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ANALYSIS OF INCENTIVE COMPATIBLE DECISIONS IN A MULTICRITERIA AUCTION

An iterative multicriteria closed-auction conducted with the use of a multi-agent computer-based system is analyzed. This system supports the submission of offers, multicriteria analysis carried out by the organizer of the auction, simulation, and analysis of the behavior of competing bidders. Analysis of incentive compatible decisions is the main subject of this research. A mathematical formulation of the decision making problem and selected results of a bidding session conducted using this system are presented and analyzed.

Keywords: *multicriteria auction, incentive compatible decisions, multi-agent systems, multicriteria optimization*

1. Introduction

This paper relates to a wider field of research dealing with the analysis of incentive compatible multicriteria decision making with the use of multi-agent computer-based systems. Situations with a number of agents acting on a market with their own interests and attempting to realize their own egoistic goals are analyzed. The effects of their decisions also depend on the decisions of other agents. Each agent has his own private information and, in general, he is reluctant to reveal information to others. The subject of the research includes decision mechanisms leading to compatibility with

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agents' motivation. Such mechanisms can be constructed by harmonizing the agents' actions to ensure the effectiveness of the whole system.

The literature dealing with incentive compatibility in decision making problems is really rich, including theory, experimental research and practical applications. It also relates to auction theory. Let us mention as examples the papers by Vickrey [19], Mayerson [11], Milgrom, Weber [13], McAfee, McMillan [12], Rutstrom [15], Chao, Wilson [3], Jurca, Faltings [5], Teich et al. [16] where other references can also be found. Incentive compatibility in market mechanisms has been analyzed by Toczyłowski [17, 18]. Experimental analysis of multicriteria incentive compatible decisions was carried out based on the example of the producer and buyers problem [7]. The ideas developed in these papers have inspired the research presented here.

This paper also deals with an experimental study based on the example of a multicriteria closed auction organized for the construction of a public object. In the study, various forms of bidding and different levels of access to information are analyzed. The decisions of the organizer of the auction and of the competing bidders are based on multiple criteria. Each of them has, of course, his own preferences. The bidders can make offers in one or more rounds. The organizer and each of the competitors in the auction, has his own knowledge about his potentialities and profitability limits. It is assumed that each of them can obtain the information required to make their own multicriteria analysis. It is also assumed that the organizer of the auction does not reveal his preferences before the offers are made. This is different to typical public auctions, where the organizer has to reveal his preferences by defining importance of the criteria valuating an offer. When the offers are made, the organizer carries out a multicriteria analysis. He decides either to open a new round in which updated offers can be made by the competitors, or to finish the auction. In this paper, an example of an auction for the construction of a bridge or a public building is analyzed in which the organizer of the auction has two criteria to be minimized: the time and price for realizing the project. In the specification of the auction he specifies several variants of the realization time. Each of competing bidders proposes the price for realizing each of these time variants in his offer.

A special multi-agent system has been constructed to simulate different variants of the auction process. The system was constructed in the AIMMS [1, 2] environment. Users of the system play the roles of the organizer of the auction and of bidders, as appropriate. The system is set up by an operator who starts the actions of a computer agent supporting the organizer and the required number of agents supporting the bidders. This system ensures the confidentiality of the information available to each user.

The experimental study analyzing mechanisms for incentive compatible multicriteria decision making with the use of multi-agent computer-based systems is still in progress. This paper presents some initial results. It includes a description of the model and mathematical formulation of the optimization problem. This problem is solved by the system using a multicriteria analysis carried out by the auction organizer. Next, the results of a simulated auction are shown and analyzed as an example. Some final remarks summarize the results and indicate further directions of research.

2. Formulation of the problem

Let us assume that an authority organizes an auction for the construction of a public object, for example a bridge. It is interested in constructing the object in the shortest possible time at the lowest possible cost. The authority, called henceforth, the auction organizer, defines a discrete set T of time variants in which the object may be constructed, with realization times d_t for $t \in T$. We assume that for each variant of the time d_t the organizer has defined the maximum acceptable price for realizing the project, so he specifies a vector of upper cost limits.

