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SUPPORT IN MULTI-CRITERIA DECISION-MAKING UNDER UNCERTAINTY IN A TRANSPORT COMPANY

The problem of finding an appropriate transportation plan for a transport company collaborating with a window manufacturer to reduce transport costs by minimizing the total distance travelled and to provide a minimum car fleet has been presented. The problem involves 3 major manufacturing bases and 15 local storehouses located in all the remaining Polish provincial capitals. Taking into account the fact that some of the parameters of the model may not be accurate due to uncertainty, fuzzy coefficients are used. Using this fuzzy model, optimistic, semi-pessimistic and pessimistic approaches are considered.

Key words: *transportation problem, fuzzy linear programming, L-R notation*

1. Introduction

Transportation is an area of business where efficiency is the result of the impact of various factors. These factors are mainly external and do not depend on the transport operators. The influence of these factors makes planning and execution of transport processes, especially the optimization of operational decisions a very difficult task. What is more, the complexity of transport processes is also an important factor. On the other hand, this complexity presents a challenge to disciplines involved in solving optimization problems.

The aim of this paper was to use methods of mathematical programming to determine the optimal storehouse supplies and transportation plan leading to minimize transport costs and the costs associated with a company car fleet. To solve this problem, a multiobjective optimization model was created. It consists of 90 decision vari-

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ables, 2 objective functions and 21 constraints. Feasible solutions were found using an appropriate computational program – Protass 2, being available for students at the Faculty of Computer Science and Information Technology in Szczecin. Because some coefficients of the objective function cannot be precisely defined, uncertainty modeling using fuzzy sets was applied.

2. The transportation problem

The transportation problem concerns mainly shipping goods from one location (called an origin) to another (called a destination). This process is carried out by a special transport system which consists of selected vehicle types such as cars, trains, aircrafts, etc. The implementation of the process requires making appropriate decisions in order to improve the efficiency of the entire enterprise.

According to Sawik [10], transportation problems can be divided into:

• the standard transportation problem (STP), where the cheapest possible way of shipping goods from suppliers to customers is sought,

• the multistage transportation problem (MTP), where in addition to suppliers and customers some intermediate points exist.

In STP, the objective is to minimize the costs of transportation, i.e.:

$$\sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x_{ij} \to \min \tag{1}$$

• constraints for suppliers:

$$\sum_{j=1}^{n} x_{ij} = D_i, \qquad \text{for} \qquad i \in \{1, ..., m\} \tag{2}$$

• constraints for customers:

$$\sum_{i=1}^{m} x_{ij} = S_j \qquad \text{for} \qquad j \in \{1, ..., n\} \tag{3}$$

• non-negativity conditions:

$$x_{ij} \geq 0 \qquad \text{for} \qquad i \in \{1, ..., m\}, \ \ j \in \{1, ..., n\} \tag{4}$$

where m is the number of suppliers (sources), n – number of customers (destinations), c_{ij} is the cost of transporting one unit of the commodity from the i-th source to the j-th destination, x_{ij} is the quantity of the commodity transported from the i-th source to the j-th destination, D_i is the total amount of commodity at the i-th source, S_j denotes the total demand for the commodity at the j-th destination. The form of (1)–(4) represents a balanced transportation problem, where the total demand equals the total supply. In reality, unbalanced tasks are more common. In the case of an unbalanced model, i.e. the total demand is not equal to the total supply, a dummy source or dummy destination can always be added to balance this difference, so the form (1)–(4) is sufficiently general. STP can be solved using the simplex algorithm, which is available in many open source applications. However, there are methods specifically designed for such problems (e.g. the potential method) which can be found in the papers of Buga and Nykowski [3], Jędrzejczak et al. [5], Kąciak et al. [7], Trzaskalik [11] or Wagner [12].

An MTP can be written as an STP with added constraints for the intermediate points, stating that the sum of the commodity volumes transported to an intermediate node equals the sum of those transported away from it. There can also be various types of transport, allowing us to create the shortest or cheapest routes via pipelines, transmission networks, access roads, etc.

Additional modifications of the problem may include:
- additional restrictions on the possible transport routes,
- the addition of production or storage costs,
- additional restrictions connected with the specific nature of a company.

