OPERATIONS RESEARCH AND DECISIONS

No. 2 2013
DOI: 10.5277/ord130204

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MULTI-CRITERIA MODEL FOR DETERMINING ORDER SIZE

A multi-criteria model for determining the order size for materials used in production has been presented. It was assumed that the consumption rate of each material is a random variable with a known probability distribution. Using such a model, in which the purchase cost of materials ordered is limited, three criteria were considered: order size, probability of a lack of materials in the production process, and deviations in the order size from the consumption rate in past periods. Based on an example, it has been shown how to use the model to determine the order sizes for polyurethane adhesive and wood in a hard-coal mine.

Keywords: *multicriteria decision support, stock control, material requirement planning*

1. Introduction

There are many models in the theory of inventory management which allow us to determine purchase and inventory policy. In most of these models, the cost function (of order, storage, and lack of inventory) is used as a criterion to evaluate a solution. Among the classical models of inventory with demand realized in a fixed period of time, the following ones should be mentioned:

- re-order point model (ROP),
- re-order cycle model (ROC).

Using the ROP model, the optimal order size is determined, along with the level of stock at which an order ought to be placed. Using the ROC model, the orders are placed at pre-determined moments of time. However, the order size is determined so as to return the inventories to a predefined level. Models of this type and their applications may be found in [21, 13, 8, 9, 23, 24].

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Under the assumption that demand is realized at determined moments (of discrete) time, the classical models are:

- the Wagner–Within model [23],
- the Silver–Meal model [22].

Using the Wagner–Within model, an optimal order policy is determined on the basis of Bellman's optimality principle from dynamic programming. The objective function is based on the total cost of material orders and storage. Using the Silver–Meal model, the optimal order policy is determined through the minimization of total inventory costs per unit time. The economic order size is usually determined using Wilson's formula (see, e.g., [21, 7, 4].

If when using one of the aforementioned models, the decision-maker cannot estimate the cost of a lack of materials, other models are constructed which include a bound, set by an expert or the decision-maker, on the probability of the event of a lack of inventory in a given cycle of material supply, called the customer service level [21]. All of the models described are used for a continuous ordering process.

In hard-coal mines such as those of the Kompania Węglowa S.A., the order sizes are generally determined about a year in advance. This is connected with the timing of the drawing up of purchase plans for all the mines and with the timing of procedures of public procurement for Kompania Węglowa S.A. Therefore, order sizes for materials required by a mine should be determined once on the basis of financial and extraction plans for the next year. When the consumption rate (demand) shows a trend or a model of demand as a function of other known factors can be built, trend models or econometric models are usually used to determine the size of an order [2].

It was assumed in the study that the demand for the materials ordered is a random variable of known probability distribution. The model for ordering only one material is similar to that of the newsboy problem (e.g., [11, 12]), the difference being that the materials that are unused in the period considered are not destroyed or returned at a lower price, but constitute the inventory. Thus, a need arises for constructing a model of a single order for numerous materials, considering the conditions mentioned. A multi-criteria model is, therefore, proposed with three criteria for each material ordered. In order to determine the order size, goal variables are introduced [10, 15] and lexicographic order-based optimization [17, 3, 10] is used.

2. Constructing the model for determining order size

Assume the consumption, per ton of the mining product, of material M_i ($i = 1, 2, \ldots, s$) to be a random variable X_i with a known distribution F_i. Let z_i be the order size for material M_i per ton of the mining product. According to the theory of inventories, there should be such an order size z_i for material M_i that, with the highest probability, covers the demand for this material. It is known that storing material increases the company's costs. Therefore, there should be such an order size for the material that the amount purchased is the lowest possible under the condition that it does not deviate considerably from the past level of demand for this material and the purchase costs of all the materials do not exceed a certain pre-determined value K. It was assumed in this study that the consumption of material M_i per ton of the mining product: x_{i1}, x_{i2}, …, x_{in} in the n previous periods (i = 1, 2, …, s) does not show any trend or periodical fluctuations. It is also assumed that the values x_{i1}, x_{i2}, …, x_{in} are the only information about mining conditions affecting the level of material consumption in the past.

