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COMPARISON OF THE VALUATIONS OF ALTERNATIVES BASED ON CUMULATIVE PROSPECT THEORY AND ALMOST STOCHASTIC DOMINANCE

There are commonly accepted and objective decision rules, which are consistent with rationality, for example stochastic dominance rules. But, as can be seen in many research studies in behavioral economics, decision makers do not always act rationally. Rules based on cumulative prospect theory or almost stochastic dominance are relatively new tools which model real choices. Both approaches take into account some behavioral factors. The aim of this paper is to check the consistency of orders of the valuations of random alternatives based on these behavioral rules. The order of the alternatives is generated by a preference relation over the decision set. In this paper, we show that the methodology for creating rankings based on total orders can be used for the preference relations considered, because they enable comparison of all the elements in a set of random alternatives. For almost second degree stochastic dominance, this is possible due to its particular properties, which stochastic dominance does not possess.

Key words: *cumulative prospect theory, almost stochastic dominance, decision analysis*

1. Introduction

For years, most research in the field of the theory and practice of decision making has been concerned with searching for new tools which could model real choices better. One such tool is the model proposed by Kahneman and Tversky [5] known as the prospect theory. The authors were criticized for its inconsistency with stochastic dominance. Its extension to cumulative prospect theory (CPT) solved this problem [13]. On the other hand, stochastic dominance rules do not predict many apparently obvious

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choices either. Such observations forced these rules to be softened and resulted in the formulation of almost stochastic dominance rules [8]. The aim of this paper is to check the consistency of orders of the valuations of random alternatives based on these behavioral rules. By the order of two alternatives, we mean the preference relation between them. In the mathematical sense, to disprove the hypothesis of the consistency of orders, it is sufficient to find a counterexample.

From the formal point of view, it is very important to indicate interrelationships between various approaches. Leshno and Levy [7, 8] have shown that first degree stochastic dominance (FSD) implies second degree stochastic dominance (SSD) and almost first degree stochastic dominance (AFSD), SSD implies almost second degree stochastic dominance (ASSD), and AFSD also implies ASSD. The veracity of "FSD implies CPT" was proved in the papers of Levy and Wiener [10] and Tversky and Kahneman [13], whereas the reverse implication does not hold (see [12]). Examples which corroborate inconsistent preferences based on CPT and SSD can be found in the paper of Michalska [12]. In the literature, there is a lack of research on the consistency of preferences based on almost stochastic dominance and on cumulative prospect theory. In the light of this research, we attempt to fill this gap.

In this paper, we also show that the method for creating rankings based on total orders can be used for the preference relations considered, because they enable comparison of all the elements in a set of random alternatives. For almost second degree stochastic dominance, this is possible due to its particular properties, which stochastic dominance does not possess.

2. Valuation of random alternatives according to cumulative prospect theory

According to cumulative prospect theory [5, 13], the phase of evaluating an alternative is preceded by the editing phase, in which the possible outcomes are written as gains and losses relative to some reference point. This point could be an actual or desirable level of assets. Such a formulation is different from the way an alternative is written using the theory of expected utility, in which levels of wealth are considered. Moreover, in the editing phase, probabilities corresponding to the same outcome are aggregated, which makes the notion and further valuation simpler. As a result of the editing phase, we obtain a random alternative L, also called lottery, which can be written as a sequence of relative outcomes x_i and their probabilities p_i.

$$L = ((x_1, p_1); \ldots; (x_{k-1}, p_{k-1}); (x_k, p_k); (x_{k+1}, p_{k+1}); \ldots; (x_n, p_n)) \quad (1)$$

where $x_1 < \ldots < x_k < 0 \le x_{k+1} < \ldots < x_n$ and $p_1 + \ldots + p_k + p_{k+1} \ldots + p_n = 1$.

In the second phase, valuation, the value of each alternative is calculated. This value depends on two functions: the value function $v(x)$ and the probability weighting function $g(p)$. The analytical form of the value function and estimates of its parameters are selected on the basis of decision makers' preferences, revealed in surveys. Researchers suggest various forms of the value function (see, e.g. [2]), but the most cited one is the following function

$$v(x) = \begin{cases} -\lambda(-x)^{\beta}, & x < 0 \\ x^{\alpha}, & x \geq 0 \end{cases} \tag{2}$$

In the above function, the estimates of parameters α, β and λ are assumed to equal 0.88, 0.88 and 2.25, respectively, as proposed by Kahneman and Tversky [13].