A set O of bidders is given. An offer p_o, $o \in O$ includes a vector of prices, which defines the prices for which bidder o is ready to construct the object for each of the specified time variants. That is to say, component p_{ot} of the vector defines the price proposed by bidder o in his offer for realizing the project for the time variant d_t, $t \in T$. It is assumed that each of the bidders has carried out a multicriteria analysis of the possible variants for realizing the project, and has defined the minimum acceptable price for which he can realize the project in a given time. This defines his profitability limits. A bidder does not know which time variant will be chosen by the organizer. Confidentiality of information is ensured. A bidder does not know the price limits of the other bidders, nor their offers. Also, he does not know the cost limits of the organizer. The bidding process should enable selection of a contractor and the realization variant giving the greatest benefits to the organizer. A general flow chart of the bidding process is presented in Fig. 1.

The bidding process can be carried out in one or several rounds. A round is started when bidders are invited by the organizer to make offers. Once the organizer has all the offers, he opens them and makes a multicriteria analysis with respect to two criteria: the time and price for realizing the project.

The analysis is carried out in an interactive way with the use of the reference point method developed by Wierzbicki [20, 21]. According to this method, the organizer can generate and analyze non-dominated offers in the space of his criteria (price and time), assuming reservation points r and aspiration points a in this space. The subscripts i of the components r_i, a_i of the vectors r and a, refer to the price and the time of realizing the project, respectively. The set of the indexes will be denoted by Y. The following optimization task is solved:

$$\max\left(z + \varepsilon \sum_{i \in Y} z_i\right) \tag{1}$$

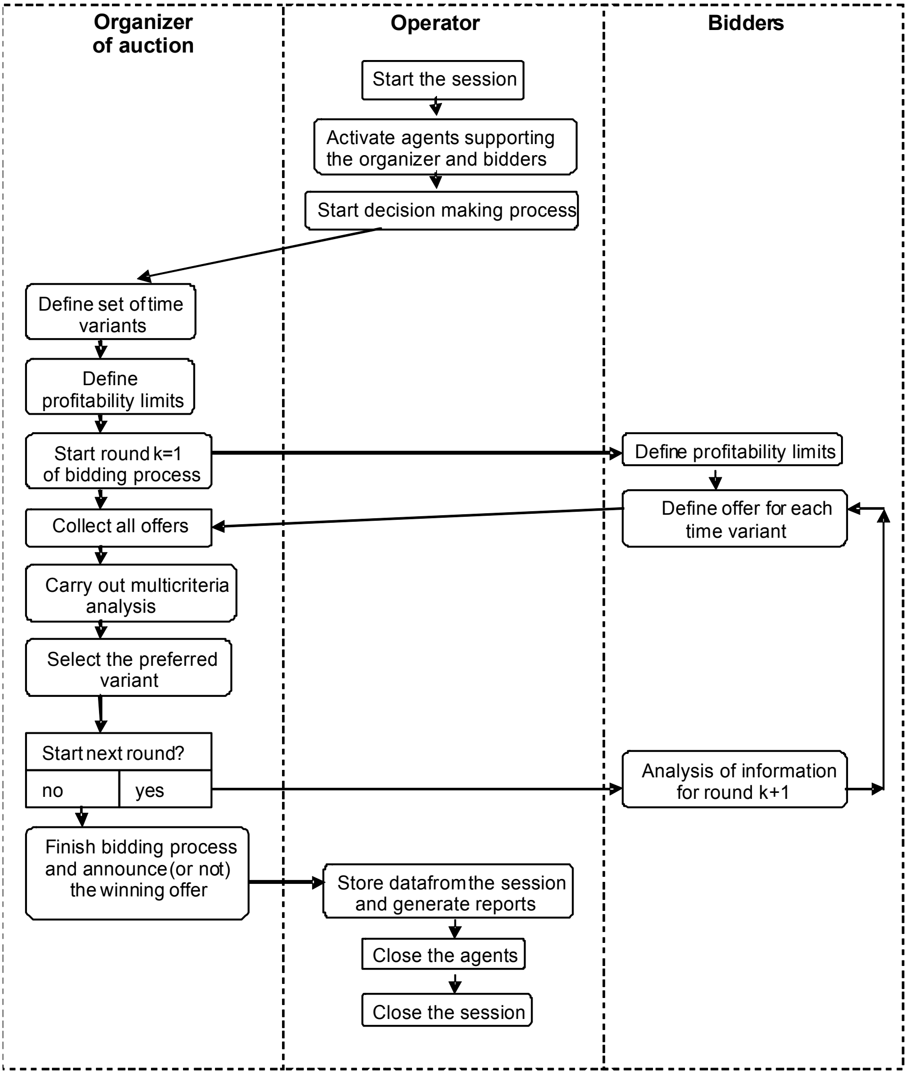

Fig. 1. Flow chart of the bidding process using the computer-based system

subject to the constraints of the reference point method

$$\begin{aligned} z &\le z_i, && \forall i \in Y \\ z_i &\le \gamma(x_i - r_i)/(a_i - r_i), && \forall i \in Y \\ z_i &\le (x_i - r_i)/(a_i - r_i), && \forall i \in Y \\ z_i &\le \beta(x_i - a_i)/(a_i - r_i) + 1, && \forall i \in Y \end{aligned} \tag{2}$$

