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ANALYSIS OF THE VOTING METHOD USED IN THE EUROPEAN CENTRAL BANK

Game theoreticians usually deal with standard voting methods such as plurality voting or approval voting. In reality however, some complicated non-standard voting methods are used. In this paper the voting method and rotation scheme have been presented used by the Governing Council of the European Central Bank (ECB), as it enlarges to accommodate new members of the economic and monetary union. We present game theoretical approaches for analyzing this method and different methods of computing the Shapley value for games connected with voting under such rotation schemes.

Key words: *voting, European Central Bank, Shapley value*

1. Introduction

A lot of different voting methods are applied in social and economic situations. The most popular are majority voting, plurality voting and ranking methods. There are also situations where very sophisticated methods are used. We analyze one of them. The method analyzed is voting with rotation, used in the European Central Bank. The construction of this method results from the predicted difficulties of voting with an increasing number of voters and guarantees voting power to the most economically important countries.

Some unexpected results were observed. There is no single method of computing power indices for voting with rotation. We analyze the Shapley value (the Shapley –Shubik index in some cases). Several methods of computing the Shapley value are

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used, one of them being proposed in this paper. We show that different methods lead to different results.

The paper is constructed as follows. In Section 2, the voting and rotation scheme, used by the European Central Bank, is presented. The game theoretical approach to analyzing this voting method is presented in Section 3. The conclusions are formulated in Section 4.

2. Scheme of voting with rotation used in the European Central Bank

On 21st March 2003 the European Council introduced a scheme of voting with rotation into the statute of the European Central Bank (ECB). This presentation of the ECB voting system is based on the ECB Monthly Bulletin [3]. The voting system of the ECB is constructed in such a way that it will still work in the case of the enlargement (or reduction, not predicted but also possible) of the Euro area. The declared goal of this construction was a wish to have a system that works without difficulties caused by a high number of voters. The solutions used are modelled on a system applied in the Federal Open Market Committee of the Federal Reserve (FOMC). In the FOMC, there are voting and non-voting members. The members without permanent voting rights are divided into groups and vote according to a rotation scheme [12].

The ECB Governing Council (GC) consists of the Executive Board of the ECB (EB) and the governors of the national central banks (NBC) of the countries that have adopted the Euro. The EB comprises 6 members: the President, the Vice-President and 4 other members. All these members are appointed by the European Council acting by a qualified majority. Each member of the GC has one vote. The GC makes decisions using a simple majority rule with the President having the casting vote in the case of a tie. The number of members of the GC who have voting rights at any given time is limited to 21. The members of the EB have permanent voting rights, So, the number of governors with voting rights cannot exceed 15. This limit was raised to 18 in 2008.

If the number of governors exceeds 18, they are allocated into two groups on the basis of a ranking determined by a composite indicator. The first group consists of the first five governors according to that ranking. The second group consists of all the other governors. The first group has 4 votes, the second group – 11.

When the number of governors exceeds 21, the governors are divided into 3 groups. The first group (consisting of 5 governors) has, again, 4 votes. The second group consists of half of the total number of the governors, rounded up to the nearest full number. It has 8 votes. The third group consists of the remaining governors. It has 3 votes.

In each group, the governors are listed in alphabetical order according to the names of the Member States in the national language written in the Latin alphabet. The rotation starts at a random point in the list. The rotation period is one month with some exceptions resulting from the seasonality of certain votes. The number of governors with voting rights is constant in the first group and is defined as the difference between the number of governors and the number of voting rights minus two for the other groups.

All the governors, voting and non-voting, participate in the GC meetings and retain the right to speak.

The composition of groups can alter when a new country enters the Euro area or the indicator changes.

The indicator is based on two parameters: the share of the country in aggregate GDP at market prices (with a weight of 5/6) and the country's share in the total aggregated balance sheet of monetary financial institutions (MFIs) (with a weight of 1/6).

The ranking of governors is changed only when the ECB's capital key is adjusted (every five years or when a new Member State joins the EU).

3. A game theoretical analysis of ECB voting with rotation

The standard way of modeling voting is cooperative game theory. Usually, a voting game is defined and power indices or values are computed. We shall use the Shapley value [10] and the Shapley–Shubik index [11]. Coalitional structures may be considered (defined in many ways) in the case of natural cooperation (or antagonism) between certain players.