The value function for changes in wealth is concave above the reference point and convex below this point. This is consistent with the diminishing marginal value of gains and losses (relative to their absolute magnitude). The concavity of the value function reflects the observed behavior of decision makers. For gains, it is concave which means that decision makers are risk averse and they prefer a certain reward to a random reward which gives the same expected gain. Whereas considering losses, decision makers are risk-prone (the value function is convex) and they prefer a random loss to a certain loss with the same expected value. Another very important feature of the value function is the difference between the slope of the curve for gains and for losses which reflects loss aversion. The value function is steeper for losses than for gains. This means that decision makers feel a loss more strongly than a gain of the same absolute value.

The authors of cumulative prospect theory have also taken into consideration the fact that decision makers do not usually use objective mathematical probabilities; instead, they transform them in some way. This observation led Kahneman and Tversky to introduce the probability weighting function, which non-linearly transforms probabilities:

$$g(p) = \frac{p^{\gamma}}{[p^{\gamma} + (1-p)^{\gamma}]^{1/\gamma}} \tag{3}$$

In cumulative prospect theory, p stands for appropriately cumulated probabilities. The estimate of the parameter γ depends on whether a given probability concerns a gain or a loss, for a gain $\gamma = 0.61$ and for a loss $\gamma = 0.69$ [13].

Regardless of its form, the probability weighting function has some general properties: it is an increasing function; it overestimates low probabilities but underestimates moderate and high probabilities; moreover, $g(0) = 0$, $g(1) = 1$, and $g(p) + g(1-p) < 1$ for all $p \in (0,1)$.

Based on the two functions $v(x)$ and $g(p)$, a measure of an alternative's value is constructed. It is the sum of the valuation of gains $\mathrm{CPT}^{+}(L)$ and the valuation of losses $\mathrm{CPT}^{-}(L)$ (see [13]):

$$\mathrm{CPT}(L) = \mathrm{CPT}^{+}(L) + \mathrm{CPT}^{-}(L) \tag{4}$$

The components $\mathrm{CPT}^{+}(L)$ and $\mathrm{CPT}^{-}(L)$ are calculated as follows:

$$\mathrm{CPT}^{+}(L) = v(x_n)g(p_n) + \sum_{i=k+1}^{n-1} v(x_i)\left[g\left(\sum_{j=i}^{n} p_j\right) - g\left(\sum_{j=i+1}^{n} p_j\right)\right] \tag{5}$$

$$\mathrm{CPT}^{-}(L) = v(x_1)g(p_1) + \sum_{i=2}^{k} v(x_i)\left[g\left(\sum_{j=1}^{i} p_j\right) - g\left(\sum_{j=1}^{i-1} p_j\right)\right] \tag{6}$$

At the end of the valuation phase, each alternative is ascribed a value CPT(L). Among all alternatives, the one with the highest CPT value is preferred. Therefore, the preference rule in the sense of cumulative prospect theory can be formulated in the following way:

CPT: Alternative $L1$ is preferred to alternative $L2$ (written as $L1 \succ_{\mathrm{CPT}} L2$) if and only if CPT($L1$) > CPT($L2$).

3. Almost stochastic dominance

For years, one of the most commonly used decision rules in the situation of risk has been the mean-variance (MV) rule proposed by Markowitz [11]. Let us consider two random alternatives, $L1$ and $L2$, their corresponding expected values $E(L1)$, $E(L2)$ and standard deviations $\sigma(L1)$, $\sigma(L2)$. The MV rule is formulated as follows:

MV: $L1$ is preferred to $L2$ according to the MV rule (written as $L1 \succ_{\mathrm{MV}} L2$) if and only if $E(L1) \geq E(L2)$ and $\sigma(L1) \leq \sigma(L2)$ and at least one inequality is strict.

