O P E R A T I O N S   R E S E A R C H   A N D   D E C I S I O N S

No. 1 2013

DOI: 10.5277/ord130103

Kjell HAUSKEN*

EXCHANGE OF GOODS WHILE INVESTING INTO PRODUCTION AND SAFETY

The tradeoff between production and safety investment is scrutinized for two agents who convert resources into production and safety investment while simultaneously exchanging goods voluntarily. We quantify how two Cobb–Douglas parameters, one scaling production versus safety, and the other scaling the relative importance of two goods, impact two agents' production, safety effort, incomes, export, import, price, and utilities. An agent's income from producing a good reaches a maximum for an intermediate value of the Cobb–Douglas parameter that scales the importance of productive effort relative to safety effort. The price of good 2 in terms of good 1, and the agents' utilities depend on both the Cobb–Douglas parameters, the productivity parameter, and both agents' resources and unit costs of production and safety effort.

Keywords: *production, safety, exchange, risk, trade, price*

1. Introduction

Safety risk has not received much attention in the economics literature**. Safety concerns are often considered as constraints imposed by law and regulations. Firms face risks due to internal factors related to production, equipment failure, human failure, due to interaction with other firms within the industry, or external factors. The latter can be societal changes in general, or targeted action such as crime, theft, espionage, hacking, blackmail, terrorism. Asche and Aven [5] argue that *safety measures*

*Faculty of Sciences, University of Stavanger, 4036 Stavanger, Norway, e-mail: kjell.hausken@uis.no

**See Calow [7], and Jones-Lee [13] for economic approaches to safety. Much literature focuses on public safety. See Feber et al. [8] for the economic effects of road safety improvements, Swinbank [22] for the economics of food safety, Thomas [24] for economic and safety pressures on nuclear power, Rose [19] for economic determinants of airline safety, Oi [15, 16] for the economics of product safety, Kotz and Schafer [14] for economic incentives to accident prevention.

have a value in an economic sense, and consider for one firm *the business incentives for investing into safety*. Similarly, Viscusi [25] considers market incentives for safety.

Recent changes in US accounting laws have made CEOs liable to legal malpractice if accounting information is found to be fraudulent. This has caused a certain panic among firms as to whether they should invest more in information assurance technologies, given that an increase in such investments could lead to a decrease in firms' productivity. Firms, most of which have finite resource constraints, are thus naturally led to determine optimal investments in information assurance technologies versus production technologies. The former can be perceived as investment to reduce the risk of legal malpractice. This p intends to understand the factors that influence the tradeoff between safety and productive investment during exchange.

Many industries suffer from incorrect allocations between production and safety, often blinded by complexity. Often the decision is as simple as increasing the allocation to safety from 10% to 30%, or lowering it from 60% to 40% if initially too high. To reflect this basic simplicity, we build a simple model that focuses on which factors impact this crucial allocation. The analysis is presented as straightforwardly as possible.

Classical exchange theory was developed by Smith [21] and Ricardo [17]. More recent accounts are Allen [1], Arrow et al. [4], Hausken and Moxnes [10, 11], Taylor [23]. Recently, exchange theory and conflict have been merged, accounting for production and fighting, see Anderton [2], Anderton et al. [3], Bowles and Gintis [6], Hausken [9], Rider [18], Skaperdas and Syropoulos [20]. The paper makes one step further accounting for safety investment in an exchange model.

2. The model

Each agent i can produce one good i, but also attaches utility to another good j, $i, j = 1, 2$, $i \neq j$. Agent i has a resource R_i (e.g. a capital good, or labor) which can be converted with unit conversion cost a_i into productive effort E_i, and with unit cost b_i into safety effort S_i, where

$$R_i = a_i E_i + b_i S_i \quad \Rightarrow \quad S_i = \frac{R_i - a_i E_i}{b_i} \tag{1}$$