Let us recall some definitions [8]. Let N denote the non-empty and finite set of players. Subsets of N are called coalitions. A pair $G = (N, v)$ is a cooperative game, where v: $2^N \to R$ such that the value of v for the empty set is zero. The function v is called the characteristic function of a game. A game is convex when $v(S \cup T) \geq v(S) + v(T) - v(S \cap T)$ A game is called a monotonic game when $S \subseteq T$ implies $v(S) \leq v(T)$. A convex game with a nonnegative valued characteristic function is a monotonic game. The Shapley value (Sh) is defined by the following formula:

$$Sh_i(G) = \sum_{S \subseteq N, i \notin S} (1/n!)s!(n-s-1)![v(\{S \cup \{i\}) - v(S)]$$

Games whose characteristic functions only take the values 0 and 1 are called simple games. A coalition S is called a winning coalition when $v(S) = 1$. When the Shapley value is applied to simple games, it is called the Shapley–Shubik index (SS).

Some game theory specialists, like Owen [8], only deal with convex games. Others, such as Mesterton [6], also deal with games which are not convex games. As Machover [5] writes, the Shapley value can also be mathematically defined for such games. However, there may be difficulties in interpreting such a value. The Shapley value measures the power of players.

Some special simple games are very often used to model voting. They are called weighted voting games. Each player has a weight w_i. There is a voting threshold t. A coalition S is a winning coalition when $\sum_{i \in S} w_i > t$.

Now we shall present and compare the properties of some, the most important, propositions for measuring the power in the ECB when voting with rotation is applied. We are purely concerned with the game theory aspects.

First we present Ulrich's model [13]. The intertemporal cooperative game (ICG) is defined by using intertemporal voting shares which are the probabilities of voting, computed for each group as the ratio of the number of voting rights to the number of members. The intertemporal voting share is defined to be the weight of a player. A weighted voting game is then played. The threshold is 50% of the sum of weights. Ulrich considers cases with a different number of countries in the European Monetary System (EMU). She analyzes different scenarios, one where EB members support governors from the same country, another where EB members care about the "average" voter in the EMU. Ulrich analyzes the voting scheme with and without rotation. She computes the Banzhaf and Shapley indices. The rotation scheme slightly diminishes the underrepresentation (in comparison to their economic and population weight) of the countries from the first group and reduces the overrepresentation of the countries from the third group.

Belke and Styczyńska [2] consider an enlargement to 27 countries. They apply the following assumptions. Similar to Ulrich's model, an intertemporal cooperative game (ICG) is considered. The EB acts as one unit and is thus treated as one player with six votes. The other players have their intertemporal weights computed in the same way as in Ulrich's model. The Banzhaf and Shapley indices are computed for the cases of rotation and no rotation. A shift is observed in the allocation of power during the early phases of accession to the Euro area. The power of the EB is strengthened by rotation.

Kosior et al. [4] analyze a generalization of the Shapley–Shubik index for voting with rotation, which is constructed as the weighted sum of the Shapley–Shubik indices computed for all the possible auxiliary games implied by the scheme of voting with rotation in the case considered. The probability with which a particular game is played gives its weight. An auxiliary game is a weighted voting game where each player has weight 1 and the threshold is 50% of the number of players. We shall call this generalization the average Shapley–Shubik index (ASS; some authors also call it the Shapley–Shubik index). The number of games considered in the case of 27 countries is 8960. This corresponds to the least common multiple of the periods of rotation of the various groups of voters. Their results show that the power of the EB is more strengthened by rotation than according to the model of Belke and Styczynska. It is assumed that the difference in the methodologies of computing the Shapley–Shubik indices and their generalization is the cause of the differences between these power indices. They also consider a situation where some countries form additional coalitions for voting. They call these coalitions precoalitions. In spite of this name and their citing of Owen's paper on games with a priori unions [7], they do not use a priori unions where a special construction of the Shapley value is needed*. They treat countries which form a precoalition as one player and sum their votes to obtain the weight of the precoalition. The weighted sum of Shapley–Shubik indices is computed analogously to the previous case. A priori unions cannot be used because the game with a priori unions cannot be defined.