3. A specific decision-making problem in a transport company

The described transport company cooperates with a factory producing windows, as one subcontractor. The transported commodities are silent windows, which have very good parameters of thermal insulation and soundproofing and are becoming a more and more competitive product on the Polish market. It is assumed that the size and weight of the transported windows are fixed, without this assumption the problem would be much more difficult. This simplification does not reduce the importance of the model, because it can also be used to plan the transportation of units of various sizes and weights as set out by the company.

The problem presented involves the shipment of a commodity from 3 major manufacturing bases to 15 local storehouses located in all the remaining Polish provincial capitals (see Fig. 1). The manufacturing bases are located in: Gdańsk, Cracow and Warsaw. Some of the weekly production of each base remains in the company warehouse to satisfy the demand of local customers. Other products can be transported from any base to any storehouse, but of course, the greater the distance between the source and destination, the greater the cost of transporting is. This transport company works six days a week – from Monday to Saturday. The subcontractor intends to minimize the distance travelled and the size of the lorry fleet because it is connected with additional costs such as: depreciation, maintenance, insurance, the number of drivers and the amount of suitable equipment required.

Fig. 1. A map of the manufacturing bases and local storehouses in Poland. Source: author's own work

The following additional assumptions made after consultation with the company have been made:

- the lorry fleet is homogeneous – the only limit is the capacity of vehicles, different for each type of vehicle;
- there is no maximum limit on the distance of a route taken by a single transport vehicle – the vehicles have an unlimited amount of fuel;
- there are three main bases and there are no intermediate points;
- each vehicle must begin and end its journey at the same base;
- in a given day, only one type of lorry may leave a base and goes only to one city;
- the optimization period is one week;
- in order to determine the distance between cities, Google Maps [13] were used – it was assumed that the distance between the cities is given as the length of the straight line between them.

The mathematical model takes into account 90 decision variables x_{ijk}. x_{ijk} denotes the number of the k-th type of lorry shipping commodity from city i to city j, where i are the cities: Gdańsk, Warsaw, Cracow, j are the cities: Szczecin, Olsztyn, Białystok, Zielona Góra, Poznań, Wrocław, Opole, Katowice, Rzeszów, Kielce, Łódź, Lublin, Bydgoszcz, Gorzów Wielkopolski, Toruń, the *k* possible types of lorry are: Mercedes (1) – capacity about 90 pcs, DAF (2) – capacity about 140 pcs.

Table 1. The distances between bases and storehouses [km]
and the demand from each storehouse

Storehouse	Gdańsk	Warsaw	Cracow	Demand (p_j) [pcs/week]
Szczecin	288	454	527	300
Olsztyn	136	176	415	200
Białystok	327	177	408	150
Zielona Góra	341	378	374	100
Poznań	245	279	335	350
Wrocław	377	301	236	400
Opole	413	276	159	180
Katowice	456	259	69	370
Rzeszów	531	253	147	140
Kielce	410	154	102	160
Łódź	294	119	192	190
Lublin	435	153	227	250
Bydgoszcz	143	226	366	220
Gorzów	289	395	442	100
Toruń	149	185	341	130

Source: author's own work based on company data and Google Maps [13].

The known parameters of the model are:
- the capacity of a single vehicle,
- the distance between cities (Table 1),
- weekly storehouse demand in the 15 provincial cities.

3.1. Functional constraints

The first fifteen constraints apply to the number of windows transported from the manufacturing bases which cannot be less than the weekly demand of a storehouse or exceed it by more than 40 pcs (an agreement between the carrier and storekeeper, designed to improve the utilization of the capacity of vehicles):

$$p_j \le \sum_{i=1}^{3} 90x_{ij1} + \sum_{i=1}^{3} 140x_{ij2} \le p_j + 40 \qquad \text{for} \qquad j \in \{1, ..., 15\} \tag{5}$$

where the coefficients p_j are given in Table 1.