The objective functions, for each material M_i, are:

a) z_i, size of order,

b) the probability of a lack of material M_i,

c) deviations of the order size z_i from the material consumption volumes x_{i1}, x_{i2}, ..., x_{in} in the last n periods.

The model may therefore be written in the following form:

$$\begin{cases} z_i \to \min & \text{(a)} \\ F_i(z_i) \to \max & \text{(b)} \\ \left|x_{it} - z_i\right| \to \min & \text{(c)} \\ \sum_{i=1}^{s} c_i z_i W \le K \\ z_i^{\text{lower}} \le z_i \ \le z_i^{\text{upper}} \\ i = 1, 2, \ ..., s \\ t = 1, 2, \ ..., n \end{cases} \tag{1}$$

where: c_i – price of one unit of material M_i, K – the funds available for purchasing materials M_1, M_2, …, M_s, W – planned volume (in tons) of mining production, $z_i^{\text{lower}} = \min(x_{it};\, t = 1,\ 2, ..., n)$, $z_i^{\text{upper}} = \max\,(x_{it};\, t = 1, 2, ..., n)$.

In order to compare the results according to the separate groups of criteria, normalization must be applied [15, 14, 3]. So let:

$$z_i^u = \frac{z_i}{A_i}, \qquad x_{it}^u = \frac{x_{it}}{A_i}$$

where $A_i = z_i^{\text{upper}} - z_i^{\text{lower}}$.

Let the variables v_i^{z-}, v_i^{F+}, v_{it}^{Q-}, $v_{it}^{Q+} \geq 0$ be goal variables so that:

$$\begin{aligned}
&z_i^u - v_i^{z-} = 0\\
&F_i(z_i) + v_i^{F+} = 1\\
&x_{it}^u - z_{it}^u - v_{it}^{Q-} + v_{it}^{Q+} = 0\\
&v_{it}^{Q-} v_{it}^{Q+} = 0\\
&i = 1,\ 2, \dots, s\\
&t = 1, 2, \dots, n
\end{aligned}$$

The problem (1) may be then written in the following form:

$$\begin{cases}
v_i^{z-} \to \min\\
v_i^{F+} \to \min\\
v_{it}^{Q-} \to \min\\
z_i^u - v_i^{z-} = 0\\
F_i(z_i^u A_i) + v_i^{F+} = 1\\
x_{it}^u - z_{it}^u - v_{it}^{Q-} + v_{it}^{Q+} = 0\\
\sum_{i=1}^{s} c_i z_{it}^u A_i W \leq K\\
z_i^u \geq 0\\
v_i^{z-}, v_i^{F+}, v_{it}^{Q-}, v_{it}^{Q+} \geq 0,\\
v_{it}^{Q-} v_{it}^{Q+} = 0\\
i = 1,\ 2, \dots, s\\
t = 1, 2, \dots, n
\end{cases} \tag{2}$$

Let:

$$f_1(v_1^{z-}, v_2^{z-}, \dots, v_s^{z-}) = (v_1^{z-}, v_2^{z-}, \dots, v_s^{z-})$$

$$f_2(v_1^{F+}, v_2^{F+}, \dots, v_s^{F+}) = (v_1^{F+}, v_2^{F+}, \dots, v_s^{F+})$$

$$f_{3t}\left(v_{1t}^{Q-}, v_{2t}^{Q-}, \dots, v_{st}^{Q-}\right) = \left(v_{1t}^{Q-} + v_{1t}^{Q+}, v_{2t}^{Q-} + v_{2t}^{Q+}, \dots, v_{st}^{Q-} + v_{st}^{Q+}\right), \qquad t = 1, 2, \dots, n$$

Multi-objective optimization problems are usually solved by scalarization, [1, 10, 3, 6, 16, 20]. If we denote by u_i the weight given by the decision-maker to material M_i based on its importance in the production process, so that $u_1, u_2, u_s > 0$ and $u_1 + u_2 + u_s = 1$, then scalarization may be performed in the following way:

$$s\left(u, f_1\left(v_1^{z-}, v_2^{z-}, ..., v_s^{z-}\right)\right) = \sum_{i=1}^{s} q_i v_i^{z-}$$

where

$$q_i = \frac{\prod_{p=1, p\neq j}^{s} u_p}{\sum_{k=1}^{s} \prod_{p=1, p\neq k}^{s} u_p}$$

$$s\left(u, f_2\left(v_1^{F+}, v_2^{F+}, ..., v_s^{F+}\right)\right) = \sum_{i=1}^{s} u_i v_i^{F+}$$

$$s\left(u, f_3\left(v_{1t}^{Q-} + v_{1t}^{Q+}, v_{2t}^{Q-} + v_{2t}^{Q+}, ..., v_{st}^{Q-} + v_{st}^{Q+}\right)\right) = \sum_{i=1}^{s} u_i \left(v_{it}^{Q-} + v_{it}^{Q+}\right)$$