Belke and Schnuberin [1] used the preference based power index introduced by Passarelli and Barr [9]. The index measures the probability that a player determines the outcome of a vote. The randomization scheme which is accommodated by this quasi-value is based on the multilinear extension of games. The authors confirmed the higher concentration of power within the EB due to rotation.

In this pape, a new concept of the Shapley–Shubik index for voting with rotation will be considered**. We shall deal with the weighted value of a coalition (WVC). As in the ASS method, we consider all possible auxiliary games. Then we construct a new game defined for the whole set of players where the value of a coalition is the weighted value of that coalition in the auxiliary games considered. The probability with which an auxiliary game is played gives its weight. For coalitions which cannot be constructed in such a way, their value is given by a parameter (usually equal to 0), with the exception of the grand coalition which has value equal to 1 or another parameter. Now, let us study two examples of voting with rotation to compare these different methodologies.

Example 1

There are 4 players. In each vote only 3 voters take part. The player who does not vote is rotated according to numerical order. Hence, four different 3-player games are played. Each voter has one vote in these games. Thus, in every 3-player game the Shapley–Shubik index of each player equals 1/3.

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*Personal communication with A. Kosior.

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1. Intertemporal cooperative game (ICG). The method is based on the Ulrich [13] and Belke–Styczynska [2] approaches. Each player has a share of 3/4. Hence, we are dealing with a weighted voting game where the vector of weights is (3/4, 3/4, 3/4, 3/4). The sum of weights equals 3. Winning coalitions are those in which the sum of the weights of the players is higher than 1.5. So, the winning coalitions are all the 3-player coalitions and the grand 4 player coalition. The Shapley–Shubik index of each player equals 1/4.

2. Average Shapley–Shubik index (ASS). The method is based on the paper by Kosior et al. [4], In each auxiliary 3-player game, the Shapley–Shubik index equals 1/3. The probability of each game is 1/4. Each player takes part in 3 games. So, the weighted sum of the Shapley–Shubik indices is equal to 3 × (1/4) × (1/3) = 1/4 for any player.

3. The weighted value of a coalition (WVC). Here we propose a new construction of the Shapley value for voting with rotation. We calculate the weighted value of a coalition. Consider all the possible systems of voting rights. For each system M we construct an auxiliary game $G(M)$. Then we construct a 4-player game where the value of a coalition is computed as the weighted sum of the values of this coalition in the auxiliary 3-player games considered. The probability with which a game is played gives its weight. This probability equals 1/4 for each game.

Let us consider 3-player games. All these games are constructed in the same way:

$V_M(A) = 1$ when $\#A \geq 2$ and $v_M(A) = 0$ when $\#A < 2$.

Let w be the characteristic function of the 4-player game with weighted values of coalitions (WVC).

Each 3-player coalition occurs only once. So, $v_w(A) = 1 \times (1/4) \times 1 = 1/4$ when $\#A = 3$.

Each 2-player coalition occurs twice. So, $v_w(A) = 2 \times (1/4) \times 1 = 1/2$ when $\#A = 2$. $v_w(A) = 0$ when $\#A < 2$. Hence, $v_w(A) = 0$ when $\#A = 1$.

It is impossible to define the value of the grand coalition in such a way. So we define $v_w(A) = a$ for A such that $\#A = 4$. We have thus obtained a non-simple, non--monotonic and non-convex game. In our game we obtain a Shapley value of $a/4$ for each player. This is the same result as obtained using the ICG and ASS methods for $a = 1$. The above results are presented in Table 1.

Table 1. Shapley–Shubik index computed using various methods (Example 1)

Method	ICG	ASS	WVC
$i = 1, 2, 3, 4$	1/4	1/4	$a/4$

Source: author's work.

We can see that all the results (with $a = 1$ for the WVC) are the same. Now, let us study an example where voting rights are not symmetric.

Example 2

We consider a situation where there are 4 voters. Voter 1 has permanent voting rights. Only 2 voters vote among voters 2, 3, 4. The voter who relinquishes their voting rights is rotated in numerical order. Four different 3-player auxiliary games are considered. We analyze the same 3 methods as in Example 1.

1. ICG. The weights of the players are their probabilities of voting. The weight of player one is 1, of players 2, 3, 4 is 2/3. So, we are dealing with a weighted voting game (1, 2/3, 2/3, 2/3). The sum of weights is 3. The threshold is 1.5.

The characteristic function is computed as follows.

