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DETERMINATION OF THE OPTIMAL EXCHANGE RATE VIA CONTROL OF THE DOMESTIC INTEREST RATE IN NIGERIA

An economic scenario has been considered where the government seeks to achieve a favourable balance-of-payments over a fixed planning horizon through exchange rate policy and control of the domestic interest rate. The dynamics of such an economy was considered in terms of a bounded optimal control problem where the exchange rate is the state variable and the domestic interest rate is the control variable. The idea of balance-of-payments was used as a theoretical underpinning to specify the objective function. By assuming that, changes in exchange rates were induced by two effects: the impact of the domestic interest rate on the exchange rate and the exchange rate system adopted by the government. Instances for both fixed and flexible optimal exchange rate regimes have been determined. The use of the approach has been illustrated employing data obtained from the Central Bank of Nigeria (CBN) statistical bulletin.

Keywords: *balance-of-payments, domestic interest rate, exchange rate, optimal control theory, statistical control*

1. Introduction

The exchange rate at the time of independence of Nigeria in 1960 up to the year 1970 was 0.7143 naira per USD [11]. Naira is the local currency in Nigeria. Due to this, the price of one USD was stable at 0.7143 naira. During this period and until 1985, the exchange rate never exceeded one naira per USD [11]. Ipso facto, the exchange rate was 0.8938 naira per USD in 1985. In 1986, the Nigerian government adopted the Structural Adjustment Programme wherein the exchange rate was deregulated. Since then, the exchange rate has continued to rise (i.e., depreciation of the

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naira). From 2011 till the date of writing this, the exchange rate fluctuated between 154 and 160 naira per USD [18]. The continued deregulation of the exchange rate and its adverse effects on the external debt stock, as well as on the monetary policy of Nigeria have become a subject of huge interest among Nigerians. This is enough motivation for us to consider this subject. We provide an alternative method to determine the optimal exchange rate based on the domestic interest rate. We demonstrate the utility of our method with MATLAB (see Appendix) using data from the Central Bank of Nigeria (CBN) statistical bulletin from 1986 up to 2010.

Earlier on, Akinlo [1] studied whether or not the continued depreciation of the naira has led to a decline in naira holdings vis-a-vis the demand for foreign currency. The finding was that it has not led to currency substitution in the country. Imimole and Enoma [10] posited that exchange rate depreciation in Nigeria has a positive effect on the inflation rate.

The need to improve the standard of living in an economy has necessitated the exchange of goods and services across national boundaries. For obvious reasons, the expansion of exports coupled with the depreciation of the local currency leads to economic growth [12]. Nonetheless, this is not a yardstick for economies to devalue (or depreciate) their local currencies [2]. Besancenot and Vranceanu [2] studied the flexibility of exchange rates from a debt management perspective and posited that governments which decide to adopt fully flexible exchange rate regimes should first reduce their foreign-currency-dominated liabilities. The volatility of the exchange rate has been discussed in the literature with respect to some macroeconomic indicators such as reserve adequacy, inflation, current account balance, debt/GDP ratio, etc., where GDP stands for gross domestic product [4]. Chen and Tsang [6] provided evidence that the nominal exchange rate is determined by both macroeconomic and financial variables. Iyoha [12] considered the determination of the exchange rate based on the demand and supply schedules for foreign exchange. An overview on the contributions to the theory of exchange rate determination is contained in [15]. Binici and Cheung [3] noted that an equation for the exchange rate based on the optimal interest rate allowing for interest rate and inflation rate inertia offers the best in-sample performance.