These scalarizing functions generate a Pareto optimal solution. Thus the model obtained in this way will be of the following form:

$$\begin{cases} \sum_{i=1}^{s} q_i v_i^{z-} \to \min & \text{(a)} \\ \sum_{i=1}^{s} u_i v_i^{F+} \to \min & \text{(b)} \\ \sum_{i=1}^{s} u_i (v_{it}^{Q-} + v_{it}^{Q+}) \to \min & \text{(c)} \\ z_i^u - v_i^{z-} = 0 & \\ F_i(z_i^u A_i) + v_i^{F+} = 1 & \\ x_{it}^u - z^u - v_{it}^{Q-} + v_{it}^{Q+} = 0 & \\ \sum_{i=1}^{s} c_i z_i^u A_i W \le K & \\ z_i^u \ge 0 & \\ v_i^{z-}, v_i^{F+}, v_{it}^{Q-}, v_{it}^{Q+} \ge 0 & \\ v_{it}^{Q-} v_{it}^{Q+} = 0 & \\ i = 1, 2, ..., s, \;\; t = 1, 2, ..., n & \end{cases} \tag{3}$$

As the goal variables $v_{it}^{Q-}, -v_{it}^{Q+}$, $t = 1, 2, \ldots, n$ appear in the set of criteria (c), there may be a process for discounting these observations, realized using weights. Let w_t be the weight for period t. For example, using linear weights $w_t = 2t/(n(n + 1))$.

In this case, the set of criteria (c) may be described using only one objective function

$$\sum_{t=1}^{n}\sum_{i=1}^{s} u_i w_t \left(v_{it}^{Q-} + v_{it}^{Q+}\right)$$

Thus, the model takes the following form:

$$\begin{cases} \sum_{i=1}^{s} q_i v_i^{z-} \to \min & \text{(a)} \\ \sum_{i=1}^{s} u_i v_i^{F+} \to \min & \text{(b)} \\ \sum_{t=1}^{n}\sum_{i=1}^{s} u_i w_t (v_{it}^{Q-} + v_{it}^{Q+}) \to \min & \text{(c)} \\ z_i^u - v_i^{z-} = 0 \\ F_i(z_i^u A_i) + v_i^{F+} = 1 \\ x_{it}^u - z_i^u - v_{it}^{Q-} + v_{it}^{Q+} = 0 \\ \sum_{i=1}^{s} c_i z_{ii}^u A_i W \le K \\ z_i^u \ge 0 \\ v_i^{z-}, v_i^{F+}, v_{it}^{Q-}, v_{it}^{Q+} \ge 0 \\ v_{it}^{Q-} v_{it}^{Q+} = 0 \\ i = 1,\ 2, \ldots, s \\ t = 1, 2, \ldots, n \end{cases} \tag{4}$$

This is a model with three linear objective functions, (a), (b), (c), and a set of non-linear constraints. In particular, this problem can be solved by lexicographic goal programming. If priorities $P1$, $P2$, $P3$ are set by the decision-maker for the particular objective functions (a), (b), (c), then we can solve (4) in three stages [3, 6, 17]. In the first stage, the decision problem with the single objective function of priority $P1$ is solved. In the second stage, the decision problem with the single objective function of priority $P2$, and the additional constraint resulting from the solution found in the first stage, is solved. In the third stage the decision problem with the single objective function of priority $P3$, and the additional constraints resulting from the solutions found in the first and second stages, is solved.