limits for the minimum values of the price and the time:

$$\begin{aligned} x_{\text{price}} &\geq p_{ot} - (p_{\max} - p_{\min})(1 - w_{ot}), \qquad \forall o \in O, \qquad t \in T \\ x_{\text{time}} &\geq d_t - (d_{\max} - d_{\min})(1 - q_t), \qquad \forall t \in T \end{aligned} \tag{3}$$

and constraints related to the discrete form of the set *T*:

$$\sum_{o \in O, t \in T} w_{ot} = 1, \qquad \sum_{o \in O} w_{ot} = q_t, \qquad \forall t \in T \tag{4}$$

The task is solved using such a formulation for the points *r* and *a*, given by the organizer. The points are defined in the space of the price and time criteria. A solution of the task, a point *x* in this space, is non-dominated in the set of variants proposed by the bidders, due to the properties of the reference point method. In this formulation there are the following additional variables: z, z_{cost}, $z_{\text{time}} \in \mathbf{R}^1$, coefficients of the reference point method ε, β, γ, where ε is an appropriately small positive number, $0 < \beta < 1 < \gamma$, $p_{\max}$ and $p_{\min}$ denote the most costly and the cheapest offer, respectively, for the given variants of time, $d_{\max}$ and $d_{\min}$ denote the shortest and the longest realization time, respectively, w_{ot} for $o \in O$ and $t \in T$, q_t for $t \in T$ denote additional binary variables.

The organizer finishes the multicriteria analysis when he has evaluated and compared all the non-dominated points in terms of his interest. Then he selects the best solution, according to his preferences and announces the selected offer, finishing the auction, or decides to continue the auction into the next round.

Suppose he decides to continue the auction. In such a case, the bidders obtain information about the cheapest offers for the indicated time variants. However, they do not know which of the bidders presented a given offer, and they do not know the preferences of the organizer. Each bidder can update his offer by decreasing the price. He cannot, however, recede from the previous offer, if he does not desire to update it. He does not know whether the auction will be continued into the next round or not. The organizer requests new offers and repeats the multicriteria analysis for the new set of offers. He can continue the process into the next round; he can stop the process in any round and cancel the auction if he has found all the offers unsatisfactory, or can finish the auction announcing the offer selected.

3. Selected results and their analysis

A number of interactive auction sessions were simulated using a computer-based system. Humans played the roles of the auction organizer and bidders. We were interested in their possible behaviors. An important question arises as to whether a multi-round and multicriteria auction mechanism can lead to exposing confidential information regarding the real cost of the bidders realizing the public project.

Selected results of one of the sessions are presented and analyzed below. The session relates to an auction for a project to construct a public object. Three bidders participated in it. The organizer of the auction defined 6 possible time variants for realizing the contract: 30, 33, 36, 39, 42 or 45 months. He also defined his profitability limit, i.e. the maximum price he can pay for realizing the project for each of the time variants. We assume that each bidder has defined his profitability limit, i.e. the minimum price for which he can construct the object for each given time variant. The profitability limits of the organizer and bidders are presented in Fig. 2. In the example presented, the profitability limits of the bidders are below the profitability limit of the organizer. Thus there exist intervals of prices for which construction is profitable for both the organizer and the winning bidder. Comparison of the profitability limits is presented only for the analysis of the bidding process. The organizer does not know the profitability limits of the bidders, and the bidders do not know the profitability limit of the organizer. The organizer is interested in the realizing the project in the shortest possible time at the minimum cost. He understands that realizing the project in a shorter time requires a greater price.