It is well-known that the performance of a country as regards international trade, during a specified period is measured by the balance-of-payments [12]. Against this background, we build our model for determination of the exchange rate for an economy where the government seeks to achieve a favourable balance-of-payments over a fixed planning horizon through its exchange rate policy and control of the domestic interest rate. The planning horizon is fixed for a year ahead as $[t_0, T]$. We assume that the foreign interest rate is given and that the domestic interest rate at time t, denoted by $r(t)$, is under statistical control. Explicitly, we set the control limits for the domestic interest rate using the idea of statistical process control, wherein periodic samples of size n are taken, so that the bounds for $r(t)$ are $\mu - 3(\sigma/\sqrt{n}) \leq r(t) \leq \mu + 3(\sigma/\sqrt{n})$, where μ is the population mean and $\sigma/\sqrt{n}$ is the standard deviation of the sample mean. These control limits are assumed to persist throughout the planning horizon. A discussion on statistical process control is found in [16]. We also assume that the level of income is exogenous, so that the balance of trade, denoted by B, measured in units of local currency, is a function of the exchange rate, i.e., $B = B(e(t))$, where $e(t) > 0$ is the price of one unit of foreign currency at time t as measured in local currency units. Let the exchange rate at time t_0 be known and denoted as $e(t_0) = e_0$. Let K denote the net capital inflow from abroad into the country. Since the foreign interest rate is given, then K depends only on $r(t)$. Moreover, $K'(r(t)) > 0$ as an increase in $r(t)$ attracts capital from abroad [12]. In particular, we assume that K is a linear function of $r(t)$. The balance-of-payments (BOP) is made up of two components, viz.: the balance of trade and net capital exports. Thus, from our aforementioned assumptions, the balance-of-payments is mathematically expressed as

$$\text{BOP} = B(e(t)) + K(r(t)) \tag{1}$$

Furthermore, we assume that changes in exchange rates are induced by two effects: the impact of the domestic interest rate that acts (via a response constant, γ_0) on the exchange rate and the exchange rate system adopted by the government (that acts via a policy constant, γ_1). More technically, we mean that

$$\dot{e}(t) = \gamma_0 r(t)e(t) + \gamma_1 e(t) = (\gamma_0 r(t) + \gamma_1)e(t), \qquad \gamma_0 \neq 0, \qquad \gamma_1 \neq 0 \tag{2}$$

The key theoretical frameworks of this paper are hinged on optimal control theory and statistical methods. We model the BOP management problem as an optimal control problem and then apply Pontryagin's maximum principle. Optimal control theory with the application of Pontryagin's maximum principle was found in the literature [5, 7– 9, 13]. We obtain estimates of the parameters of the state equation (2) by transforming the non-linear component using a Taylor series expansion [17] and then apply the least squares method [14].

This study is important to economists and policy makers, as well as ministries, departments and agencies involved in the formulation of a national budget where the exchange rate is projected for the financial year ahead.

2. Methodology

We first derive estimators for the parameters, γ_0 and γ_1 in Eq. (2). We do this by linearizing the term $r(t)e(t)$ using a Taylor series expansion about $(r_0, 1)$, where r_0 is an arbitrary point, and neglecting terms of degree two and higher. Moreover, data on the real domestic interest rate and exchange rate are published annually by the regulatory agency of the government in Nigeria. As a consequence, we use a discrete-time model as a proxy for the continuous-time process. For instance, we define $\dot{e}(t)$ to be the first-order difference, $\Delta e_t = e_t - e_{t-1}$, where e_t is the discrete-time analogue of $e(t)$. Thus, we obtain the linear specification

$$e_t = \beta_0 + \beta_1 r_t + \beta_2 e_{t-1} + \xi_t, \qquad t = 1, 2, ..., \xi \tag{3}$$

where $\beta_0 = -\dfrac{\gamma_0 r_0}{1 - r_0 - \gamma_1}$, $\beta_1 = \dfrac{\gamma_0}{1 - r_0 - \gamma_1}$, $\beta_2 = \dfrac{1}{1 - r_0 - \gamma_1}$, $t = 1, 2, ..., \xi$ are the historical periods for which data are available, and ζ_t is the error due to linearizing the product term of Eq. (2). By minimizing the sum of squares of ξ_t using the least squares criterion, we have