$v(S) = 1$ when $\#S \geq 3$ or ($\#S = 2$ and $1 \in S$).

$SS_1 = 1/2$, $SS_i = 1/6$ for $i = 2, 3, 4$.

2. ASS. The following triples of voters may form the electorate: {1, 2, 3}, {1, 3, 4}, {1, 2, 4}, In each auxiliary 3-player game, $SS_i = 1/3$ for each player. The probability of each game is 1/3. The probability that player 1 takes part in such a 3-player game equals 1. For the other players this probability is equal to 2/3. So the weighted average of the Shapley–Shubik indices equals $3 \times (1/3) \times (1/3) = 1/3$ (number of games $\times$ probability $\times$ SS_1) for player 1. For players $i = 2, 3, 4$ the weighted sum of the Shapley–Shubik indices equals $2 \times (1/3) \times (1/3) = 2/9$.

3. WVC. We use the method defined in point 3. of Example 1. The sets of players in the 3-player auxiliary games G_1, G_2, G_3 are {1, 2, 3}, {1, 3, 4} and {1,2,4}, respectively. Each game is a weighted voting game with weights 1 for each player and threshold 1.5. The winning coalitions are presented in the following table.

Table 2. Winning coalitions in the 3-player games defined according to the WVC method (Example 2)

Winning coalitions	G_1	G_2	G_3
	{1, 2, 3}, {1, 2}, {2, 3}, {1, 3}	{1, 3, 4}, {1, 4}, {3, 4}, {1, 3}	{1, 2, 4}, {1, 2}, {1, 4}, {2, 4}

Source: author's work.

Each auxiliary game is played with the probability 1/3. So the weighted values of coalitions are as follows:

$v_w(\{1, 2, 3\}) = v_w(\{1, 3, 4\}) = v_w(\{1, 2, 4\}) = (1/3) \times 1 = 1/3$,

$v_w(\{1, c\}) = 2 \times (1/3) \times 1 = 2/3$, $c = 2, 3, 4$,

$v_w(\{c, d\}) = 1 \times (1/3) \times 1 = 1/3$ where $c, d = 2, 3, 4, c \neq d$;

$v_w(\{c\}) = 0$ for $c = 1, 2, 3, 4$.

So, we construct a game w with a set of players $\{1, 2, 3, 4\}$ where $w(S) = v_w(S)$ if S is a coalition of the electorate from one of the games G_1, G_2, G_3. Some subsets of $\{1, 2, 3, 4\}$ do not form parts of the electorate in any of the games G_i, $i = 1, 2, 3$, so we have to define the values of such a coalition. We introduce parameters a and b, and define $v_w(\{1, 2, 3, 4\}) = a$ and $v_w(\{2, 3, 4\}) = b$. The most intuitive way of defining a and b is $a = 1$ (because this is the grand coalition) and $b = 0$ (because such a coalition never exists). We compute the Shapley value for the game $G = (\{1, 2, 3, 4\}, v_w)$.

$Sh_1(G) = (1/24)[6(a - b) + 3 \times (4/3)] = (a - b)/4 + 1/6$.

$Sh_i(G) = (1/24)[6a - 2 + 2b - 2/3] = a/4 + b/12 - 1/18$ for $i = 2, 3, 4$.

The Shapley values obtained using the WVC method are presented in Table 3.

Table 3. The Shapley values obtained using the WVC method (Example 2)

Player	Method		
	ICG	ASS	WVC
1	1/2	1/3	$(a - b)/4 + 1/6$
2, 3, 4	1/6	2/9	$a/4 + b/12 - 1/18$

Source: author's work.

This time the results obtained using the three different methods are not the same. Comparing the results presented in Table 5, for parameters satisfying $a - b < 4/3$ we see that the power index of player 1, the one with permanent voting rights, is greatest when we use the ICG method. The power index of player 1 is the smallest when the WVC method is used (for parameters satisfying $a - b \leq 2/3$). Player 1 is stronger than each of the players 2, 3, 4 according to the ICG and ASS methods and for $b \leq 2/3$ according to the WVC method. Let us consider $a = 1$ and $b = 0$. Player 1 is stronger than each of the players 2, 3, 4 according to the WCV method. We have $Sh_1 = 5/12$ and $Sh_i = 7/36$ for $i = 2, 3, 4$. In this case, the results obtained using the different methods are compared in the following table.