3. Indices used in the calculation of solutions

It was assumed that all the information about the production process, and conditions in the mine influencing material consumption, are included in the historical data concerning the level of extraction and material consumption: $x_{i1}, x_{i2}, \ldots, x_{in}$. It was also assumed that in the period considered these conditions will not change. To determine the optimal order size z_i of material M_i, the following indices, based on the historical data, were proposed:

- ratio of occurrences of surpluses of material M_i to occurrences of shortages W_{bi},
- ratio of total surpluses of material M_i to total shortages W_{sbi}.

These indices were defined on the basis of simple profit and loss indices [5] applied on the financial markets:

$$W_{bi} = \frac{\text{count}\, G_i}{\text{count}\, L_i}, \qquad \text{count}\, L_i \neq 0 \tag{5}$$

$$W_{sbi} = \frac{\sum_{t=1}^{n} G_{it}}{\sum_{t=1}^{n} L_{it}}, \qquad \sum_{t=1}^{n} L_{it} \neq 0, \tag{6}$$

where:

$$G_{it} = \begin{cases} 0 & \text{when } x_{it} - z_i > 0 \\ |x_{it} - z_i| & \text{when } x_{it} - z_i \leq 0 \end{cases} \tag{7}$$

$$L_{it} = \begin{cases} x_{it} - z_i & \text{when } x_{it} - z_i > 0 \\ 0 & \text{when } x_{it} - z_i \leq 0, \end{cases} \tag{8}$$

x_{it} are the observations of the consumption of material M_i in the periods $t = 1, 2, \ldots, n$ countG_i is the number of values of G_{it} different from zero, countL_i is the number of L_{it} values different from zero. $W_{bi} \geq 1$ means that the order size z_i was generally associated with a surplus of material M_i rather than a shortage in the past periods. $W_{sbi} \geq 1$ means that when the order size is z_i, then it is expected that the inventory of material M_i will tend to increase.

4. Example of the use of the multi-criteria model to determine the order size for wood and polyurethane adhesive

Wood and polyurethane adhesive are consumed in hard-coal mines [18]. The hypothesis that there is no trend of monthly wood consumption per ton of mining production is not rejected at a significance level of 0.05 (median run test). We obtain the same result for the consumption of polyurethane adhesive per ton of mining production. It was also demonstrated that the volume of consumption of neither material shows periodical fluctuations. Tables 1 and 2 show the basic parameters of wood and polyurethane adhesive consumption, determined based on monthly data from the period of 2008–2010.

Table 1. Basic parameters of the distribution of monthly wood consumption

Parameter	Wood consumption [m^3/t]
Maximum value	0.006892
Minimum value	0.002427
Average value	0.003580
Standard deviation	0.00086
Coefficient of variation	0.238
Median	0.003506
Kurtosis	4.796
Skewness	1.687

Source: Author's own work based on the data of a subsidiary mine of Kompania Węglowa S.A.

The volumes of wood consumption are generally close to the mean and have a slightly right-skewed distribution. The hypothesis that wood consumption is a random variable with normal distribution $N(0.00358; 0.0086)$ is not rejected at a significance level of 0.05 (Kolmogorov–Smirnov test).

Table 2. Basic parameters of the monthly distribution of the consumption of polyurethane adhesive

Parameter	Adhesive consumption [kg/t]
Maximum value	0.201134
Minimum value	0.016772
Average value	0.122
Standard deviation	0.045472
Coefficient of variation	0.371583
Median	0.120728
Kurtosis	–0.140458099
Skewness	–0.368422971

Source: Author's own work based on the data of a subsidiary mine of Kompania Węglowa S.A.

The volumes of polyurethane adhesive consumption are not generally so close to the mean and have a slightly left-skewed distribution. The hypothesis that polyurethane adhesive consumption is a random variable with normal distribution $N(0.00358; 0.0086)$ is not rejected at a significance level of 0.05 (Kolmogorov–Smirnov test). It can also be noted that both materials show a relatively high variation of consumption. The hypothesis that there is no linear correlation between the consumption of wood and of polyurethane adhesive is not rejected at a significance level of 0.05.

Table 3 presents the average unit prices of wood and adhesive, together with planned annual mining production and the planned annual level of expenditure on wood and polyurethane adhesive.