The production cost coefficient a_i, where $1/a_i$ is the productive efficiency, measures the resources required to maintain the agent and machinery he uses in production. Analogously, $1/b_i$ is the safety efficiency. The different unit costs a_i and b_i reflect how productive effort and safety effort have different weights, denominations, and are different in nature, thus ensuring that the two equations signs in Eq. (1) are valid. As a practical aid it may be convenient to think of good i as a consumption good such as oil, and the resource R_i as a capital good such as oil drilling equipment. Alternatively, the product may be a consumption good such as fish, and the resource R_i a capital good such as fishing nets. The productive effort E_i is designed to generate good i, i.e. extract income from resources currently employed. Without risk, the production function for good i or income Y_i takes the simple form $Y_i = E_i^h$, where h is the productivity parameter, with no need for safety effort [12]. An example is production of potatoes in a stable climate with proven conditions and manual labor with minimal technology where risk is negligible and safety effort can be largely ignored. In contrast, with unstable conditions and high reliance on uncertain technology where many unforeseen adverse consequences may follow, investment in safety is required. Examples are protective gear including, e.g. helmets for manual workers on construction sites, blowout preventers in the oil industry, alternative exit routes and food rations for miners. In such conditions, no safety investment causes substantial losses such as costly head injuries, costly blowouts, costs associated with stranded miners who may die before getting rescued, legal costs, and reputation loss. To reflect how production depends jointly on productive effort and safety effort in a risky environment, we model the income as

$$Y_i = \left(E_i^{\beta_i} S_i^{1-\beta_i}\right)^h, \qquad \beta_i \in (0,1) \tag{2}$$

where β_i is a parameter that scales the importance of productive effort relative to safety effort. The parameters β_i and h can be interpreted as parameters, reflecting deterministic investments in production and safety facing a risky environment, or β_i and h can be interpreted as stochastic random variables to incorporate added uncertainty into the optimal investment choices into production and safety. $\beta_i = 1$ means no safety effort. As β_i decreases from 1, safety effort gains increased importance, and the agent faces a tradeoff between E_i and S_i. $\beta_i = 0$ means no productive effort.

Agent 1 exports an amount X_1 of good 1 to agent 2 in exchange for an amount X_2 in return. The agents have equivalent Cobb–Douglas preferences for the two goods, with utilities

$$U_1 = \left(Y_1 - X_1\right)^{\alpha} X_2^{1-\alpha}, \qquad U_2 = X_1^{\alpha}\left(Y_2 - X_2\right)^{1-\alpha}, \qquad X_1 = P_2 X_2, \qquad \alpha \in (0,1) \tag{3}$$

where α is the relative preference parameter for good 1 for both agents, and P_2 is an interior terms-of-exchange price denoting the price of good 2 in terms of good 1. The parameter α can alternatively be interpreted as a random variable to reflect preference fluctuations for the two goods.

3. Analysis of the model

To determine the first order conditions, we let agent 1 choose E_1 and X_1, and agent 2 choose E_2 and X_2, simultaneously and independently, to maximize utility. This gives

$$\frac{\partial U_i}{\partial E_i}=0 \;\Rightarrow\; E_i=\frac{R_i\beta_i}{a_i},\qquad S_i=\frac{R_i(1-\beta_i)}{b_i}\qquad Y_i=\left[\left(\frac{R_i\beta_i}{a_i}\right)^{\beta_i}\left(\frac{R_i(1-\beta_i)}{b_i}\right)^{1-\beta_i}\right]^h \tag{4}$$

Proposition 1. The productive effort E_i increases in the resource R_i and in β_i, and decreases in the unit cost a_i of production. Proof follows from (4).

Proposition 2. The safety effort S_i increases in the resource R_i, and decreases in β_i and in the unit cost b_i of safety effort. Proof follows from Eq. (4).

Proposition 3. The income Y_i increases in the resource R_i, decreases in both unit costs a_i and b_i, and reaches maximum for an intermediate β_i. Proof follows from Eq. (4).

We next substitute $X_2=P_2^{-1}X_1$ into the first equation in (3) and differentiate U_1 with respect to X_1, and thereafter substitute $X_1=P_2X_2$ into the second equation in (3) and differentiate U_2 with respect to X_2. This gives

$$\frac{\partial U_1}{\partial X_1}=0 \;\Rightarrow\; X_1=Y_1(1-\alpha),\qquad \frac{\partial U_2}{\partial X_2}=0 \;\Rightarrow\; X_2=Y_2\alpha \tag{5}$$

Proposition 4. Agent 1's export of good 1 to agent 2 equals agent 1's income multiplied by one minus the Cobb–Douglas preference parameter for good 1 for both agents. Agent 2's export of good 2 to agent 1 equals agent 2's income multiplied by the Cobb–Douglas preference parameter for good 1 for both agents. Proof follows from Eq. (5).