$$\hat{\beta} = \left(\begin{bmatrix} \mathbf{1} \\ \mathbf{R} \\ \mathbf{E}_{-1} \end{bmatrix} \begin{bmatrix} \mathbf{1} & \mathbf{R} & \mathbf{E}_{-1} \end{bmatrix} \right)^{-1} \begin{bmatrix} \mathbf{1} \\ \mathbf{R} \\ \mathbf{E}_{-1} \end{bmatrix} \mathbf{E} \tag{4}$$

where $\hat{\beta} = \begin{bmatrix} \hat{\beta}_0 \\ \hat{\beta}_1 \\ \hat{\beta}_2 \end{bmatrix}$, $\mathbf{1}$ is a $\zeta \times 1$ vector of ones, $\mathbf{R}$ is a $\zeta \times 1$ vector of the current domestic interest rates, $\mathbf{E}_{-1}$ is a $\zeta \times 1$ vector of the one-period lagged exchange rates and $\mathbf{E}$ is a $\zeta \times 1$ vector of the current exchange rates. The parameters, γ_0 and γ_1 are therefore estimated as

$$\hat{\gamma}_0 = \frac{\hat{\beta}_1 - \hat{\beta}_0}{\hat{\beta}_2 \left(1 - \dfrac{\hat{\beta}_0}{\hat{\beta}_1} \right)} \tag{5}$$

and

$$\hat{\gamma}_1 = 1 + \frac{\hat{\beta}_0}{\hat{\beta}_1} - \frac{1}{\hat{\beta}_2} \tag{6}$$

For convenience, we suppress the argument t in the functions $r(t)$ and $e(t)$. We express the BOP management problem in accordance with the economic scenario under consideration for an arbitrary time interval $[t_0, T]$, $t_0 \geq \zeta$, as follows.

Objective function:

$$\max_r \int_{t_0}^{T} \left(B(e) + K(r)\right) dt$$

subject to:

- the state equation

$$\dot{e} = (\hat{\gamma}_0 r + \hat{\gamma}_1) e \tag{8}$$

- the control limits for the domestic interest rate

$$\mu - 3\frac{\sigma}{\sqrt{n}} \leq r \leq \mu + 3\frac{\sigma}{\sqrt{n}} \tag{9}$$

The initial value is

$$e(t_0) = e_0 \tag{10}$$

The existence of an optimal control r^* with a corresponding exchange rate e^* for the BOP problem follows from Pontryagin's maximum principle. The Hamiltonian H is

$$H = B(e) + K(r) + \lambda(\hat{\gamma}_0 r + \hat{\gamma}_1) e \tag{11}$$

where λ is the marginal valuation of the exchange rate at time t. Since $K(r)$ is a linear function of r, then H is linear in r. The influence function is given by

$$\dot{\lambda} = -\frac{\partial H}{\partial e} = -(B'(e) + \lambda(\hat{\gamma}_0 r + \hat{\gamma}_1)) \tag{12}$$

The Lagrange function, L, for the Hamiltonian subject to the control limits on the domestic interest rate is

$$L = B(e) + K(r) + \lambda(\hat{\gamma}_0 r + \hat{\gamma}_1) e + \rho_1 \left(\mu + 3\frac{\sigma}{\sqrt{n}} - r \right) + \rho_2 \left(r - \left(\mu - 3\frac{\sigma}{\sqrt{n}} \right) \right) \tag{13}$$

where ρ_1 and ρ_2 are the Lagrange multipliers. To obtain more information about the optimal control r^* and its corresponding exchange rate e^*, we employ the necessary conditions for optimality:

$$\frac{\partial L}{\partial r} = K'(r) + \lambda\hat{\gamma}_0 e - \rho_1 + \rho_2 = 0\,, \qquad \rho_1 \geq 0,\ \rho_1\left(\left(\mu + 3\frac{\sigma}{\sqrt{n}}\right) - r\right) = 0$$

$$\rho_2 \geq 0, \qquad \rho_2\left(r - \left(\mu - 3\frac{\sigma}{\sqrt{n}}\right)\right) = 0$$