Table 4. The Shapley values according to the WVC method for $a = 1$ and $b = 0$ method (Example 2)

Player	Method		
	ICG	ASS	WVC
1	1/2	1/3	5/12
2, 3, 4	1/6	2/9	7/36

Source: author's work.

The above results are connected with some general properties of the WVC method. We consider the most intuitive case where we ascribe a value of 1 to the value of the grand coalition and 0 to other coalitions which do not occur in the auxiliary games. Our results follow from the symmetry of the construction of the rotation scheme and anonymity of the Shapley value. First, let us deal with the situation presented in Example 1. Let us consider a voting system with rotation where there are n voters and k voting rights, $n > k$. We assume that voters are ordered in such a way that the electorate in the first vote consists of the first k voters. In the second vote, the first voter relinquishes his voting rights to the first voter who did not vote in the previous turn. So, in the second vote, voters with numbers from 2 to $k + 1$ have voting rights. Thereafter, the same method of relinquishing voting rights as after the first vote is used. In the m-th vote, voters with numbers m, $(m + 1)(\mathrm{mod}n)$, …, $(m + k - 1)(\mathrm{mod}n)$, where $m = 1, …, n$, have voting rights. In the last considered vote voters with numbers $n, 1, …, k - 1$ have voting rights. We denote this system $V(n, k)$. The following lemma holds.

Lemma 1. In the game $G = (N, v)$ constructed for the system $V(n, k)$, where $N = \{1, …, n\}$ and v is constructed using the WVC method, the Shapley values of all the players are equal.

Proof. Assume $i, j \in N, j > i$.

Let us consider $f: N \to N, f(h) = (h + j - i)(\mathrm{mod}n)$. f is a bijection and $f(i) = j$.

Let us define $N^m = \{m, (m + 1)\mathrm{mod}n, …, (m + k - 1)(\mathrm{mod}n)\}$ with $n(\mathrm{mod}n) = n$, $m = 1, …, n$. N^m is the set of voters with voting rights in the m-th voting. It can be seen that

i) $f(N^m) = N^{(m+j-i)(\mathrm{mod}\,n)}$,

ii) if $i \notin S$ then $j \notin f(S)$ for $S \subseteq N$,

iii) $\#S = \#f(S)$,

iv) if $S \subseteq N^m$ then $f(S) \subseteq N^{(m+j-i)(\mathrm{mod}\,n)}$.

Let us construct the game $G = (N, v_w)$ defined using WVC in the following way.

(WVC1) If there exists m such that $S \subseteq N^m$, then $v_w(S) = (1/n) \sum_{m, S \subseteq N_m} v_m(S)$, where v_m is a characteristic function defined on N^m, $v_m(S) = 1$ if $\#S > k/2$ and $v_m(S) = 0$ otherwise.

(WVC2) $v_w(N) = 1$.

(WVC3) $v_w(S) = 0$ for $S \neq N$ when there does not exist an N^m such that $S \subseteq N^m$.

Because f is a bijection, from properties i–iv we obtain $v_w(\mathrm{S}) = v_w(f(S))$. The Shapley value is anonymous, so

$$\sum_{i \notin S, S \subseteq N} s!(n-s-1)![v_w(S \cup \{i\}) - v_w(S)]$$

$$= \sum_{j \notin f(S), f(S) \subseteq N} s!(n-s-1)![v_w(f_{i,j}(S) \cup \{\mathrm{j}\}) - v_w(\mathrm{f}_{\mathrm{i,j}}(\mathrm{S}))],$$ and $Sh_i(G) = Sh_j(G)$. □

If the condition (WVC3) were to be replaced by $v_w(S) = a_S$ for $S \neq N$ when there does not exist an N^m such that $S \subseteq N^m$, then for the generalization of the above lemma the following consistency condition is needed:

$v_w(S) = v_w(f(S))$ for every bijection f such that $f(i) = j$ and every $i, j \in N$.

Now, let us study an adaptation of Example 2. It is a generalization of the situation in Example 1 and Lemma 2 is a generalization of Lemma 1.