Stochastic dominance rules are commonly accepted and objective non-parametric decision rules. Let the symbols F_{L1} and F_{L2} stand for the distribution functions of random alternatives $L1$ and $L2$, respectively, and S stand for the set of all outcomes of $L1$

and $L2$. First degree stochastic dominance (FSD) and second degree stochastic dominance (SSD) can be formulated as follows [4]:

FSD: $L1$ dominates $L2$ according to first degree stochastic dominance (written as $L1 \succ_{FSD} L2$) if and only if the inequality $F_{L1}(r) - F_{L2}(r) \leq 0$ is satisfied for each $r \in S$ and for at least one value $r \in S$ this inequality is strict.

SSD: $L1$ dominates $L2$ according to second degree stochastic dominance (written as $L1 \succ_{SSD} L2$) if and only if the inequality $F_{L1}^{(2)}(r) - F_{L2}^{(2)}(r) \leq 0$ is satisfied for each $r \in S$ and for at least one value $r \in S$ this inequality is strict, where $F_{L1}^{(2)}(r) = \int_{-\infty}^{r} F_{L1}(t)\, dt$ and $F_{L2}^{(2)}(r) = \int_{-\infty}^{r} F_{L2}(t)\, dt.$

These stochastic dominance rules do not require any assumptions about the distribution functions and properties of the utility function, except for the following classical assumptions [6]:

- a decision maker prefers more to less (the utility function is increasing),
- a decision maker is risk averse (the utility function is concave).

This means that if FSD holds, then the alternative with higher mean is preferred, and if SSD holds then the alternative with higher mean and lower variance is preferred.

The mean-variance rule and the stochastic dominance rules do not always lead to a conclusion as to which one of the considered alternatives is better, even when the choice seems to be obvious. Such a situation is shown in the following simple example.

Example 1

Let us assume that by choosing alternative $L1$ we obtain \$1 with probability 0.01 and \$100 with probability 0.99, and by choosing alternative $L2$ we obtain \$2 for sure. These alternatives can be written as $L1 = ((1;0.01);(100;0.99))$ and $L2 = ((2;1))$. It is easy to show that neither does $L1$ dominate $L2$ nor $L2$ dominate $L1$ based on the MV rule. Also, neither does $L1$ nor $L2$ dominate the other based on either first degree or second degree stochastic dominance rule, but most "reasonable" decision makers (if not all) prefer $L1$ to $L2$. Moreover, analyzing the graphs of both distribution functions shown in Fig. 1, we can notice that the area A, corresponding to the range in which $L2$ dominates $L1$, is much smaller than the area B, corresponding to the range in which $L1$ dominates $L2$. Therefore, we can say that $L1$ "almost" dominates $L2$ according to first degree stochastic dominance.

By considering similar examples, Leshno and Levy [7] proposed a "relaxation" of the stochastic dominance rules to almost stochastic dominance (ASD). The conditions for almost first degree and second degree stochastic dominance can be formulated as follows:

AFSD: $L1$ dominates $L2$ according to almost first degree stochastic dominance (written as $L1 \succ_{\text{AFSD}} L2$) if and only if there exists ε, $0 \le \varepsilon < \varepsilon^*$, such that

$$\int_{S_1} (F_{L1}(r) - F_{L2}(r))\, dr \le \varepsilon \int_{S} | F_{L1}(r) - F_{L2}(r) |\, dr$$

where S is the set of all possible outcomes of $L1$ and $L2$ and

$$S_1 = \{ r \in S : F_{L2}(r) < F_{L1}(r) \}$$

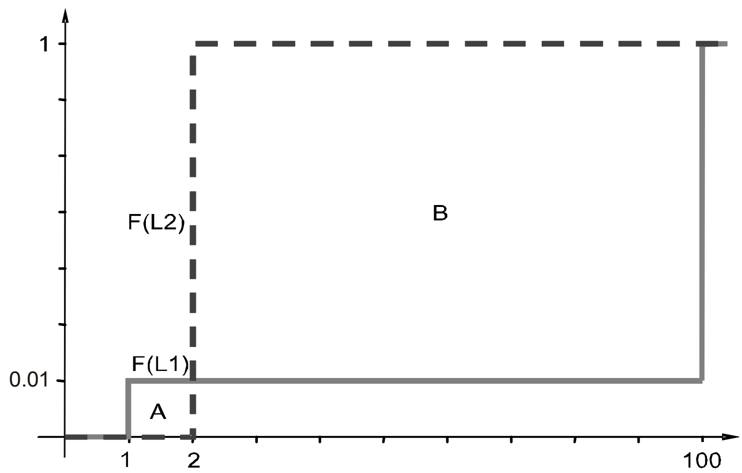

Fig. 1. Distribution functions for alternatives $L1$ and $L2$

ASSD: $L1$ dominates $L2$ according to almost second degree stochastic dominance (written as $L1 \succ_{\text{ASSD}} L2$) if and only if there exists ε, $0 \le \varepsilon < \varepsilon^*$, such that