To determine the market equilibrium condition, substituting Eq. (5) into Eq. (3) gives the price equation

$$P_2=\frac{X_1}{X_2}=\frac{Y_1(1-\alpha)}{Y_2\alpha}=\frac{\left[\left(\frac{R_1\beta_1}{a_1}\right)^{\beta_1}\left(\frac{R_1(1-\beta_1)}{b_1}\right)^{1-\beta_1}\right]^h(1-\alpha)}{\left[\left(\frac{R_2\beta_2}{a_2}\right)^{\beta_2}\left(\frac{R_2(1-\beta_2)}{b_2}\right)^{1-\beta_2}\right]^h\alpha} \tag{6}$$

Proposition 5. The price of good 2 in terms of good 1 equals agent 1's export divided by agent 2's export, which depends on both the Cobb–Douglas parameters, the productivity parameter, and both agents' resources and unit costs of production and safety effort. Proof follows from Eq. (6).

The price P_2 of good 2 in terms of good 1 is determined endogenously on a supply-demand basis. When agent 1 acquires more resources (R_1 increases), he produces more (Y_1 increases), exports more (X_1 increases), and the price $P_2 = X_1/X_2$ increases. Conversely, when the relative preference parameter α for good 1 increases so that both agents attach higher utility to good 1 than to good 2, the demand for good 1 increases, causing a lower price P_2 of the less valuable good 2 in terms of the more valuable good 1. Substituting (5) into (3) gives the utilities

$$
\begin{aligned}
U_1 &= Y_1^{\alpha} Y_2^{1-\alpha}\alpha = \left[\left(\frac{R_1\beta_1}{a_1}\right)^{\beta_1}\left(\frac{R_1(1-\beta_1)}{b_1}\right)^{1-\beta_1}\right]^{\alpha h} \\
&\times \left[\left(\frac{R_2\beta_2}{a_2}\right)^{\beta_2}\left(\frac{R_2(1-\beta_2)}{b_2}\right)^{1-\beta_2}\right]^{(1-\alpha)h} \alpha \\
U_2 &= Y_1^{\alpha} Y_2^{1-\alpha}(1-\alpha) = \left[\left(\frac{R_1\beta_1}{a_1}\right)^{\beta_1}\left(\frac{R_1(1-\beta_1)}{b_1}\right)^{1-\beta_1}\right]^{\alpha h} \\
&\times \left[\left(\frac{R_2\beta_2}{a_2}\right)^{\beta_2}\left(\frac{R_2(1-\beta_2)}{b_2}\right)^{1-\beta_2}\right]^{(1-\alpha)h} (1-\alpha)
\end{aligned} \tag{7}
$$

Proposition 6. Both agents' utilities depend on $Y_1^{\alpha} Y_2^{1-\alpha}$ which is agent 1's income from producing good i raised to the Cobb–Douglas preference parameter α for good 1 for both agents, multiplied with agent 2's income from producing good i raised to one minus the Cobb–Douglas preference parameter for good 1 for both agents. Multiplying $Y_1^{\alpha} Y_2^{1-\alpha}$ with α gives agent 1's utility. Multiplying $Y_1^{\alpha} Y_2^{1-\alpha}$ with 1– α gives agent 2's utility. The two utilities depend on both the Cobb–Douglas parameters, the productivity parameter, and both agents' resources and unit costs of production and safety effort. Proof follows from (7).

4. Conclusion

The paper quantifies how two Cobb–Douglas parameters, one scaling production versus safety, and the other scaling the relative importance of two goods, impact two agents' production, safety effort, incomes, export, import, price, and utilities. Using Cobb–Douglas preferences for production versus safety effort, we show how two agents strike a balance between converting resources into production and safety while simultaneously exchanging two goods voluntarily. First, and intuitively, both productive effort and safety effort increase in the agents' resources and decrease in the respective unit costs of these efforts. Second, and also intuitively, productive effort increases in the Cobb–Douglas parameter that scales the importance of productive effort relative to safety effort, while safety effort decreases in this parameter. Third, an agent's income from producing a good reaches a maximum for an intermediate value of this Cobb–Douglas parameter. Fourth, agent 1's export of good 1 to agent 2 equals agent 1's income multiplied by one minus the Cobb–Douglas preference parameter for good 1 for both agents. Agent 2's export of good 2 to agent 1 equals agent 2's income multiplied by this Cobb–Douglas parameter. The two agents' exports determine the price of good 2 in terms of good 1. Fifth, the price of good 2 in terms of good 1 equals agent 1's export divided by agent 2's export, which depends on both the Cobb –Douglas parameters, the productivity parameter, and both agents' resources and unit costs of production and safety effort. Sixth, we show how the agent's utilities depend on their incomes from producing the two goods and their Cobb–Douglas preference parameter for good 1, which also depend on both the Cobb–Douglas parameters, the productivity parameter, and both agents' resources and unit costs of production and safety effort.