Let the set of voters N be divided into q disjoint groups of voters N_i, each with k_i voting rights, $i = 1, \ldots, q$. $n_i = \#N_i$. We assume that the voters are ordered and

$N_1 = \{1, \ldots, n_1\}$, $N_2 = \{n_1 + 1, \ldots, n_1 + n_2\}$, …, $N_i = \{n_1 + \ldots + n_{i-1} + 1, \ldots, n_1 + \ldots n_{i-1} + n_i\}$.

Let us define $m_0 = 0$, $m_i = n_1 + \ldots + n_i$, $i = 1, \ldots, q$, Then $N_i = \{m_{i-1} + r\colon r = 1, \ldots, n_i\}$, $i = 1, \ldots, q$. In the following, we shall use the convention that $m(\mathrm{mod}\, m) = m$.

In what follows we shall define for numbers x, y a function $d(x, y)$ where $d(x, y) = 1$ if $x \leq y$ and $d(x, y) = 0$ if $x > y$.

Voters in group N_i have k_i voting rights, $k_i \leq n_i$, $k = k_1 + \ldots + k_q$. The rotation scheme works as follows. We assume that in the first round the first k_i voters in N_i vote, $i = 1, \ldots, q$. They are numbered $m_{i-1} + r$, $r = 1, \ldots, k_i$. In the second round, the first voter relinquishes his vote to the voter who is the next voter after the voter $m_{i-1} + k_i$. If this voter is the last in N_i, then the new voter with voting rights is the first voter in N_i. So his number is

$(m_{i-1} + k_i + 1)(\mathrm{mod}\, m_i) + d(m_{i-1} + (m_{i-1} + k_i + 1)(\mathrm{mod}\, m_i), m_{i-1})m_{i-1}$.

Applying this scheme, the first voter in round s, the voter $m_{i-1} + s(\mathrm{mod}\, n_i)$, relinquishes his voting rights in round $s + 1$ to the voter following the k_i th voter in round s. If this voter is the last in N_i, then the new voter with voting rights is the first in N_i. So, in round s the set of voters with voting rights is $N_i^s = ((m_{i-1} + s(\mathrm{mod}\, n_i) + t](\mathrm{mod}\, m_i) + m_{i-1}d([m_{i-1} + s(\mathrm{mod}\, n_i) + t](\mathrm{mod}\, m_i), m_{i-1})\colon t = 0, \ldots, k_i - 1\}$, where $s = 1, \ldots, K$ and K is the lowest common multiple of the numbers $n_1, \ldots, n_q$. We define K in this way, because we consider all the possible combinations in which the k_i voting rights in group N_i appear under this scheme. We shall denote such system of voting rights $V(n_1, \ldots, n_q, k_1, \ldots, k_q)$.

For the system $V(n_1, \ldots, n_q, k_1, \ldots, k_q)$ let us define the game $G^s = (N^s, v^s)$, $s = 1, \ldots, K$, where $N^s = \bigcup_{i=1}^{q} N_i^s$ and v^s is defined as follows.

For $T \subseteq N_s$ $v(T) = 1$ if $\#T > k/2$ and $v(T) = 0$ otherwise. The family of games G^s is constructed from all the games which may be played using the rotation scheme. The probability of each game is $1/K$. Now we define the game $G = (N, v)$ where $v(T) = 1/K \sum_{s:T \subseteq N^s} v^s(T)$ if there exists s such that $T \subseteq N^s$, $v(T) = 1$ when $T = N$ and $v(T) = 0$ otherwise. $v(T)$ is equal to the expected value of v^s in the case where it is possible to compute $v^s(T)$, 1 for the grand coalition N and 0 for other coalitions where the expected value cannot be computed.

The following lemma holds.

Lemma 2. Let us consider a system of voting rights $V(n_1, \ldots, n_q, k_1, \ldots, k_q)$ and $h, j \in N_i$. Then $Sh_h(G) = Sh_j(G)$.

Proof. Let $j, h \in N$, $j > h$. We define f_i: $N_i \to N_i$, $p = m_{i-1} + r \in N_i$,

$f_i(p) = (p + (j - h)(\text{mod}\, n_i))(\text{mod}\, m_i) + d((p + (j - h)(\text{mod}\, n_i)](\text{mod}\, m_i), m_{i-1})m_{i-1}$,

f_i is a translation on N_i by $j - h$. If $j, h \in N_i, f_i(h) = j$. We combine all the f_i to form a function f defined on N. $f: N \to N$, $f(p) = f_i(p)$ when $p \in N_i$, $i = 1, \ldots, q$.