$$\int_{S_2} (F_{L1}(r) - F_{L2}(r))\, dr \le \varepsilon \int_{S} | F_{L1}(r) - F_{L2}(r) |\, dr$$

and

$$E(L1) \ge E(L2),$$

where S is the set of all possible outcomes of $L1$ and $L2$ and

$$S_2 = \{ r \in S_1 : F_{L2}^{(2)}(r) < F_{L1}^{(2)}(r) \}$$

For almost first degree and second degree stochastic dominance, it is assumed that the value of the parameter ε, which corresponds to the region in which strict domination is violated, should be lower than $\varepsilon^* = 0.5$. In [9] you can also find the notation ε^*–AFSD and ε^*–ASSD, where ε^* denotes the "allowed" degree of violation, and $0 \leq \varepsilon < \varepsilon^* \leq 0.5$.

In Example 1, the alternative $L1$ does not dominate $L2$ according to first degree and second degree stochastic dominance, but it dominates $L2$ according to AFSD for $\varepsilon \approx 0.000103$ (for AFSD, the value of ε is calculated as the quotient of area A to the sum of area A and area B). The fundamental advantage of using almost stochastic dominance is that the set of non-comparable (according to other criteria or rules) alternatives can be reduced. Moreover, rules based on almost stochastic dominance reveal preferences consistent with intuition, whereas stochastic dominance may not corroborate the intuitive choices of decision makers.

4. Consistency between choices made according to cumulative prospect theory and almost stochastic dominance

Cumulative prospect theory, as well as almost stochastic dominance are assumed to describe the preferences of a decision maker. To check whether objective consistency of the preference relation based on these behavioral approaches exists, we will analyze some examples.

For alternatives $L1$ and $L2$ (considered in Example 1), the choice of the best alternative made on the basis of cumulative prospect theory is consistent with the choice based on almost stochastic dominance (Tables 1 and 2 show all the calculations). The question arises as to whether this will always be so. In the mathematical sense, to disprove the hypothesis of consistency of preferences, it is sufficient to find a counterexample in which such preference relations are inconsistent according to the rules considered.

Example 2

Let us consider the following two random alternatives: $L3 = ((25;0.3); (38;0.3); (49;0.4))$ and $L4 = ((5;0.1); (28;0.5); (61;0.4))$.

In the case of alternatives $L3$ and $L4$ (similarly as for $L1$ and $L2$), no one alternative is preferred to the other by the MV rule (see Tables 1 and 2). There is also no dominance according to FSD (as shown in Fig. 2) or SSD. The condition for almost first degree stochastic dominance is satisfied but the preferences on the basis of CPT and AFSD do not coincide. According to CPT, the preferred alternative is *L*3, whereas according to AFSD (with $\varepsilon^* = 0.5$) the preferred alternative is *L*4 (see Tables 1 and 2).

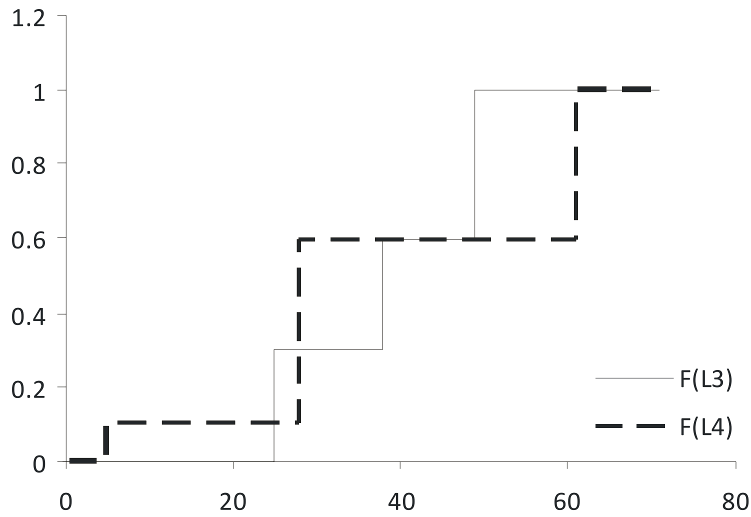

Fig. 2. Distribution functions for alternatives *L*3 and *L*4

Example 2 is a counterexample, which shows that there is no objective consistency between the CPT and ASD rules.