Table 3. Planned and unit costs of wood and adhesive

Subject	Cost
Price per kg of adhesive, c_1	13.75 PLN
Price per m^3 of wood, c_2	296 PLN
Planned mining production, W	4 000 000 t
Maximum expenditure on materials, K	10 970 000 PLN

Source: Author's own work based on the data of a subsidiary mine of Kompania Węglowa S.A.

In model (4), linear weights w_t ($t = 1, 2, \ldots, 36$) were used and it was assumed that both materials are equally important in the mining process, so the weights satisfy $u_1 = u_2 = 0.5$.

Moreover, it was assumed that the decision-maker has not set the priorities of the particular objective functions (a, b, c). Table 4 presents solutions to problem (4) with various sets of priorities. These sets of priorities are:

• solution 1: $P1$ – function (b), $P2$ – function (a), $P3$ – function (c), or $P1$ – function (b), $P2$ – function (c), $P3$ – function (a),

• solution 2: $P1$ – function (a), $P2$ – function (c), $P3$ – function (b), or $P1$ – function (a), $P2$ – function (b), $P3$ – function (c),

• solution 3: $P1$ – function (c), $P2$ – function (b), $P3$ – function (a),

• solution 4: $P1$ – function (c), $P2$ – function (a), $P3$ – function (b).

The SPSS software package and EXCEL have been used to solve the problem.

Table 4. Solutions to problem (4) with various sets of priorities given to the objective functions

Solution	z_1	z_2	$F(z_1)$	$F(z_2)$	Value of objective function			Size of the order		Total cost of the order
					a	*b*	*c*	Adhesive [kg]	Wood [m^3]	
1	0.0953	0.0048	0.28	0.93	0.049	0.397	32.766	381 397.6	19 344.5	1 0970 000.0
2	0.0941	0.0030	0.27	0.23	0.048	0.751	7.741	376 436.8	11 811.2	8 672 122.3
3	0.1011	0.0031	0.32	0.28	0.052	0.699	7.451	404 284.4	12 349.7	9 214 416.8
4	0.1011	0.0033	0.32	0.36	0.051	0.659	7.451	404 201.8	13 124.8	9 442 710.7

Source: Author's own work based on the data of a subsidiary mine of Kompania Węglowa S.A.

To assess the solutions obtained, the W_{bi}, W_{sbi} indices can be used. The values of these indices, determined from formulas (5) and (6), are presented in Table 5.

Table 5. The solutions and values of the indices W_{bi}, W_{sbi}

Size of the order		W_{bi} adhesive	W_{sbi} adhesive	W_{bi} wood	W_{sbi} wood
Adhesive [kg]	Wood [m^3]				
381 397.6	19 344.5	0.333	0.253	11.000	20.433
376 436.8	11 811.2	0.333	0.238	0.286	0.099
404 284.4	12 349.7	0.333	0.334	0.385	0.175
404 201.8	13 124.8	0.333	0.333	0.636	0.362

Source: Author's own work based on the data of a subsidiary mine of Kompania Węglowa S.A.

As it may be seen, with the restrictions set, i.e. not more than 10 970 000 PLN can be spent on the purchase of both materials, based on the values of the indices W_{bi}, W_{sbi}, the following solution should be chosen: $z_1 = 0.1011$ kg/t; $z_2 = 0.0033$ m^3/t, for which the values of the indices W_{bi}, W_{sbi} are closest to 1.

5. Conclusions

A multi-criteria model has been proposed which may be helpful in determining the order sizes for materials needed in a hard-coal mine. The use of the model was illustrated based on the example of determining the order sizes for polyurethane adhesive and wood with an expenditure constraint. A lexicographic order was suggested and given a lack of decision-maker's preferences, a finite set of efficient solutions was found. Based on the analysis of the values of indices describing material surplus and shortage, calculated on the grounds of historical data, one final solution was chosen. The method proposed should be used in an interactive form, in which the decision-maker has the possibility of determining priorities and controlling parameter values when comparing purchase costs, the probability of a lack of materials and the values of W_{sb}, W_b during the calculations. The model presented may serve to assist in the annual planning of material orders for production.

Acknowledgements

The work reported here was carried out within the framework of the research project No. N N524 552038 *Multi-criteria support of material demand planning and control in a mining enterprise* financed by the Ministry of Science and Higher Education.

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