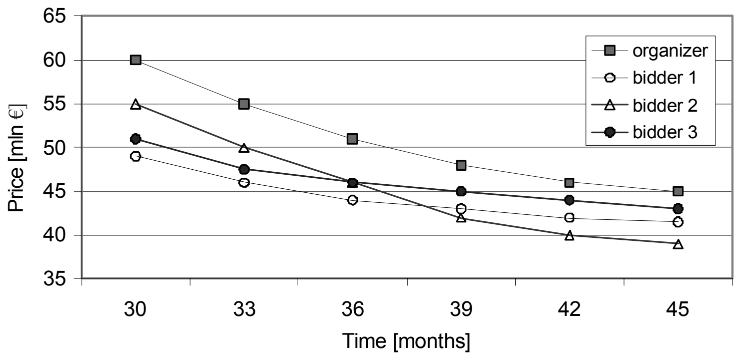

Fig. 2. Profitability limits of the organizer and bidders

Figure 3 presents the initial offers of the bidders, compared to the cost limits of the organizer. Each of the bidders tries to obtain a significant increase in price in comparison to his profitability limit. They do not know the profitability limits of the organizer. The offers of the first and the second bidder are so close to the organizer's profitability limits that they are not satisfactory for him. Note that the offers of the third bidder for the longer time variants are higher than the profitability limits of the organizer. The organizer decided to continue the auction into the next round. In the next round, each bidder has information about the lowest prices offered in the previous round, but does not know who presented it.

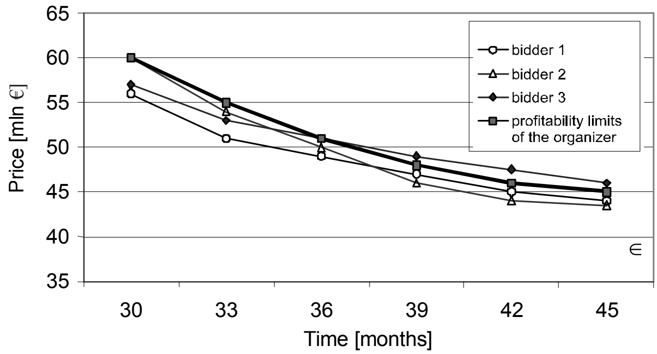

Fig. 3. Initial offers and the profitability limits of the organizer

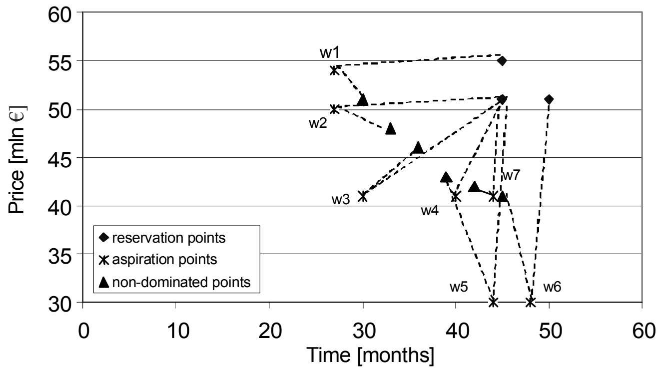

Fig. 4. Results of the multicriteria analysis carried out by the organizer

In each round, the organizer carries out a multicriteria analysis in the criteria space of the price and the time for realizing the project. The interval reference point method is used. An example of such analysis is illustrated in Fig. 4. The analysis is carried out over a number of iterations. In each iteration, the organizer assumes a reservation and aspiration point in his criteria space. The computer-based system solves the optimization task formulated in section 2 and derives the appropriate non-dominated point. Figure 4 presents several variants of reservation and aspiration points and the appropriate non-dominated points linked by dashed lines. The non-dominated points are represented in the figure by black triangles. Note that the same non-dominated point has been derived by the system for variants w4 and w5 of the aspiration and reservation points. The organizer can derive a representation of the set of non-dominated points assuming different aspiration and reservation points, and then select the point closest to his preferences. The organizer can select the most preferred non-dominated point and informs the bidders about his decision when he decides to finish the auction. In this example, he decided to continue the process until the 4th round.

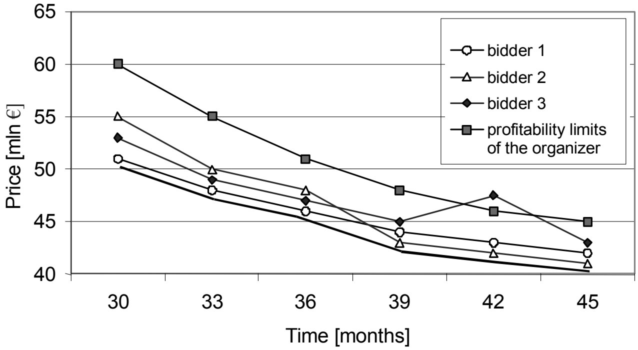

Fig. 5. Offers in the fourth round

Figure 5 presents the offers in the fourth round. The best offers are underlined by the bold line. In the case of a realization time of 30, 33 or 36 months, the best offers are the ones by bidder 1, while in the case of a realization time of 39, 42 or 45 months, the best offers are the ones by bidder 2. The organizer obtained a significant improvement in the offers compared to the best initial offers given in round 1. The most competitive offers for each of the time variants were revealed.

