^*)\), \((N^*, \xi^*, \mu^*)\) and \((N^*, \lambda^*, \mu^*)\) have been obtained and the total optimum profit presented against total expected cost in all three cases.

The results obtained in this paper are useful for any firm operating in the field of finance, supply chain, manufacturing, etc.

References


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