When r^* is selected to satisfy

$$K'(r) + \lambda\hat{\gamma}_0 e = 0, \qquad \rho_1 = \rho_2 = 0\,, \qquad \mu - 3\frac{\sigma}{\sqrt{n}} < r^* < \mu + 3\frac{\sigma}{\sqrt{n}}$$

then we determine the exchange rate policy by taking the time derivative of

$$K'(r) + \lambda\hat{\gamma}_0 e = 0$$

to get

$$\dot{K}'(r) + \dot{\lambda}\hat{\gamma}_0 e + \lambda\hat{\gamma}_0 \dot{e} = 0$$

Using the influence function in Eq. (12) and the state equation (8), we obtain

$$\hat{\gamma}_0 e B'(e) = \dot{K}'(r) \tag{14}$$

Note that $K(r)$ is a linear function of r, so that $\dot{K}'(r) = 0$. In addition, $e \neq 0$ and, by a priori consideration, $\hat{\gamma}_0 \neq 0$. Thus, $B'(e) = 0$. This implies that $B(e)$ is constant. It follows that the exchange rate is fixed. Therefore, $\dot{e} = 0$ and $(\hat{\gamma}_0 r + \hat{\gamma}_1)e = 0$ so that

$$r^* = -\frac{\hat{\gamma}_1}{\hat{\gamma}_0}$$

provided that

$$-\frac{\hat{\gamma}_1}{\hat{\gamma}_0} \in \left(\left(\mu - 3\frac{\sigma}{\sqrt{n}}\right), \left(\mu + 3\frac{\sigma}{\sqrt{n}}\right)\right)$$

The economic implication of this is that the government should adopt a fixed exchange rate policy over the period $[t_0, T]$ when

$$\mu - 3\frac{\sigma}{\sqrt{n}} < r^* < \mu + 3\frac{\sigma}{\sqrt{n}}$$

The value of the fixed exchange rate is determined from the model fitted to Eq. (3) by setting $e_{t-1} = e_0$. Thus, we get

$$e^* = \hat{\beta}_0 + \hat{\beta}_1 r^* + \hat{\beta}_2 e_0 \tag{15}$$

However, when $-\hat{\gamma}_1/\hat{\gamma}_0 < \mu - 3(\sigma/\sqrt{n})$ then we set $r^* = \mu - 3(\sigma/\sqrt{n})$, so that $(\mu + 3(\sigma/\sqrt{n})) - r^* > 0$ and $\rho_1 = 0$. Therefore,

$$K'\left(\mu - 3\frac{\sigma}{\sqrt{n}}\right) + \lambda\hat{\gamma}_0 e + \rho_2 = 0$$

For $\rho_2 > 0$

$$K'\left(\mu - 3\frac{\sigma}{\sqrt{n}}\right) + \lambda\hat{\gamma}_0 e < 0$$

The exchange rate function is therefore obtained from Eqs. (8) and (10) as

$$e^* = e_0 \exp\left(\left(\hat{\gamma}_0\left(\mu - 3\frac{\sigma}{\sqrt{n}}\right) + \hat{\gamma}_1\right)(t - t_0)\right) \tag{16}$$

On the other hand, when $-\hat{\gamma}_1/\hat{\gamma}_0 > \mu + 3(\sigma/\sqrt{n})$, we set $r^* = \mu + 3(\sigma/\sqrt{n})$, so that

$$r^* - \left(\mu - 3\frac{\sigma}{\sqrt{n}}\right) > 0, \qquad \rho_2 = 0$$

and

$$K'\left(\mu + 3\frac{\sigma}{\sqrt{n}}\right) + \lambda\hat{\gamma}_0 e > 0$$

Similarly, the exchange rate function is obtained from Eqs. (8) and (10) as

$$e^* = e_0 \exp\left(\left(\hat{\gamma}_0\left(\mu + 3\frac{\sigma}{\sqrt{n}}\right) + \hat{\gamma}_1\right)(t - t_0)\right) \tag{17}$$