Table 1. Offers of bidders in consecutive rounds

Round	Bidder	Time					
		30	33	36	39	42	45
1	1	56	51	49	47	45	44
	2	60	54	50	46	44	43.5
	3	57	53	51	49	47.5	46
2	1	56	51	49	45	43	42
	2	55	50	48	46	44	43.5
	3	55	49	48.5	45	47.5	43
3	1	54	48	47	44	43	42
	2	55	50	48	44	42	41
	3	53	49	47	45	47.5	43
4	1	51	48	46	44	43	42
	2	55	50	48	43	42	41
	3	53	49	47	45	47.5	43
Profitablity limits							
Organizer		60	55	51	48	46	45
Bidder 1		49	46	44	43	42	41.5
Bidder 2		55	50	46	42	40	39
Bidder 3		51	47.5	46	45	44	43

All the offers given during the 4 rounds of the auction are presented in Table 1. We can observe interesting aspects in the behavior of bidders and their strategies. Let us note the offer given by bidder 3 in the 4-th round for a realization time of 42 months, also presented in Fig. 4. This bidder has not decided to update his offer given in round 1, because the most competitive offer was equal to his profitability limit and he could not beat it. He had no incentive to decrease his price, though he could do so, as his initial price was significantly greater than his profitability limit. Some bidders updated their offers in successive rounds by minimally decreasing the cheapest offer from the previous round. However, one of them decided to make a greater decrease in price to clearly beat the competitors.

In this and other sessions, we can observe that the final offers converged to the second smallest profitability limit of the bidders. It is understandable that the bidder with the lowest profitability limit for a given time variant has no incentive to decrease the price of such an offer and the other bidders cannot beat it. In general, a large number of rounds may be required to obtain such a result, especially if bidders only make small decreases in their offer prices in successive rounds.

4. Conclusions

An approach of a multicriteria open auction has been proposed. Such an auction is carried out over a number of rounds in which updated offers are proposed, evaluated and compared. A mathematical model has been constructed, including the formulation of an optimization problem. This problem is solved to support the multicriteria analysis carried out by the auction organizer. A multi-agent computer-based system has been designed and implemented. Using this system, a number of sessions have been carried out with humans to assess different variants of multicriteria auctions and to analyze the incentive compatibility of the decisions made by the bidders and by the organizer of the auction. Different behaviors of the organizer and bidders, their decisions and mutual relations were assessed. It has been found that in a multi-round auction, under the full confidentiality of individual information, the prices converged to the second smallest cost limit of the bidders.

Planned research includes development of the model and appropriate reconstruction of the multi-agent computer-based system. Different rules for the multicriteria auction and various bidding strategies will be analyzed. Full confidentiality of individual information has been assumed in the model proposed. This confidentiality relates to cost limits and to the preferences of the organizer, as well as of the bidders. It would be interesting to check how the access of bidders to some selected information, for example to information on the preferences of the organizer, influences the behavior of bidders and their strategies during the auction process. In the present model, bidders introduce data about their cost limits, as well as proposed offers, into the system. Appropriate multicriteria analysis leading to this data has to be carried out outside the system. An additional module supporting such analysis is planned in a new version of the system. The cost limits of the organizer and bidders state natural reservation points for the multicriteria analysis each of them carries out. The cost limits can be calculated using the concept of BATNA (Best Alternative to a Negotiated Agreement) in an analogical way to the approach by Kruś [8–10]. The BATNA concept [4] is commonly used in international negotiation processes.

This example relates to a public project in which two criteria based on the price and time for realizing a project were assessed. Examples with a greater number of criteria are also analyzed in the research. For example, a project for the construction of a number of blocks in an energy plant in given time periods. In such a case, the organizer of the auction has a set of criteria based on, among other things, the price for constructing successive blocks. The approach presented in Ogryczak, Wierzbicki [14] and Kozłowski, Ogryczak [6] could be then used.

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