Note that f_i is a bijection on N_i. So f is a bijection on N. Let g: $\{1, ..., K\} \to \{1, ..., K\}$, $g(s) = (s + j - h)(\text{mod}K)$. g is a bijection on $\{1, \ldots, K\}$. Let us note that $f_i(N_i^s) = N_i^{g(s)}$. So $T \subseteq N^s$ if and only if $f(T) \subseteq N^{g(s)}$ and $v^s(T) = v^{g(s)}(f(T))$. Therefore, $v(T) = v(f(T))$ and for $h, j \in N_i$, $v(T \cup \{h\}) = v(f(T) \cup \{j\})$. From the anonymity of the Shapley value we obtain

$$Sh_h(G) = \sum_{T \subseteq N, h \notin T} (1/n!)t!(n-t-1)!(v(\{T \cup \{h\}) - v(T))$$

$$= \sum_{f(T) \subseteq N, j \notin f(T)} (1/n!)t!(n-t-1)!(v(\{f(T) \cup \{j\}) - v(f(T))) = Sh_j(G). \square$$

4. Conclusions

The example of a non-standard voting method considered is constructed with the goal of being effective, transparent and simple. Well known measures of voting power are applied to study the properties of such a voting method. One of the most popular measures of voter power is the Shapley value (the Shapley–Shubik index in the case of simple games). We show that the scheme of voting with rotation used in the European Central Bank leads to inconsistencies in defining the Shapley value for such a voting scheme. Different methods give different results. This is especially important when we compute how rotations of voters affect the enlargement of the Euro zone. The measured impact of using a rotation scheme may depend on the method used to compute the Shapley value. Thus, it is difficult to measure the power of voters in the case of voting with rotation. We propose a new method, called weighted value of coalition (WVC). This method is compared to other methods using the game defined with the aid of a rotation scheme. This method has good computational properties, in spite of the complicated form of the game.

References

[1] BELKE A., VON SCHUBERIN B., *European Monetary Policy and the EBC Rotation Model. Voting Power of the Core versus Periphery*, Discussion Paper 983, DIV, Berlin 2010.
[2] BELKE A., STYCZYNSKA B., *The allocation of power in the enlarged EBC Govering Council: An Assessment of the EBC Rotation Model*, Journal of Common Market Studies, 2006, 44 (5), 865–897.
[3] EBC Monthly Bulletin, European Central Bank, 2009, July, Frankfurt/Main.
[4] KOSIOR A., ROZKRUT M., TOROJ A., *Rotation Scheme of the EBC Governing Council: Monetary Policy Effectiveness and Voting Power Analysis*, [in:] *Report on full membership of the Republic of Polish in the third stage of Economic and Monetary Union*, National Bank of Poland, 2008, 53–102 (in Polish).
[5] MACHOVER M., *Notions of a priori voting power. Critique of Holler and Windgren*, Homo Oeconomicus, 2000, 16, 415–425.
[6] MESTERTON-GIBBONS M., *An Introduction to Game Theoretic Modelling*, American Mathematical Society, 2001.
[7] OWEN G., *Values of games with a priori unions*, [in:] *Essays in Mathematical Economics and Game Theory*, R. Hein, O. Moeshlin (Eds.), Springer-Verlag, Berlin 1977, 76–88.
[8] OWEN G., *Game Theory*, Third Edition, Academic Press, San Diego, 1995.
[9] PASSARELLI F., BARR J., *Preferences, the agenda setter, and the distribution of power in the EU*, Social Choice and Welfare, 2007, 28, 41–60.
[10] SHAPLEY L.S., *A value for n-person games*, [in:] *Contributions to the Theory of Games*, Vol. II, H.W. Kuhn, A.W. Tucker (Eds.), Annals of Mathematical Studies, 1953, 28, 307–317.
[11] SHAPLEY L.S., SHUBIK M., *A method for evaluating the distribution of power in a committee system*, American Political Science Review, 1954, 48, 787–792.
[12] TILMANN P., *Strategic Forecasting of the FOMC*, European Journal of Political Economy, 2011, 27, 547–553.
[13] ULRICH K., *Decision making of the EBC. Reform and voting power*, Discussion Paper, No. 04-70, ZEW, Mannheim 2004.