In either case, when $r^* = \mu - 3(\sigma/\sqrt{n})$ or $\mu + 3(\sigma/\sqrt{n})$, the exchange rate is an exponential function of time. This means that the government should adopt a flexible exchange rate policy over the period $[t_0, T]$. Notice that the floating exchange rate models in Eq. (16) and Eq. (17) enable us to obtain the exchange rate at any instant of time – daily, weekly, monthly or yearly. This is an improvement over the existing methods in [15].

In all, we propose that the government should adopt a fixed exchange rate regime as determined by Eq. (15) when $(\mu - 3(\sigma/\sqrt{n})) < r^* < (\mu + 3(\sigma/\sqrt{n}))$; otherwise, a flexible exchange rate should be adopted wherein r^* is set to be either $(\mu - 3(\sigma/\sqrt{n}))$ or $(\mu + 3(\sigma/\sqrt{n}))$ as determined from Eq. (16) or Eq. (17), respectively.

It is worth mentioning here that our proposed approach to determining the exchange rate is not based on the demand and supply schedules for foreign exchange as in [12]. Since the optimal exchange rate, e^*, is not market-determined, it may not be the equilibrium exchange rate. The equilibrium exchange rate, e^δ, is the exchange rate at the point of intersection of the demand and supply schedules for foreign exchange. By this consideration, the Central Bank should intervene whenever $e^* \neq e^\delta$. Specifically, the Central Bank should buy up excess foreign exchange in the country when $e^* > e^\delta$, and sell foreign exchange in the foreign exchange market when $e^* < e^\delta$.

3. Numerical illustration

To illustrate the use of our model, we extract data on the domestic interest rate (real interest rate in %) and exchange rate (in naira per USD), for a twenty-five-year period (from 1986 to 2010) from the CBN statistical bulletin and assume that the exchange rate at the beginning of 1986 coincided with that of 1985. We use data from 1986, because exchange rate deregulation in Nigeria commenced in 1986. The exchange rate and the real interest rate are given in Table 1 and Table 2, respectively, as a dataset of five subgroups, each of size $n = 5$.

Table 1. Subgroups for the exchange rate from 1986 to 2010

Subgroup	Exchange rate [naira/USD]				
1	2.0206	4.0179	4.5367	7.3916	8.0378
2	9.9095	17.2984	22.0511	21.8861	21.8861
3	21.8861	21.8861	21.8860	92.3428	100.8016
4	111.7010	126.2577	134.0378	132.3704	130.6016
5	128.2796	118.2147	126.4820	148.8155	153.6500

Table 2. Subgroups for the real interest rate (in %) from 1986 to 2010

Subgroup	Real interest rate [%]				
1	12.00	19.20	17.60	24.60	27.70
2	20.80	31.20	36.09	21.00	20.79
3	20.86	23.32	21.34	27.19	21.55
4	21.34	30.19	22.88	20.82	19.49
5	18.70	18.24	21.18	22.15	20.50

To determine the optimal exchange rate for the year 2011, we choose the year 2010 as the base year, so that t_0 corresponds to year 2010 and T corresponds to the end of year 2011. We estimate the model in Eq. (3) that minimizes the sum of squares of ξ_t, using the data in Tables 1 and 2, as

$$e_t = -16.7029 + 1.0044 r_t + 1.0046 e_{t-1}, \qquad t = 1, 2, \ldots, 25 \tag{18}$$

The variance-covariance matrix is

$$\mathbf{cov}(\hat{\beta}) = \begin{bmatrix} 222.4527 & -8.7471 & -0.2881 \\ -8.7471 & 0.3761 & 0.0051 \\ -0.2881 & 0.0051 & 0.0028 \end{bmatrix}$$

The coefficient of determination and the F-statistic are computed as $R^2 = 0.9420$ and F-value $= 178.7557$, respectively. These results indicate that, although it appears that only the one-period lagged exchange rate is significant and the domestic interest rate is non-significant, the model in Eq. (18) is significant at the 5% level and about 94.20% of the variation in the exchange rate is explained by the model.