The next six constraints consider the number of days in which windows can be transported. Because working days are from Monday to Saturday, there cannot be more than six departures of one type of car from any manufacturing base

$$\sum_{j=1}^{15} x_{ijk} \le 6 \quad \text{for} \quad i \in \{1, 2, 3\}, k \in \{1, 2\} \tag{6}$$

3.2. Objective functions

The first objective function requires minimization of the total distance because in the real world this has a proportional impact on the amount of petrol consumed:

$$f_1(\mathbf{x}) = \sum_{k=1}^{2}\sum_{i=1}^{3}\sum_{j=1}^{15} c_{ij} x_{ijk} \to \min \tag{7}$$

where the coefficients c_{ij} are the distances between the cities presented in Table 1.

The second objective function concerns minimization of the number of shipments by each type of lorry. Because the Mercedes has a smaller capacity, it is associated with the coefficient of 1 and the DAF is associated with the coefficient 1.5, due to the fact that the costs of fuel are higher for a more loaded lorry and also insurance depends on the lorry size:

$$f_2(\mathbf{x}) = \sum_{i=1}^{3}\sum_{j=1}^{15} x_{ij1} + 1.5\sum_{i=1}^{3}\sum_{j=1}^{15} x_{ij2} \to \min \tag{8}$$

Table 2 shows a feasible solution obtained by the STEM method [2], implemented using the Protass 2 application. With this transportation plan, the total distance travelled by the company fleet is 11 052 km (back and forth) and the value of the second objective function is $f_2(\mathbf{x}) = 13 + 1.5 \times 17 = 38.5$. From the manufacturing base in Gdańsk, two Mercedes lorries travel to Szczecin, one – to Olsztyn, Poznań and Bydgoszcz and one DAF to Szczecin, Zielona Góra, Bydgoszcz, Toruń and Gorzów Wielkopolski. The departures from the manufacturing bases in Warsaw and Cracow can be interpreted in a similar way. Based on these results, it can be concluded that in each of the manufacturing bases just one lorry of each type is needed.

A sample transportation schedule was created (Table 3). It should be noted that the manager of the company did not impose restrictions on the daily supply of the commodity to each storehouse. However, the presented schedule satisfies the required supply of the commodity within a maximum of two days. What is more, there are no more than 6 transports by one type of lorry from each base and all bases work 6 days per week, so it can be assumed that one DAF and one Mercedes lorry in each base minimizes the number of lorries required.

Table 2. Feasible solution of the problem

j	Storehouse	Gdańsk Mercedes	Gdańsk DAF	Warsaw Mercedes	Warsaw DAF	Cracow Mercedes	Cracow DAF
1	Szczecin	2	1				
2	Olsztyn	1			1		
3	Białystok			2			
4	Zielona Góra		1				
5	Poznań	1			2		
6	Wrocław						3
7	Opole					2	
8	Katowice					1	2
9	Rzeszów						1
10	Kielce					2	
11	Łódź			1	1		
12	Lublin				2		
13	Bydgoszcz	1	1				
14	Gorzów		1				
15	Toruń		1				
The number of lorry transports		5	5	3	6	5	6

Source: author's own work based on the results from the Protass 2 application.

Table 3. Transportation schedule

Day	Gdańsk	Warsaw	Cracow
Monday	Mercedes – Szczecin DAF – Szczecin	Mercedes – Białystok DAF – Lublin	Mercedes – Opole DAF – Wrocław
Tuesday	Mercedes – Szczecin DAF – Zielona Góra	Mercedes – Białystok DAF – Lublin	Mercedes – Opole DAF – Wrocław
Wednesday	Mercedes – Poznań DAF – Gorzów	DAF – Poznań	Mercedes – Kielce DAF – Wrocław
Thursday	Mercedes– Bydgoszcz DAF – Bydgoszcz	DAF – Poznań	Mercedes – Kielce DAF – Katowice
Friday	Mercedes – Olsztyn DAF – Toruń	DAF – Olsztyn	Mercedes – Katowice DAF – Katowice
Saturday		Mercedes – Łódź DAF – Łódź	DAF – Rzeszów

Source: author's own work.

The minimum weekly supply of each manufacturing base are:

- manufacturing base located in Gdańsk: 300 + 100 + 90 + 100 + 220 + 90 + 130 = 1030 pcs + local customers,
- manufacturing base located in Warsaw: 150 + 250 + 260 + 110 + 190 = 960 pcs + local customers,
- manufacturing base located in Cracow: 180 + 400 + 160 + 370 + 140 = 1250 pcs + local customers.