We can also observe inconsistency between the MV and CPT rules, which can be seen in the following example.

Example 3

Let us consider another two random alternatives: $L5 = ((26;0.2);(30;0.5);(56;0.3))$ and $L6 = ((10;0.1);(28;0.3);(46;0.6))$.

Alternative *L*6 dominates *L*5 according to the MV rule but for this pair there is no dominance according to FSD (see Fig. 3), SSD or AFSD (see Tables 1 and 2). The choices made on the basis of CPT and ASSD do not coincide. According to CPT, the preferred alternative is *L*5, whereas according to ASSD (with $\varepsilon^* = 0.5$) the preferred one is alternative *L*6. What is interesting, is that for ASSD the value of the parameter ε is lower than 0.5 both when dominance of *L*6 by *L*5 and when dominance of *L*5 by *L*6 is considered. Moreover, $E(L5) = E(L6)$. In such a situation, the dominant alternative is the one with the smaller value of ε. From this it follows that *L*6 dominates *L*5.

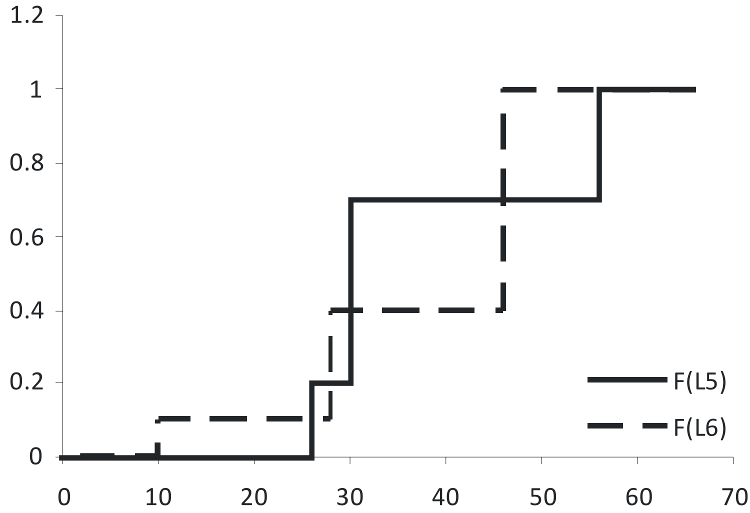

Fig. 3. Distribution functions for alternatives *L*5 and *L*6

Table 1. Values of parameters for alternatives *L*1–*L*6

Parameter	Alternative					
	*L*1	*L*2	*L*3	*L*4	*L*5	*L*6
$E(L)$	99.01	2.00	38.50	38.90	37.00	37.00
$\sigma(L)$	9.85	0.00	9.94	19.23	12.53	12.07
CPT(L)	52.54	1.84	23.31	21.38	23.67	20.42
$\varepsilon_{\mathrm{AFSD}}$	0.000103 for (*L*1, *L*2)		0.519231 for (*L*3, *L*4)		0.5 for (*L*5, *L*6)	
	0.999897 for (*L*2, *L*1)		0.480769 for (*L*4, *L*3)		0.5 for (*L*6, *L*5)	
$\varepsilon_{\mathrm{ASSD}}$	0.000103 for (*L*1, *L*2)		0.038462 for (*L*3, *L*4)		0.3 for (*L*5, *L*6)	
	0.999794 for (*L*2, *L*1)		0.480769 for (*L*4, *L*3)		0.2 for (*L*6, *L*5)	

Table 2. Preferences for alternatives *L*1–*L*6

Criterion of decision	Example 1	Example 2	Example 3
MV	–	–	$L6 \succ L5$
CPT	$L1 \succ L2$	$L3 \succ L4$	$L5 \succ L6$
FSD	–	–	–
SSD	–	–	–
AFSD	$L1 \succ L2$	$L4 \succ L3$	–
ASSD	$L1 \succ L2$	$L4 \succ L3$	$L6 \succ L5$

Interrelationships between the decision rules considered here are shown in Fig. 4. For example, the arrow FSD $\Rightarrow$ SSD means that for any two different alternatives *Li* and *Lj*, if $Li \succ_{\mathrm{FSD}} Lj$, then $Li \succ_{\mathrm{SSD}} Lj$. If the arrow is crossed out, then such an implication is not true (i.e. there exist at least one pair of alternatives for which such an implication is not true).