From the dataset in Table 2, we compute the mean of each of the subgroups as

$$\bar{X} = \{20.2200, 25.9760, 22.8520, 22.9440, 20.1540\}$$

We obtain the lower and upper control limits, respectively as

$$\mu - 3(\sigma/\sqrt{n}) = 15.6634 \qquad \text{and} \qquad \mu + 3(\sigma/\sqrt{n}) = 29.1950$$

None of the entries in the set $\overline{X}$ require active control as $\overline{x}_i \in \overline{X}$, $i = 1, \ldots, 5$ satisfies the relation

$$15.6634 \leq \overline{x}_i \leq 29.1950, \qquad \forall \overline{x}_i \in \overline{X}$$

From Equations (5) and (6), we estimate the parameters γ_0 and γ_1 as $\hat{\gamma}_0 = 0.9998$ and $\hat{\gamma}_1 = -16.6246$. These results indicate that the exchange rate seems to be increasing in the domestic interest rate, and that the exchange rate policy adversely affects changes in the exchange rate. The optimal control is

$$r^* = -\frac{\hat{\gamma}_1}{\hat{\gamma}_0} = 16.6279, \qquad r^* \in (15.6634, 29.1950)$$

Since r^* is under statistical control, we propose that the optimal domestic interest rate for the Nigerian economy in 2011 was 16.6279% and that the government should adopt a fixed exchange rate policy throughout the year. We determine the fixed exchange rate for the year from Eq. (15) as $e^* = 154.3605$ naira per USD. This exchange rate lies within the current range of exchange rate fluctuation in Nigeria since 2011.

4. Conclusions

This study has provided an insight into the determination of exchange rate via control of the domestic interest rate. Our approach to determination of the exchange rate is novel in as much as it is not based on the demand and supply schedules for foreign exchange. Instead, we determine the exchange rate by solving an optimal control problem wherein the objective is to maximize the balance-of-payments and the domestic interest rate is under statistical control. We have provided such a normative approach as an alternative to an economist's perspective on exchange rate determination. At this point of the study, it is premature to conclude that our model is the most appropriate mathematical formulation for the determination of exchange rate policy. Further work should be undertaken so as to incorporate additional macroeconomic indicators and political factors that may affect the objective function, as well as the constraints. This will go a long way to refining the model.