4. Modeling uncertainty using fuzzy sets

The model described above gives a solution which does not take into account random situations that may occur during transportation. The owner of the transport company should consider that many obstacles can appear such as storehouse demand increasing due to additional orders; roadwork causing delays or additional mileage; there might be even a car accident, etc.

In order to take into consideration the lack of precision in the determination of certain factors in the presented model, fuzzification will be used [14]. There are many approaches to fuzzy multiobjective optimization [6, 8, 9]. This transformation will be defined based on the task to be carried out using the parametric approach proposed by Banaś [1]. This fuzzy model will consider variations in some coefficients of the objective function defining the distance between cities. Such deviations may occur to the variables $x_{15k}, x_{16k}, x_{111k}, x_{113k}, x_{115k}$, $x_{25k}, x_{26k}, x_{211k}, x_{36k}$ because of repair work carried out on the national roads which lead to the cities where Euro 2012 matches were played. According to the Ministry of Transport, one of the solutions was to change the organization of movement or plan detours.

For further discussion, it is assumed that there may be (with truth value equal to 1) a deviation in the range of 6–10% from the value determined in the model but it is not allowed to deviate by more than 15%.

The possible deviations depend on the route:

highway A1 – 10%,	highway A4 – 6%,
highway A2 – 8%,	highway A8 – 7%.

Taking into account these limits of deviations, the coefficients of the objective function can be presented in the form of trapezoidal fuzzy numbers, stored in *L-R* notation as proposed by Dubois and Prade [4].

To present the mechanism of this method, the fuzzy coefficient for the route between Gdańsk and Poznań $\tilde{c}_{15k}$ is:

- lower bound of the core – $c_{15k}^{L} = 245$,
- upper bound of the core – $c_{15k}^{R} = 245 + 245 \times 0.10 = 269.5$,

- right spread of the fuzzy number – $\gamma_{15k}^{R} = c_{15k}^{L} - (245 - 245 \times 0.15) = 36.75$,
- left spread of the fuzzy number – $\gamma_{15k}^{L} = 0$.

The coefficient $\tilde{c}_{15k}$ denoted in the *L-R* notation can be described in the following way: $\tilde{c}_{15k} = (245; 269.5; 0; 36.75)_{LR}$.

Modeling uncertainty using fuzzy sets takes into consideration various degrees of compromise:

- optimistic approach (1^L; 0),
- semi-pessimistic approach (1^R; 0),
- pessimistic approach (0.5^R; 0).

According to the Banaś proposal [1], the corresponding fuzzy coefficient in the objective function is replaced by the following numbers:

- $c_{15k(1;0)}^{L} = 245 + (269.5 - 245) \times 0 = 245$,
- $c_{15k(1;0)}^{R} = 245 + (269.5 - 245) \times 1 = 269.5$,
- $c_{15k(0.5;0)}^{R} = 269.5 + 36.75 \times (1 - 0.5) = 287.875$.

The other coefficients are calculated in a similar way and all these coefficients are summarized in Table 4.

Table 4. The fuzzy coefficients of the objective function depending on the degree of compromise assumed

Decision variables	Determined coefficients c_{ij}	Degree of compromise		
		$(1^L; 0)$	$(1^R; 0)$	$(0{,}5^R; 0)$
x_{15k}	245	245	269.5	287.875
x_{16k}	377	377	414.7	442.975
x_{111k}	294	294	323.4	345.45
x_{113k}	143	143	157.3	168.025
x_{115k}	149	149	163.9	175.075
x_{25k}	279	279	301.32	322.245
x_{26k}	301	301	322.07	344.645
x_{211k}	119	119	128.52	137.445
x_{36k}	236	236	250.16	267.86

Source: author's own work based on Google Maps [13].

Feasible solutions obtained from the Protass 2 application are classified and shown in Tables 5–7.

Total distance travelled by company fleet is:

- optimistic approach: 11 052 km,

- semi-pessimistic approach: 11 664.68 km,
- pessimistic approach: 11 952.98 km.