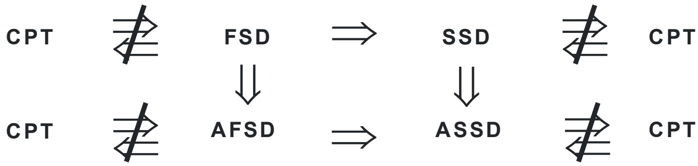

Fig. 4. Interrelationships between the decision rules FSD, SSD, AFSD, ASSD and CPT

The interrelationships indicated between FSD, SSD, AFSD and ASSD are corroborated in the literature [7, 8]. Using the counterexamples above, we proved that there is no consistency of preferences based on cumulative prospect theory and almost first degree and second degree stochastic dominance.

The veracity of the implication FSD $\Rightarrow$ CPT was proved in the papers of Levy and Wiener [10] and Tversky and Kahneman [13], whereas the reverse implication does not occur (see [12]). Examples which corroborate the inconsistency of preferences based on CPT and SSD can be found in the paper of Michalska [12].

5. Rankings of random alternatives based on almost stochastic dominance

The comparison of a number of alternatives based on stochastic dominance of the first, second or higher degree gives a partial order (see [1]). In the set of all alternatives, there may exist non-comparable elements which make it impossible to construct a full ranking. Such a comparison based on almost second degree stochastic dominance for $\varepsilon^* = 0.5$ also gives a partial order. Moreover, analyzing the relations based on the almost first degree and second degree stochastic dominance on the set of random alternatives, we noticed some interesting properties regarding the values of the parameter ε. For any two different alternatives, Li and Lj, we have:

1) $\varepsilon_{\text{AFSD}}(Li, Lj) = \varepsilon_{\text{AFSD}}(Lj, Li) = 1$,

2) $\varepsilon_{\text{ASSD}}(Li, Lj) = \varepsilon_{\text{ASSD}}(Lj, Li) = \max\{\varepsilon_{\text{AFSD}}(Li, Lj), \varepsilon_{\text{AFSD}}(Lj, Li)\}$,

where $\varepsilon_{\text{AFSD}}(Li, Lj)$ denotes the share of the region where Li does not dominate Lj according to FSD in the area between the distribution functions F_{Li} and F_{Lj} and the parameter $\varepsilon_{\text{ASSD}}(Li, Lj)$ denotes the share of the region where Li does not dominate Lj according to SSD and FSD in the area between the distribution functions F_{Li} and F_{Lj}.

For any two different alternatives *Li* and *Lj*, we have $Li \succ_{\text{ASSD}} Lj$, $Lj \succ_{\text{ASSD}} Li$ or $Li \sim_{\text{ASSD}} Lj$, where $Li \sim_{\text{ASSD}} Lj$ means that a decision maker using the almost second degree stochastic dominance rule is indifferent between alternatives *Li* and *Lj*, i.e., $E(Li) = E(Lj)$ and $\varepsilon_{\text{ASSD}}(Li, Lj) = \varepsilon_{\text{ASSD}}(Lj, Li) = 0.25$. The properties indicated above mean that only for $\varepsilon^* = 0.5$ does the relation of the almost second degree stochastic dominance allow us to define a full ranking of all the elements of a set of random alternatives. Using the almost stochastic dominance approach, the smaller the area of violation we have the better. However, when the area of violation allowed is small, some alternatives can be incomparable and then it is impossible to construct a ranking. Therefore, for the purpose of constructing a full ranking, we have to assume $\varepsilon^* = 0.5$.