References

[1] ALLEN B., *The future of microeconomic theory*, Journal of Economic Perspectives, 2000, 14, 1, 143–150

[2] ANDERTON C.H., *Appropriation possibilities in a simple exchange economy*, Economics Letters, 1999, 63, 77–83.

[3] ANDERTON C.H., ANDERTON R.A., CARTER J.R., *Economic activity in the shadow of conflict*, Economic Inquiry, 1999, 37 (1), 166–179.

[4] ARROW K.J., CHENERY H.B., MINHAS B.S., SOLOW R.M., *Capital-labor substitution and economic efficiency*, Review of Economics and Statistics, 1961 43 (August), 225–250.

[5] ASCHE F., AVEN T., *On the economic value of safety*, Risk Decision and Policy, 2004, 9, 253–267.

[6] BOWLES S., GINTIS H., *The revenge of homo economicus: Contested exchange and the revival of political economy*, Journal of Economic Perspectives, 1993, 7 (1), 83–102.

[7] *Handbook of Environmental Risk Assessment and Management*, P. Calow (Ed.), Blackwell Sciences, Oxford 1998.

[8] FEBER D.J., FELDMEIER J.M., CROCKER K.J., *The economic effects of road safety improvements: An insurance claims analysis*, Journal of Risk and Insurance, 2003, 70, 4, 651–664.

[9] HAUSKEN K., *Mutual Raiding and the Emergence of Exchange*, Economic Inquiry, 2004, 42, 4, 572–586.

[10] HAUSKEN K., MOXNES J.F., *The dynamics of bilateral exchange and division of labor*, International Journal of Modern Physics C, 2005, 16, 1, 117–137.

[11] HAUSKEN K., MOXNES J.F., *The dynamics of multilateral exchange*, International Journal of Modern Physics C, 2005, 16, 4, 607–632.
[12] HIRSHLEIFER J., *Anarchy and its breakdown*, Journal of Political Economy, 1995, 103, 1, 26–52.
[13] JONES-LEE M.W., *The Economics of Safety and Physical Risk*, First Blackwell, Oxford 1989.
[14] KOTZ H., SCHAFER H.-B., *Economic incentives to accident prevention: An empirical study of the German sugar industry*, International Review of Law and Economics, 1993, 13, 19–33.
[15] OI W.Y., *Economics of product safety*, Bell Journal of Economics, 1974, 5, 689–695.
[16] OI W.Y., *Safety at what price*, American Economic Review, 1995, 85, 67–71.
[17] RICARDO D., *On the principles of political economy and taxation*, [in:] *The Works and Correspondence of David Ricardo*, P. Sraffa (Ed.) with the collaboration of M. Dobb, Cambridge University Press, London 1817, 1951–1955.
[18] RIDER R., *Conflict, the sire of exchange: Violence is the sire of all the world's values*, Journal of Economic Behavior and Organization, 1999, 40, 217–232.
[19] ROSE N.L., *Profitability and product quality – economic determinants of airline safety performance*, Journal of Political Economy, 1990, 98, 5, 944–964.
[20] SKAPERDAS S., SYROPOULOS C., *Guns, butter, and openness: On the relationship between security and trade*, American Economic Review Papers and Proceedings, 2001, 91 (2), 353–357.
[21] SMITH A., *The Wealth of Nations*, The University of Chicago Press, Chicago 1976.
[22] SWINBANK A., *The economics of food safety*, Food Policy, 1993, 18, 2, 83–94.
[23] TAYLOR M.S., *Quality ladders' and Ricardian trade*, Journal of International Economics, 1993, 34, 225–243.
[24] THOMAS S., *Economic and safety pressures on nuclear power: A comparison of Russia and Ukraine since the break-up of the Soviet Union*, Energy Policy, 1999, 27, 13, 745–767.
[25] VISCUSI W.K., *Market incentives for safety*, Harvard Business Review, 1986, 63, 133–138.