The value of the second objective function is the same for all approaches – $f_2(\mathbf{x}) = 13 + 1.5 \times 17 = 38.5$ but there is a difference in the number of particular transports by one type of car from each base. For example, using the optimistic approach there are 5 DAF transports from Gdańsk and 6 DAF transports from Cracow and using the pessimistic approach there are 4 DAF transports from Gdańsk and 7 DAF transports from Cracow.

Table 5. Comparison of the results from each model for the base in Gdańsk

j	Storehouse	Optimistic		Semi-pessimistic		Pessimistic	
		Mercedes	DAF	Mercedes	DAF	Mercedes	DAF
1	Szczecin	2	1	2	1	2	1
2	Olsztyn	1		1		1	
3	Białystok						
4	Zielona Góra		1		1		1
5	Poznań	1		1		1	
6	Wrocław						
7	Opole						
8	Katowice						
9	Rzeszów						
10	Kielce						
11	Łódź						
12	Lublin						
13	Bydgoszcz	1	1	1		1	
14	Gorzów		1		1		1
15	Toruń		1		1		1
The number of lorry transports		5	5	5	4	5	4

Source: author's own work.

The results illustrate that the total distance travelled and the transportation plan of each base change depending on the degree of compromise.

Table 6. Comparison of the results from each model for the base in Warsaw

j	Storehouse	Optimistic		Semi-pessimistic		Pessimistic	
		Mercedes	DAF	Mercedes	DAF	Mercedes	DAF
1	Szczecin						
2	Olsztyn		1		1		1
3	Białystok	2		2		2	
4	Zielona Góra						
5	Poznań		2		2		2
6	Wrocław						

j	Storehouse	Optimistic		Semi-pessimistic		Pessimistic	
		Mercedes	DAF	Mercedes	DAF	Mercedes	DAF
7	Opole						
8	Katowice						
9	Rzeszów						
10	Kielce						
11	Łódź	1	1	1		1	
12	Lublin		2		2		2
13	Bydgoszcz				1		1
14	Gorzów						
15	Toruń						
The number of lorry transports		3	6	3	6	3	6

Source: author's own work.

A difference only appears in two places: using the semi-pessimistic or pessimistic approach, the supplier for Bydgoszcz is Warsaw, not Gdańsk, as in the case of the optimistic approach and the supplier for Łódź is not Warsaw – it is Cracow.

Table 7. Comparison of the results from each model for the base in Cracow

j	Storehouse	Optimistic		Semi-pessimistic		Pessimistic	
		Mercedes	DAF	Mercedes	DAF	Mercedes	DAF
1	Szczecin						
2	Olsztyn						
3	Białystok						
4	Zielona Góra						
5	Poznań						
6	Wrocław		3		3		3
7	Opole	2		2		2	
8	Katowice	1	2	1	2	1	2
9	Rzeszów		1		1		1
10	Kielce	2		2		2	
11	Łódź				1		1
12	Lublin						
13	Bydgoszcz						
14	Gorzów						
15	Toruń						
The number of lorry transports		5	6	5	7	5	7

Source: author's own work.

Access to such plans allows us to predict the scale of the costs which will be incurred. The owner of this transport company may expect that the fleet weekly travels a distance of no less than 11 052 km, but which should not exceed 11 952,98 km.

5. Conclusions

The presented results illustrate that the total distance travelled and the transportation plan of each base change depending on the degree of compromise. This information allows us to predict the costs associated with transportation. Taking all the factors into account, analyzing the problem considered in rapidly changing conditions is a very difficult task. Nowadays, qualified planners are responsible for creating optimal transportation schedules. Their knowledge and qualifications are irreplaceable in most cases. Models defined in such a way can be implemented using an appropriate computer package and significantly improve the work of planners and increase their effectiveness. As a result, a company will be able to deliver more orders in less time, which automatically generates benefits, both tangible (higher profits) and intangible (better image).

The described example allows us to minimize the number of vehicles and mileage, cutting operating costs while maintaining high quality services. It also enables the optimal allocation of orders to individual vehicles and their execution. In order to increase effectiveness, precise mapping of market and business conditions must be included in the corresponding mathematical optimization models. The presented models are a good basis for further research. Effective use of the information which these models provide depends on the experience, knowledge and intuition of the decision maker, as well as the use of knowledge acquisition tools.

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