To order the set of alternatives, the agreeing ordinal value function proposed by French [3] can be applied. Let **L** stand for the set of alternatives considered. The agreeing ordinal value function is a function h: $\mathbf{L} \to \mathbf{N} \cup \{0\}$ defined as follows:

$$h(Li) = \{\text{the number of alternatives } Lj \in \mathbf{L} \text{ such that } Li \succ Lj \text{ or } Li \sim Lj\}.$$

The value of this function for a given alternative is the number of dominated or indifferent alternatives from the set **L**. The value of the function $h(\cdot)$ determines the rank of an indifference class, which consists of alternatives with the same value of the function $h(\cdot)$. The top rank is assigned to the indifference class with the highest value of the function $h(\cdot)$, whereas the bottom rank is assigned to the indifference class with the lowest value of the function $h(\cdot)$. Example 4 illustrates the method for constructing a ranking based on almost stochastic dominance. This ranking will be compared with the ranking based on CPT.

Example 4

In this example, we will analyze comparisons based on almost stochastic dominance AFSD and ASSD (for $\varepsilon^* = 0.5$) for the set containing nine random alternatives:

$\mathbf{L} = \{L1, L2, L3, L4, L5, L6, L7, L8, L9\}$:

$L1$ = ((20;0.2);(30;0.5);(56;0.3)), $L2$ = ((5;0.1);(28;0.5);(60;0.4)),
$L3$ = ((15;0.4);(35;0.1);(50;0.5)), $L4$ = ((15;0.5);(35;0.2);(50;0.3)),
$L5$ = ((25;0.3);(38;0.3);(49;0.4)), $L6$ = ((30;0.5);(38;0.3);(42;0.2)),
$L7$ = ((10;0.1);(28;0.5);(52;0.4)), $L8$ = ((5;0.1);(28;0.5);(61;0.4)),
$L9$ = ((15;0.3);(44;0.3);(53;0.4)).

All the ε parameters calculated using the almost first degree and second degree stochastic dominance are shown in Tables 3 and 4. For any two alternatives $Li, Lj \in$ **L,** we have:

1) $\varepsilon_{\text{AFSD}}(Li, Lj) = \varepsilon_{\text{AFSD}}(Lj, Li) = 1$,

2) $\varepsilon_{\text{ASSD}}(Li, Lj) = \varepsilon_{\text{ASSD}}(Lj, Li) = \max\{\varepsilon_{\text{AFSD}}(Li, Lj), \varepsilon_{\text{AFSD}}(Lj, Li)\}$.

This means that all the elements in the considered set **L** are comparable according to the ASSD rule with $\varepsilon^* = 0.5$.

Table 3. Values of the parameter $\varepsilon_{\text{AFSD}}(Li, Lj)$ for alternatives *L*1–*L*9

$\varepsilon_{\text{AFSD}}(Li, Lj)$	*L*1	*L*2	*L*3	*L*4	*L*5	*L*6	*L*7	*L*8	*L*9
*L*1		0.68493	0.43689	0.12048	0.67089	0.17308	0.5	0.70130	0.65657
*L*2	0.31507		0.33051	0.13710	0.5	0.34959	0.05882	1	0.51786
*L*3	0.56311	0.66949		0	0.77778	0.51282	0.58025	0.68033	0.92771
*L*4	0.87952	0.86290	1		0.96875	0.74312	0.86207	0.86719	1
*L*5	0.32911	0.5	0.22222	0.03125		0.13766	0.28571	0.51923	0.53125
*L*6	0.82692	0.65041	0.48718	0.25688	0.86239		0.55814	0.66142	0.65649
*L*7	0.5	0.94118	0.41975	0.13793	0.71429	0.44186		0.94737	0.68675
*L*8	0.29870	0	0.31967	0.13281	0.48077	0.33858	0.05263		0.5
*L*9	0.34343	0.48214	0.07229	0	0.46875	0.34351	0.31325	0.5	

Table 4. Values of the parameter $\varepsilon_{\text{ASSD}}(Li, Lj)$ for alternatives *L*1–*L*9