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Appendix
MATLAB Program for the model of exchange rate determination

```
% Data declaration.
   x1=[12.00 19.20 17.60 24.60 27.70];
   x2=[20.80 31.20 36.09 21.00 20.79];
   x3=[20.86 23.32 21.34 27.19 21.55];
```

```
    x4=[21.34 30.19 22.88 20.82 19.49];
    x5=[18.70 18.24 21.18 22.15 20.50];
    R=[x1 x2 x3 x4 x5]';

    e1=[2.0206 4.0179 4.5367 7.3916 8.0378];
    e2=[9.9095 17.2984 22.0511 21.8861 21.8861];
    e3=[21.8861 21.8861 21.8860 92.3428 100.8016];
    e4=[111.7010 126.2577 134.0378 132.3704 130.6016];
    e5=[128.2796 118.2147 126.4820 148.8155 153.6500];
    E=[e1 e2 e3 e4 e5]';

% The X-bar control chart.
    m1=mean(x1),
    m2=mean(x2),
    m3=mean(x3),
    m4=mean(x4),
    m5=mean(x5),

    n=5;
    m=[m1 m2 m3 m4 m5];
    mu=mean(m),
    f1=length(x1); f2=length(x2); f3=length(x3); f4=length(x4); f5=length(x5);
    f=sum([length(x1) length(x2) length(x3) length(x4) length(x5)]);
    sd=sqrt((length(x1)*std(x1)^2+length(x2)*std(x2)^2+length(x3)*std(x3)^2+...
    length(x4)*std(x4)^2+length(x5)*std(x5)^2)*(f)+f1*f2*(m1-m2)^2+...
    f2*f3*(m2-m3)^2+f3*f4*(m3-m4)^2+f4*f5*(m4-m5)^2)/f,

    e=ones(1,length(m));
    L=mu-3*(sd/sqrt(n))*e,
    U=mu+3*(sd/sqrt(n))*e,
    CL=mu*e,

    x=1:length(m);
    clf
    plot(x,m,'ro-')
    xlabel('Subgroups')
    ylabel('Sample means (in %)')
    hold on
    plot(x,L,'b+--')
    hold on
    plot(x,U,'g+--')
```

```
    hold on
    plot(x,CL,'k*--')
    hold off
    legend ('Sample means', 'Lower Control Limit', 'Upper Control Limit', 'Mean of
    sample means')

%Least squares estimates of the regression parameters.
    et0=0.8938; a=ones(f,1);
    el5=[e5(1,n-4) e5(1,n-3) e5(1,n-2) e5(1,n-1)];
    Elag=[et0 e1 e2 e3 e4 el5]';

    X=[a R Elag];

    beta=inv(X'*X)*X'*E,

%t-test for the significance of parameters.
I=eye(f); N=length(E); p=length(beta); s=inv(X'*X);
se=sqrt((E'*(I-X*s*X')*E)/(N-p)),
covbeta=(se^2)*s,
tcal0=([1 0 0]*beta)/sqrt(covbeta(1,1)),
tcal1=(([0 1 0]*beta))/sqrt(covbeta(2,2)),
tcal2=(([0 0 1]*beta))/sqrt(covbeta(3,3)),

% Decision rule.
if abs(tcal0)>2.06
    disp('Reject H0: the constant term is significant at 5% level')
else
    disp('We do not reject H0: the constant term is not significant at 5% level')
end

if tcal1>2.06
    disp('Reject H0: the coefficient of interest rate is significant at 5% level')
else
    disp('We do not reject H0: the coefficient of interest rate is not significant at 5%
    level')
end

if tcal2>2.06
    disp('Reject H0: the coefficient of lagged exchange rate is significant at 5% level')
else
```

```
    disp('We do not reject H0: the coefficient of lagged exchange rate is not significant
    at 5% level')
end

Rsquare=(beta'*X'*X*beta-N*(mean(E))^2)/(E'*E-N*(mean(E))^2),

Fcal=((beta'*X'*X*beta-N*(mean(E))^2)*(N-p))/((E-X*beta)'*(E-X*beta)*(p-1)),
Ftab=3.44;

if Fcal>Ftab
    tdisp('Reject H0: the model is significant at 5% level')
else
    if Fcal<Ftab
    disp('We do not reject H0: the model is not significant at 5% level')
    end
end

%Estimate of the state equation coefficients.
    gamma0=(beta(2,1)-beta(1,1))/(beta(3,1)*(1-beta(1,1)/beta(2,1))),

    gamma1=1+beta(1,1)/beta(2,1)-1/beta(3,1),

%The monentary policy.
    r=-gamma1/gamma0,

% The exchange rate policy.
if r>(mu-3*sd/sqrt(n)) & r<(mu+3*sd/sqrt(n))
    disp('Fixed exchange rate policy')
e=beta(1,1)+beta(2,1)*r+beta(3,1)*e5(1,5),
else
    disp('Flexible rate policy')
for t=(1/52):(1/52):1
    e=e5(1,n)*exp((gamma0*(mu-3*sd/sqrt(n))+gamma1)*t),
end
end
```

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