$\varepsilon_{\text{ASSD}}(Li, Lj)$	*L*1	*L*2	*L*3	*L*4	*L*5	*L*6	*L*7	*L*8	*L*9
*L*1		0.36986	0.04854	0	0.67089	0.17308	0.2	0.40260	0.40404
*L*2	0.31507		0.08475	0.08065	0.5	0.34959	0.05882	1	0.375
*L*3	0.51456	0.58475		0	0.77778	0.51282	0.51852	0.59836	0.92771
*L*4	0.87952	0.78226	1		0.96875	0.74312	0.80460	0.78906	1
*L*5	0	0	0	0		0.13762	0	0.03846	0.0625
*L*6	0.65385	0.30081	0	0	0.72477		0.11628	0.32284	0.31298
*L*7	0.3	0.88235	0.06173	0.05747	0.71429	0.44186		0.89474	0.43374
*L*8	0.29870	0	0.08197	0.07813	0.48077	0.33858	0.05263		0.36207
*L*9	0.25253	0.14286	0	0	0.46875	0.34351	0.25301	0.13793	

The corresponding comparisons based on almost second degree stochastic dominance and the values of $h(Li)$ for all the alternatives are shown in Table 5.

The calculated values of the function $h(Li)$ are the basis for constructing a ranking of the alternatives. The distinct values of $h(Li)$ obtained for each alternative mean that each indifference class includes only one element. The alternative with the highest value of $h(Li)$ takes the first place in the ranking, while the last place is taken by the alternative with the lowest value of $h(Li)$. For comparison, we construct the ranking of alternatives *L*1–*L*9 according to the CPT criterion. Both rankings are presented in Table 6.

Both rankings differ significantly (the Spearman's rank correlation coefficient equals 0.533 and is insignificant, p-value = 0.15). Thus the selection of the decision criteria has a strong influence on the rank of an alternative in the ranking. Despite the fact that cumulative prospect theory and almost stochastic dominance take into account some behavioral aspects of decision making, they evaluate the same alternative in different ways.

Table 5. Comparisons based on ASSD (for $\varepsilon^* = 0.5$) and values of the function $h(Li)$ for alternatives $L1$–$L9$

ASSD	$L1$	$L2$	$L3$	$L4$	$L5$	$L6$	$L7$	$L8$	$L9$	$h(Li)$
$L1$		$\prec$	$\succ$	$\succ$	$\prec$	$\succ$	$\succ$	$\prec$	$\prec$	4
$L2$	$\succ$		$\succ$	$\succ$	$\prec$	$\succ$	$\succ$	$\prec$	$\prec$	5
$L3$	$\prec$	$\prec$		$\succ$	$\prec$	$\prec$	$\prec$	$\prec$	$\prec$	1
$L4$	$\prec$	$\prec$	$\prec$		$\prec$	$\prec$	$\prec$	$\prec$	$\prec$	0
$L5$	$\succ$	$\succ$	$\succ$	$\succ$		$\succ$	$\succ$	$\prec$	$\prec$	6
$L6$	$\prec$	$\prec$	$\succ$	$\succ$	$\prec$		$\prec$	$\prec$	$\prec$	2
$L7$	$\prec$	$\prec$	$\succ$	$\succ$	$\prec$	$\succ$		$\prec$	$\prec$	3
$L8$	$\succ$	$\succ$	$\succ$	$\succ$	$\succ$	$\succ$	$\succ$		$\prec$	7
$L9$	$\succ$	$\succ$	$\succ$	$\succ$	$\succ$	$\succ$	$\succ$	$\succ$		8

Table 6. Ranking of alternatives $L1$–$L9$ based on the ASSD and CPT criteria

Rank	ASSD	CPT
1	$L9$	$L5$
2	$L8$	$L6$
3	$L5$	$L1$
4	$L2$	$L9$
5	$L1$	$L8$
6	$L7$	$L2$
7	$L6$	$L7$
8	$L3$	$L3$
9	$L4$	$L4$

6. Conclusions

There is no objective (complete, mathematical) consistency of preferences based on the behavioral approaches considered: cumulative prospect theory and almost stochastic dominance. We have also observed some particular properties of almost second degree stochastic dominance, which allow us to construct a ranking. In the case of stochastic dominance and almost stochastic dominance with $\varepsilon^* < 0.5$, some alternatives can be incomparable, which makes it impossible to construct a complete ranking of all the alternatives.

Although our examples showed a lack of consistency between the preferences based on CPT and ASD, it would be worthwhile to explore the degree of consistency for a larger set of alternatives (e.g. mutual funds or stock portfolios, as suggested by a Referee) in future research using the approach based on Spearman's rank correlation coefficient. It would also be worthwhile to check how the selection of the value function and the probability weighting function influences preferences and rankings based on CPT.

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